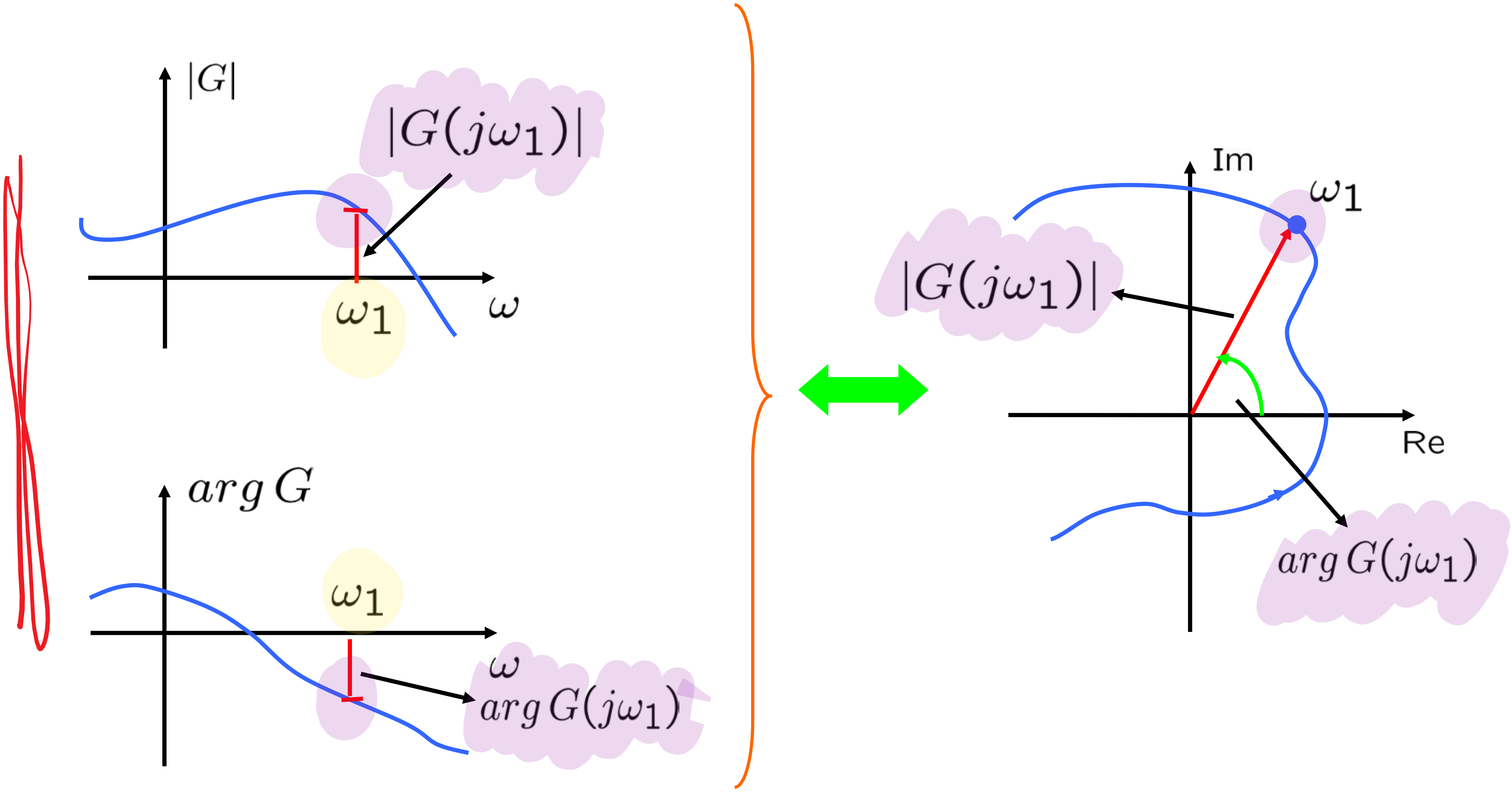
