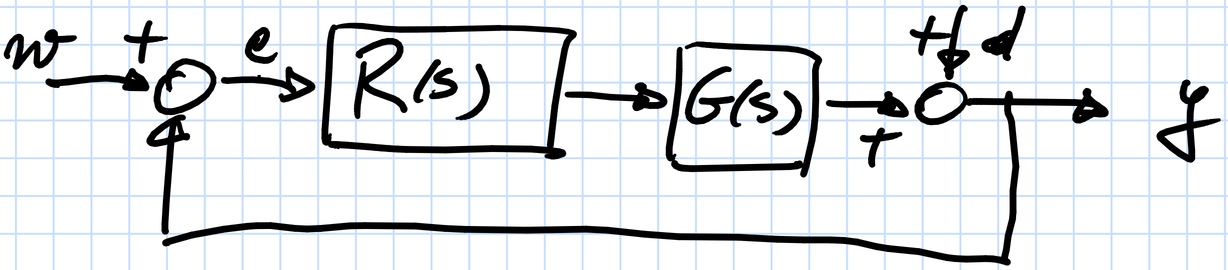
Esercizi sul progetto di regolatori

11 giugno 2020

Fondamenti di Automatica
a.a. 2019/2020

Progetto per sistema con ritardo finito 18/7/2014



$$G(s) = \frac{10}{1+10s}\, e^{-2s}$$

Specifiche di progetto

(a) $e_\infty \to 0$ per $w(t) = A \cdot 1(t)$ $\quad A, B \in \mathbb{R}$
$\quad d(t) = B \cdot 1(t)$

(b) considerando $d=0$, $\omega_c \geq 0{,}5$ rad/s

(c) considerando $d=0$, $\varphi_m \geq 30°$

(a) si desidera un sistema di "tipo 1"

$$R_1(s) = \frac{\mu}{s} \qquad R(s) \overset{?}{=} R_1(s) \cdot R_2(s)$$

$$G(s) = G'(s) \cdot e^{-2s}$$

→ influenza/modifica fase della risposta in frequenza ma NON il modulo

Se $\bar{\omega}$ fosse la pulsazione critica soluzione allora

$$|G(j\bar{\omega})| = |G'(j\bar{\omega})| \cdot \underset{|e^{-2j\bar{\omega}}|}{\underbrace{1}}$$

Ma

$$\angle G(j\bar{\omega}) = \angle G'(j\bar{\omega}) + \angle e^{-2j\bar{\omega}}$$

Se voglio rispettare le specifiche (b), (c) per la f.d.T $G(s)$, allora devo modificarle per poter progettare il regolatore $R(s)$ utilizzando $G'(s)$

$\omega_c \rightarrow$ resta invariata

$$\varphi_M \geq 30° \Longrightarrow \varphi_M \geq 30° + |\angle e^{-2j\omega_c}|$$

1° analisi

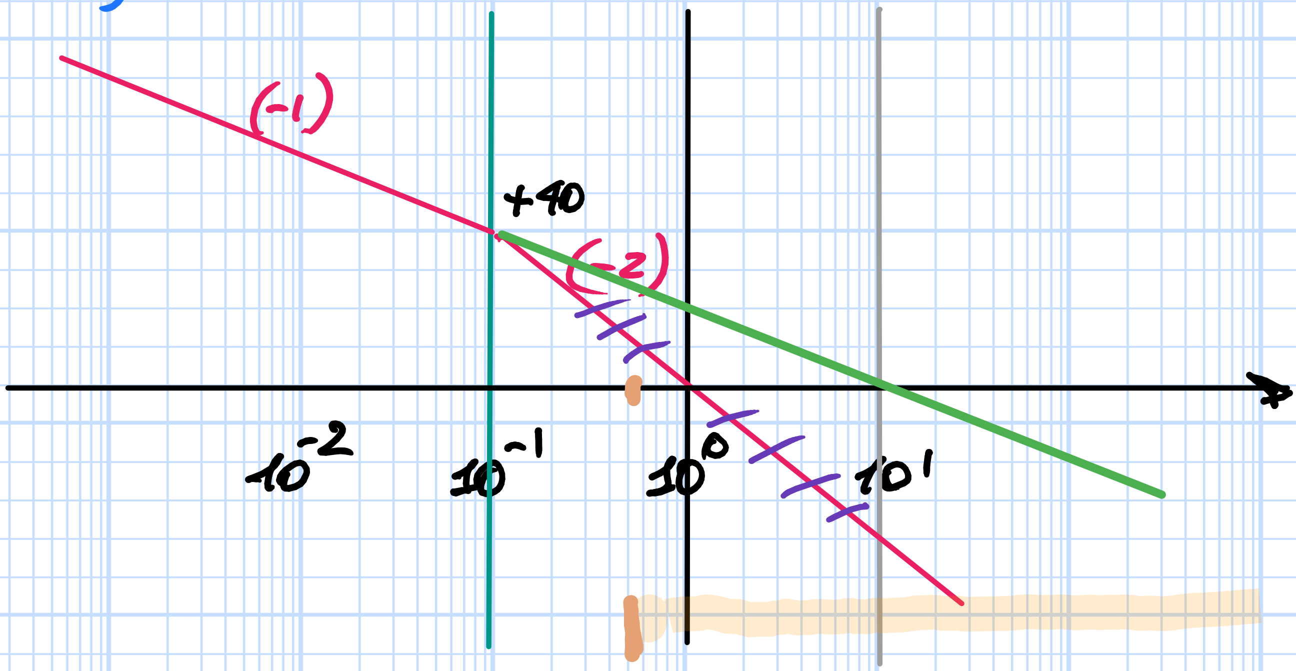
(a) $\Longrightarrow R_1(s) = \frac{\mu}{s}$ $\qquad G'(s)$