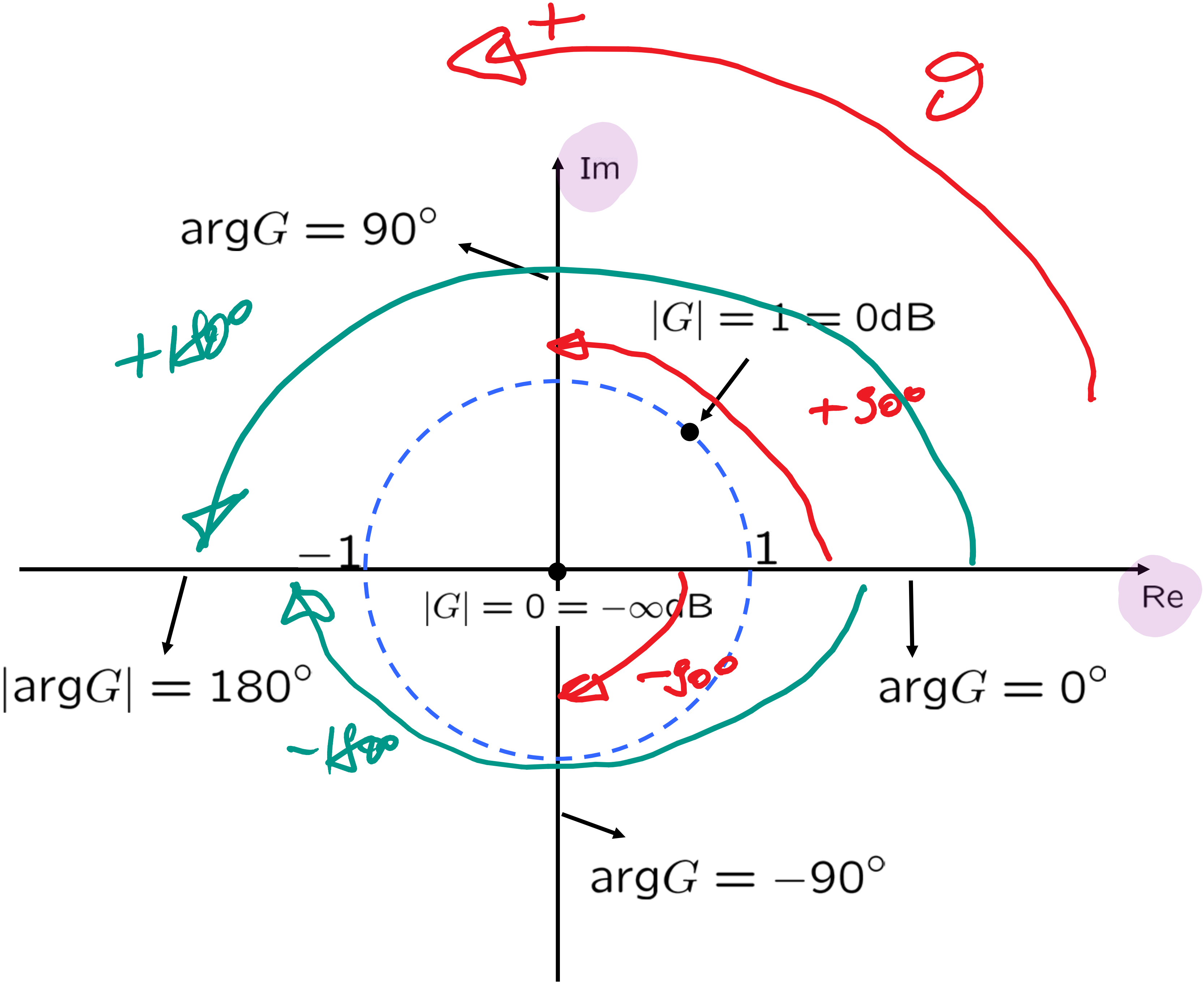
Studio di sistemi dinamici tramite FdT

Risposta transitoria e risposta a regime

- Diagrammi polari

$$G(j\omega)\,, \qquad \omega \geq 0$$




+
φ
Im
$\arg G = 90°$
+180°
$|G| = 1 = 0\text{dB}$
+90°
-1
1
Re
$|G| = 0 = -\infty\text{dB}$
$|\arg G| = 180°$
-90°
$\arg G = 0°$
-180°
$\arg G = -90°$

- Esempio 1

(•) esprimere le FdT in termini di K, τ, T

$$G(s) = \frac{10}{(1+10s)(1+2s)}$$

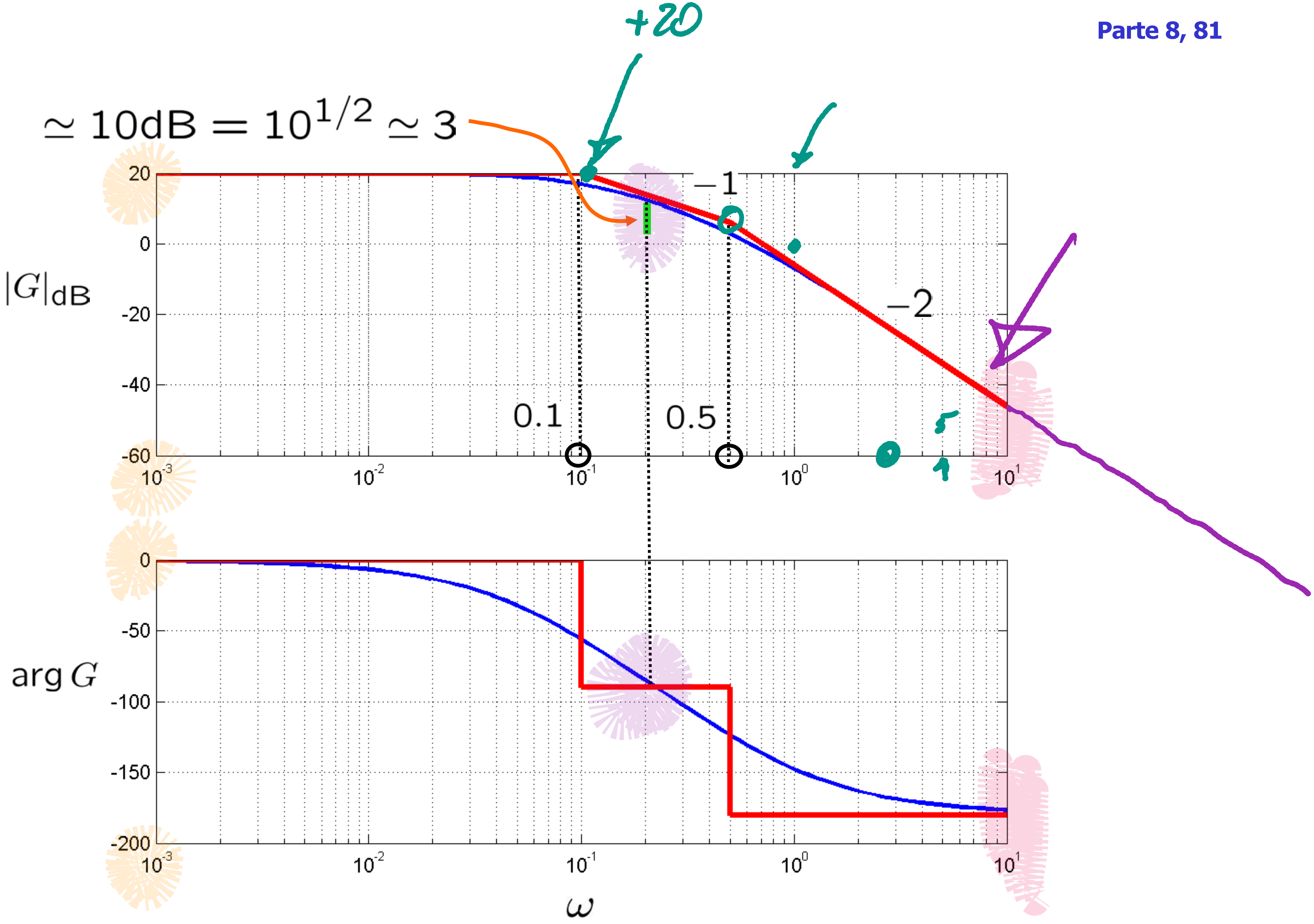
$$\mu = 10 \implies \mu_{\mathrm{dB}} = 20dB$$