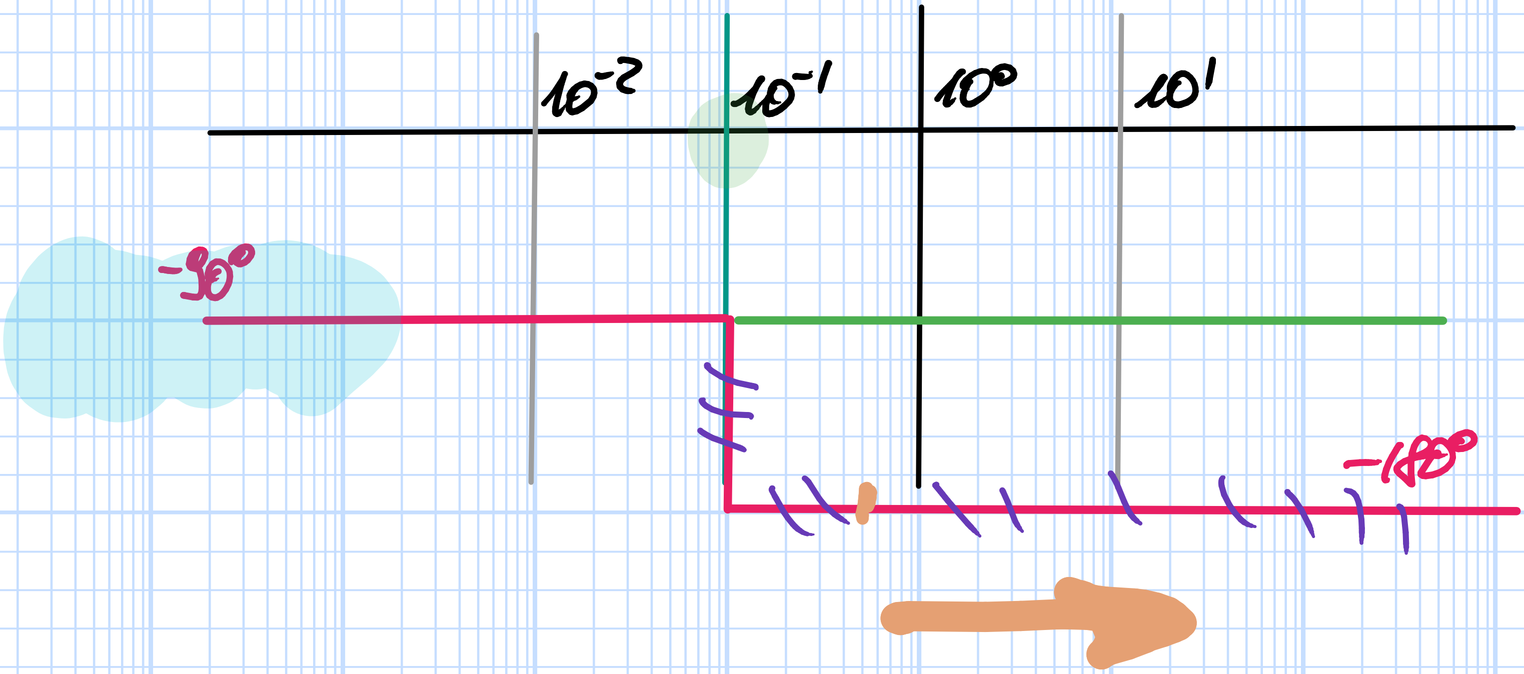
$$L_1'(s) = \frac{10\mu}{s(1+10s)}$$

$$L'_1(s) = \frac{10\mu}{s(1+10s)}$$

$\mu = 1$

$$\frac{10\mu}{s \cdot 1}$$

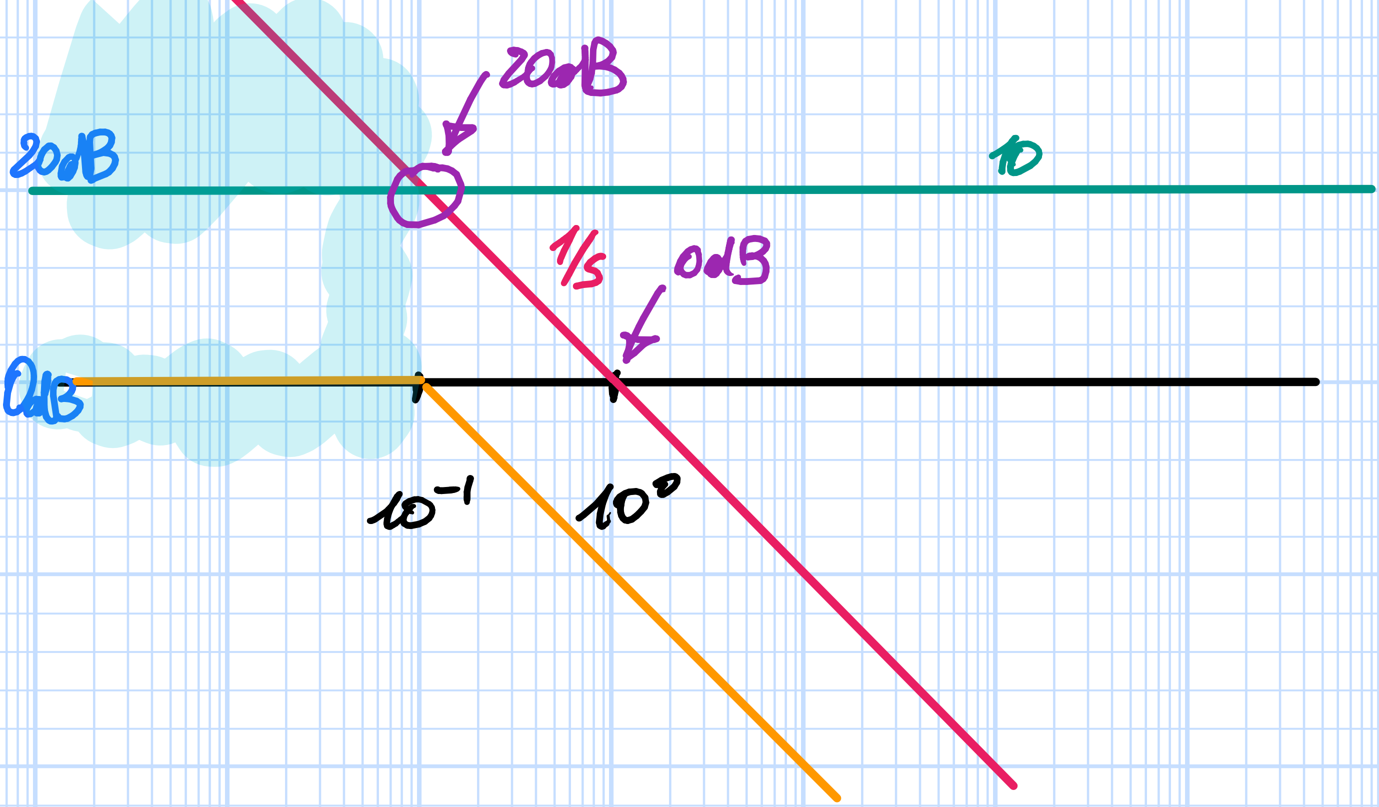




10

$\frac{1}{s}$

$\frac{1}{1+10s}$



$$R(s) = R_1(s) \cdot R_2(s) \qquad G'(s) = \frac{10}{1+10s}$$

$$= \frac{\mu}{s} \cdot (1+10s) \qquad PI \quad \frac{k(1+\tau_1 s)}{s}$$

$$L_2'(s) = \frac{10\mu\,\cancel{(1+10s)}}{s\,\cancel{(1+10s)}} = \frac{10\mu}{s}$$