$$g = 0$$

$$\tau_1 = 10 \qquad \omega_1 = 0.1$$

$$\tau_2 = 2 \qquad \omega_2 = 0.5$$

$$\simeq 10\text{dB} = 10^{1/2} \simeq 3$$



Tracciamento diagramma polare $L(j\omega)$

① per $\omega = 0 \Rightarrow$ $|L(j0)|$ $\angle L(j0)$

② per $\omega \to +\infty \Rightarrow$ $|L(j\omega)| \xrightarrow[\omega\to+\infty]{} ?$

$\angle L(j\omega) \longrightarrow ?$

③ quanti quadranti occupa il diagramma polare? $\longleftarrow \angle L(j\omega)$

④ intersezioni con gli assi del diagramma polare

⑤ gli particolari / asintoto, valori scelti per ω)

$$L(j\omega) = \frac{a+jb}{c+jd} = \frac{a(\omega)+jb(\omega)}{c(\omega)+jd(\omega)}$$

$$= \frac{a+jb}{c+jd} \cdot \frac{c-jd}{c-jd} = \frac{(ac+bd)+j(bc-ad)}{c^2+d^2}$$

$$= \underset{\mathrm{Re}}{\frac{ac+bd}{c^2+d^2}} + j \underset{\mathrm{Im}}{\frac{bc-ad}{c^2+d^2}}$$

$$L(s) = \frac{10}{(1+10s)(1+2s)} \rightarrow L(j\omega) = \frac{10}{(1-20\omega^2)+12j\omega}$$

$$= \frac{10}{1+12s+20s^2}$$

$$= \frac{a+jb}{c+jd}$$

$$\Re e = 0 \qquad \frac{ac+bd}{c^2+d^2} = 0 \qquad 10(1-20\omega^2)+0 = 0$$

$$20\omega^2 = 1$$

$$\omega = \begin{cases} -\frac{\sqrt{20}}{20} & \text{non acc. } (<0) \\ +\frac{\sqrt{20}}{20} \end{cases}$$

$$L\left(j\frac{\sqrt{20}}{20}\right) = \frac{10}{+12j\frac{\sqrt{20}}{20}} = -j\frac{10}{12\frac{\sqrt{20}}{20}}$$

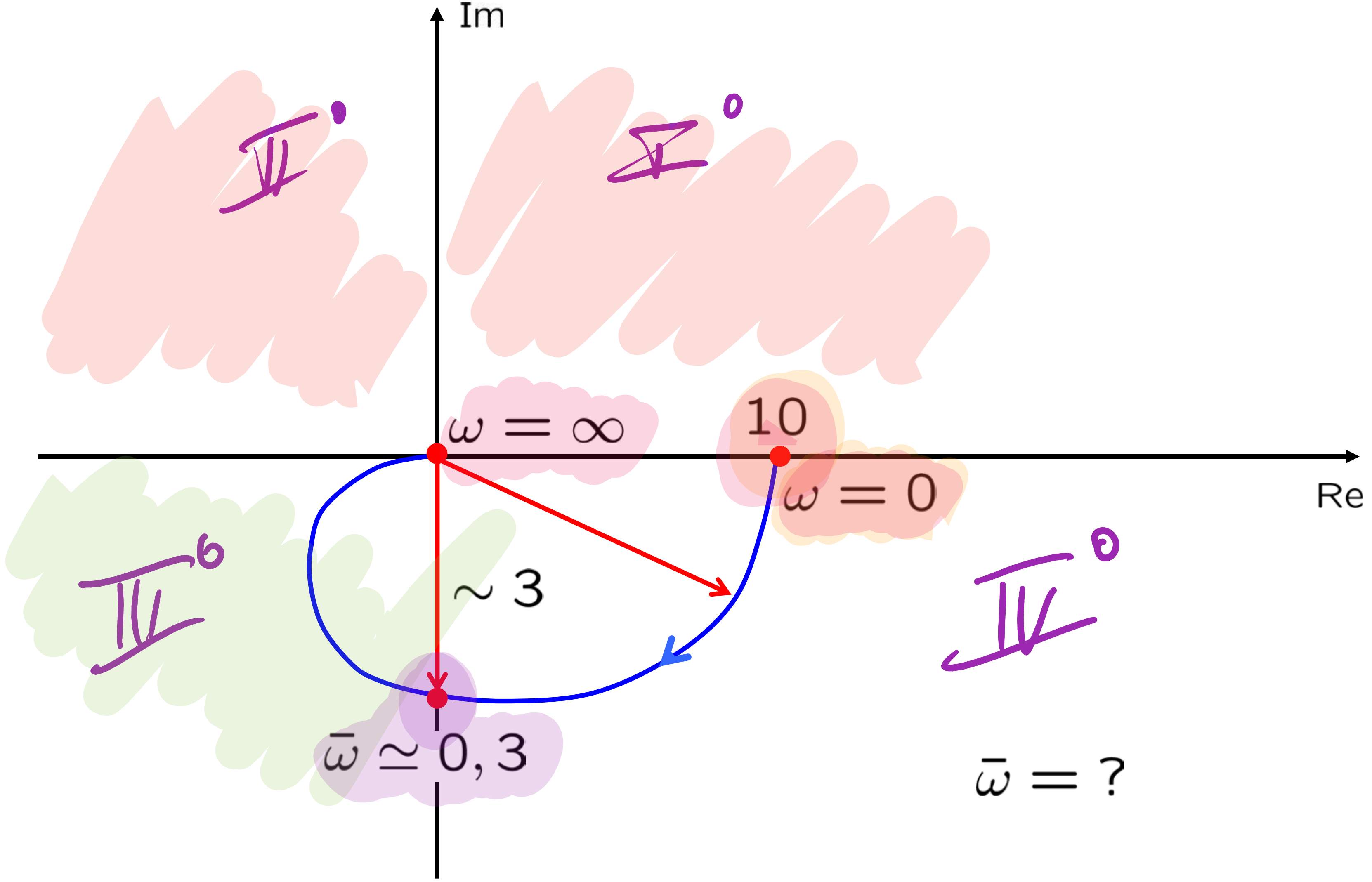
$$\frac{bc-ad}{c^2+d^2}=0$$

$$L(j\omega)=\frac{10}{(1-70\omega^2)+12j\omega}$$

$$-10\cdot 12\omega=0 \implies \omega=0$$

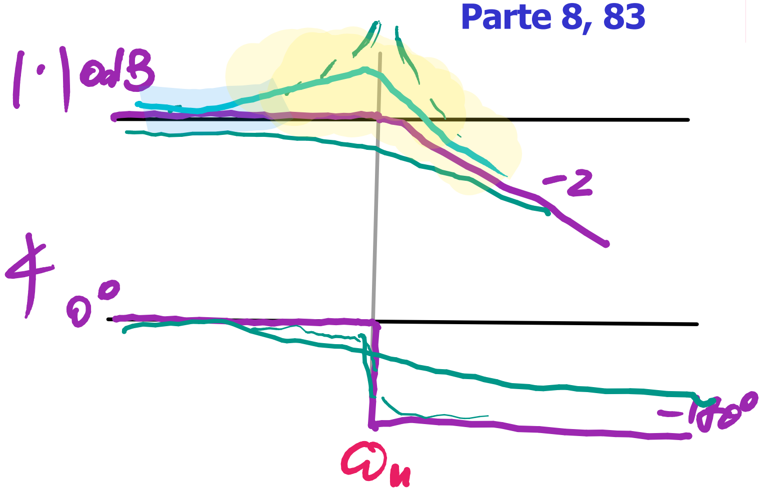
$$\downarrow$$