Adesso tengo conto del ritardo finito

$$\measuredangle L_2'(j\bar{\omega}) + \measuredangle e^{-2j\bar{\omega}} \geq -150° \qquad \varphi_m \geq 30°$$

$$-\frac{\pi}{2} - 2\bar{\omega} \geq -\frac{5}{6}\pi$$

$$2\bar{\omega} \leq \frac{\pi}{3}$$

$$\left[\frac{1}{2} \leq\right] \bar{\omega} \leq \frac{\pi}{6} \simeq 0{,}5236\ldots$$

$$\frac{10\mu}{\bar{\omega}} = 1 \Rightarrow \mu = \frac{\bar{\omega}}{10}$$

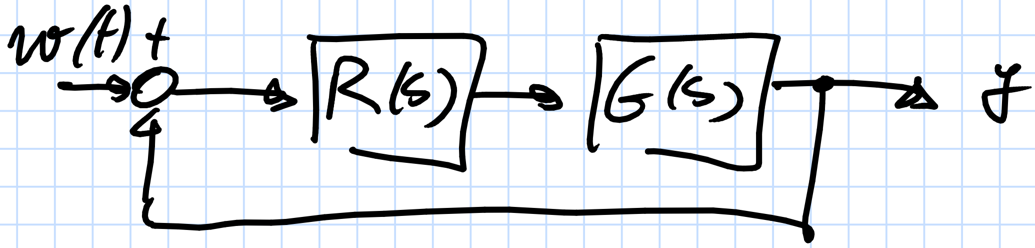
per es $\bar{\omega} = 0{,}52$

$$\varphi_m \simeq 30°\,27'$$

$$\measuredangle L_2'(j\bar{\omega}) = -90°$$

$$\measuredangle L_2(j\bar{\omega}) = -90° - 2 \cdot 0{,}52 \cdot \frac{180°}{\pi} \simeq -149°\,32'$$

Discretizzazione con formula di Tustin



il regolatore $R(s) = 75 \dfrac{1}{s(1+0{,}5s)}$

garantisce le prestazioni

(a) $\omega_c \simeq 2{,}3$ rad/s

(b) $\varphi_m \simeq 74°$

Determinare $R_{TU}(z)$ in modo che

(P1) sia rispettato il teorema del campionamento

(P2) T_s tale che la diminuzione del margine di fase (stimata) sia

$$\left|\delta_{\varphi_m}\right| \leq 4°$$

$$\left|\delta_{\varphi_M}\right| = \left| -\frac{\bar{\omega}_c T_s}{2} \cdot \frac{180}{\pi} \right|$$

$\bar{\omega}_c$ pulsazione critica

T_s periodo di campionamento

$$\frac{\bar{\omega}_c T_s}{2} \frac{180^\circ}{\pi} \le 4^\circ \Rightarrow T_s \le \frac{8\pi}{180 \cdot \bar{\omega}_c} \simeq 6 \cdot 10^{-2} s$$

$\bar{\omega}_c = 7{,}3 \text{ rad/s}$

$$\ldots \omega_c \le \omega_s \le \ldots \omega_c$$

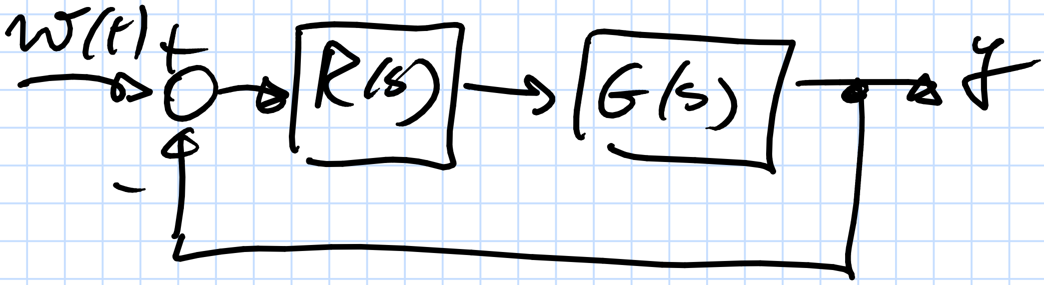
per es. $T_s = 5 \cdot 10^{-2} s$

$$s = \frac{2}{T_s} \frac{z-1}{z+1}$$

$$R_{TO}(z) = 25 \cdot \frac{1}{\frac{2}{T_s}\frac{z-1}{z+1} \cdot \left[1 + \frac{1}{\cancel{2}_1} \frac{\cancel{2}_1}{T_s} \frac{z-1}{z+1}\right]}$$