$$L(j0)=\frac{10}{1+j0}=+10$$



$\bar{\omega} = ?$

- Esempio 2



$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

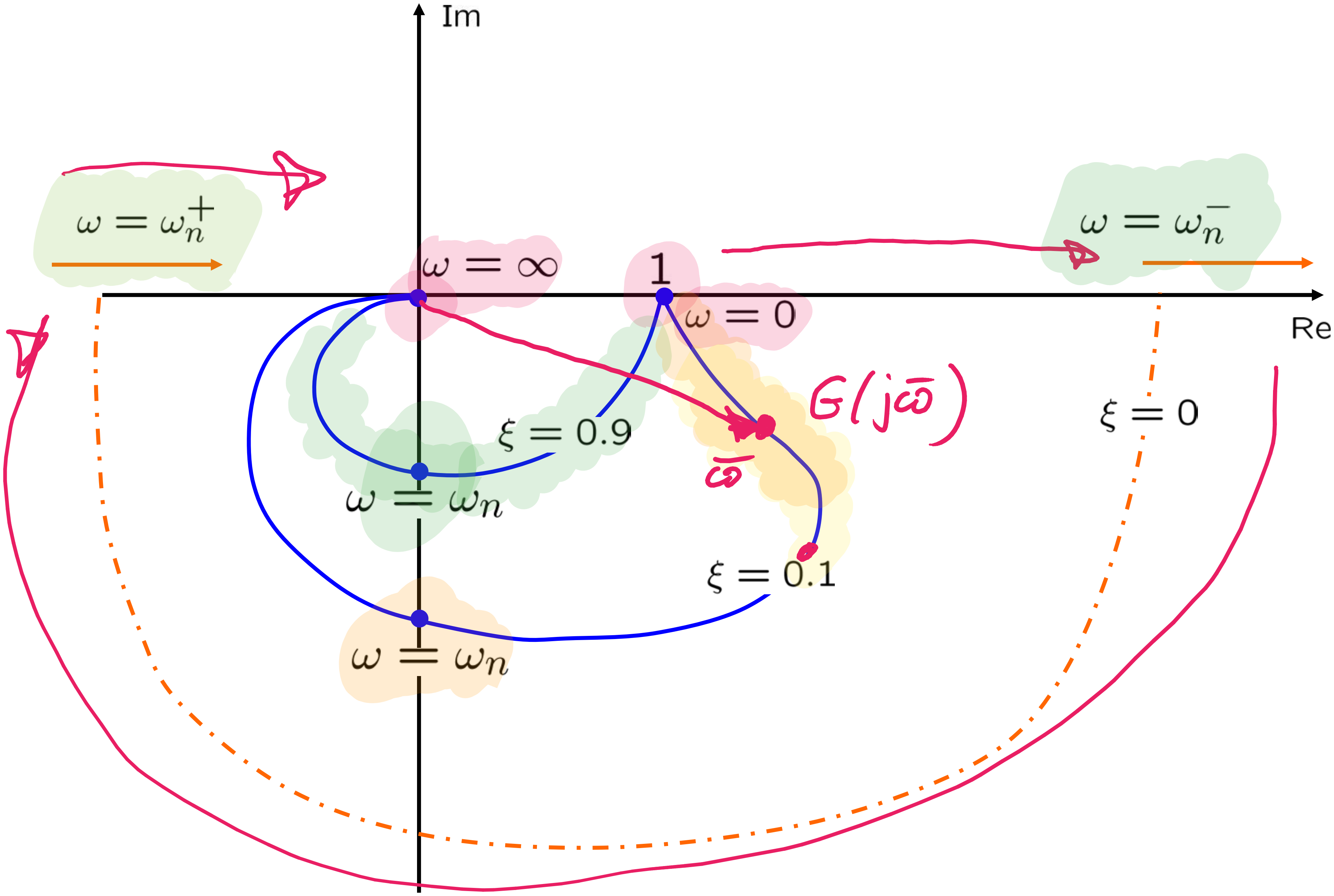
$$\mu = G(0) = 1$$

$$g = 0$$

$$\xi = 0.9$$

$$\xi = 0.1$$

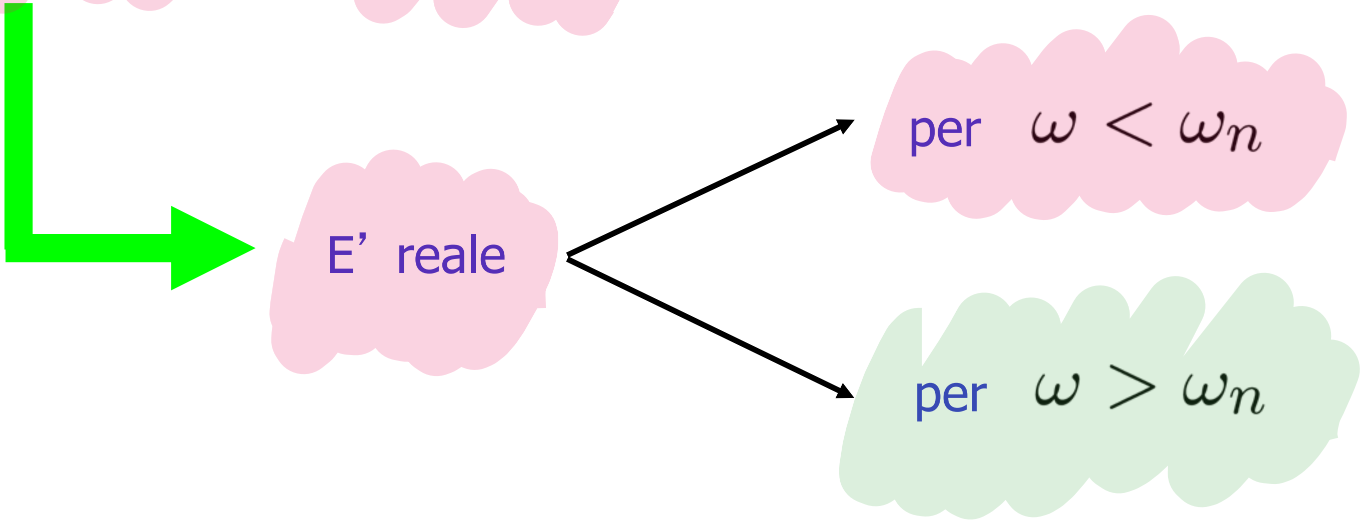
$$\xi = 0$$


Im
Re
$\omega = \omega_n^+$
$\omega = \omega_n^-$
$\omega = \infty$
1
$\omega = 0$
$\xi = 0.9$
$\xi = 0.1$
$\xi = 0$
$\omega = \omega_n$
$\omega = \omega_n$
$G(j\bar{\omega})$
$\bar{\omega}$