$$= \frac{25 T_s^2}{2} \frac{(z+1)^2}{(z-1)\left[(T_s+1)z + (T_s-1)\right]}$$

Progetto per sistema instabile   3/7/2012



$$G(s) = \frac{1+2s}{(1-s)(1+10s)}$$

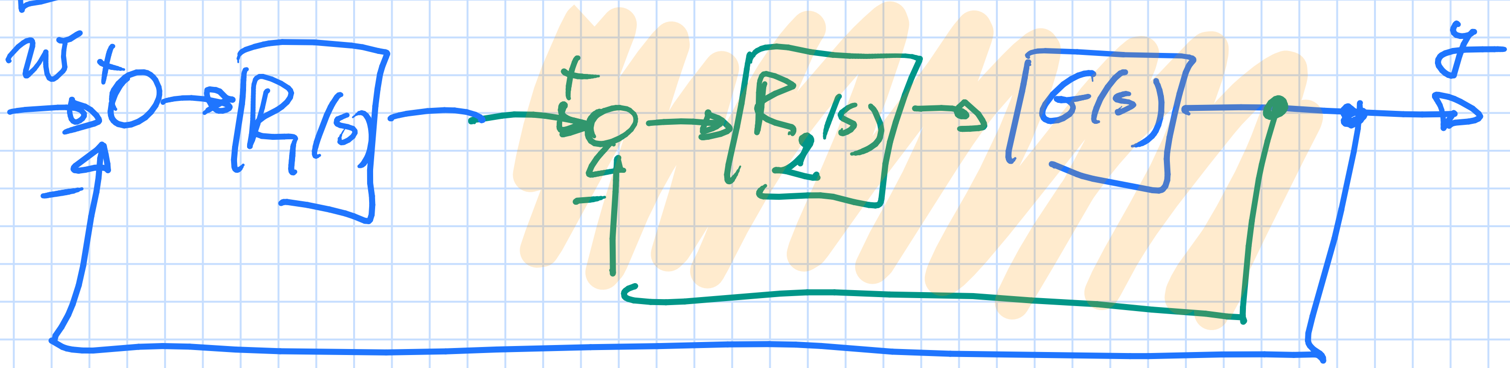
Richieste:

(a) sistema as. stabile a ciclo chiuso

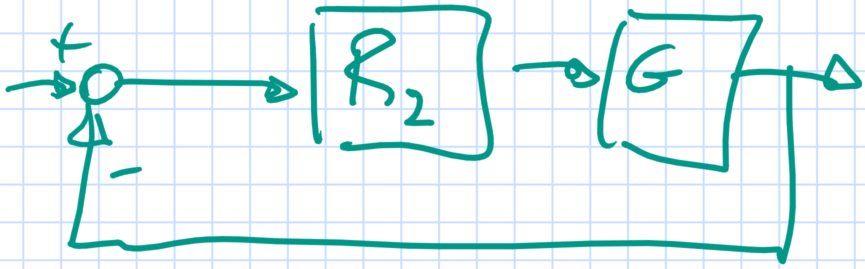
(b) $\omega_c \geq 10$ rad/s

(c) $\varphi_m \geq 60°$

Strategia a doppio anello



$\boxed{F_2(s)}$ as. stabile



$R_2(s) = \mu$

$$F_2(s) = \frac{R_2(s)G(s)}{\underbrace{1 + R_2(s)G(s)}}$$

$$(1-s)(1+10s) + \mu(1+2s) = \tilde{P}_{F_2}(s)$$

$$-10s^2 + (9+2\mu)s + \mu = \tilde{P}_{F_2}$$

$$\begin{array}{c|cc} 2 & -10 & \mu \\ 1 & 9+2\mu & \\ 0 & \mu & \end{array}$$

$$\begin{cases} 9+2\mu < 0 \\ \mu < 0 \end{cases}$$

$$\boxed{\mu < -\frac{9}{2}}$$