$\xi = 0$

$$G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2}$$



E' reale

per $\omega < \omega_n$

per $\omega > \omega_n$

- Esempio 3

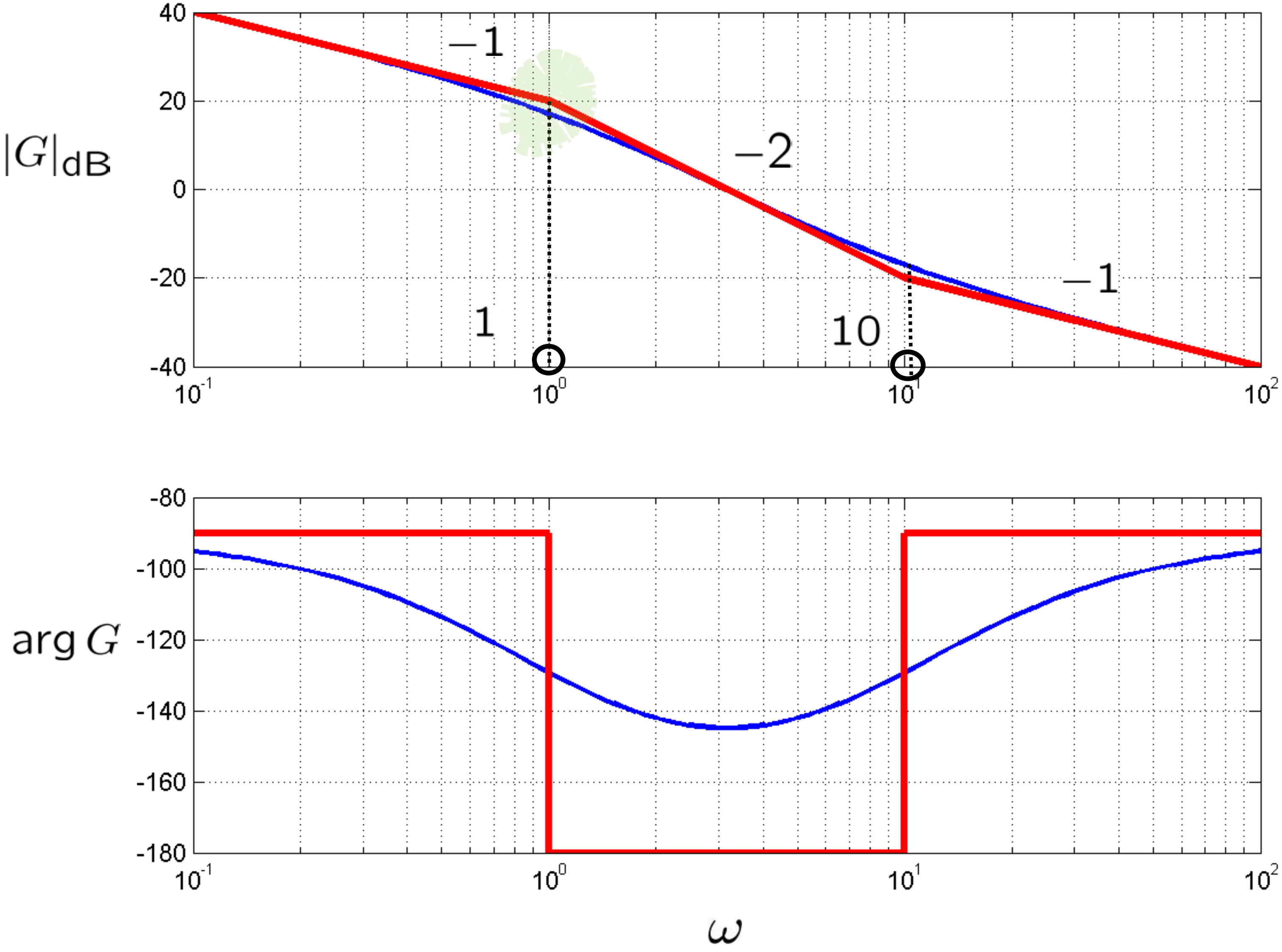
$$\omega = 10$$

$$G(s) = \frac{s+10}{s(s+1)} = \frac{10(1+\frac{s}{10})}{s(1+s)}$$

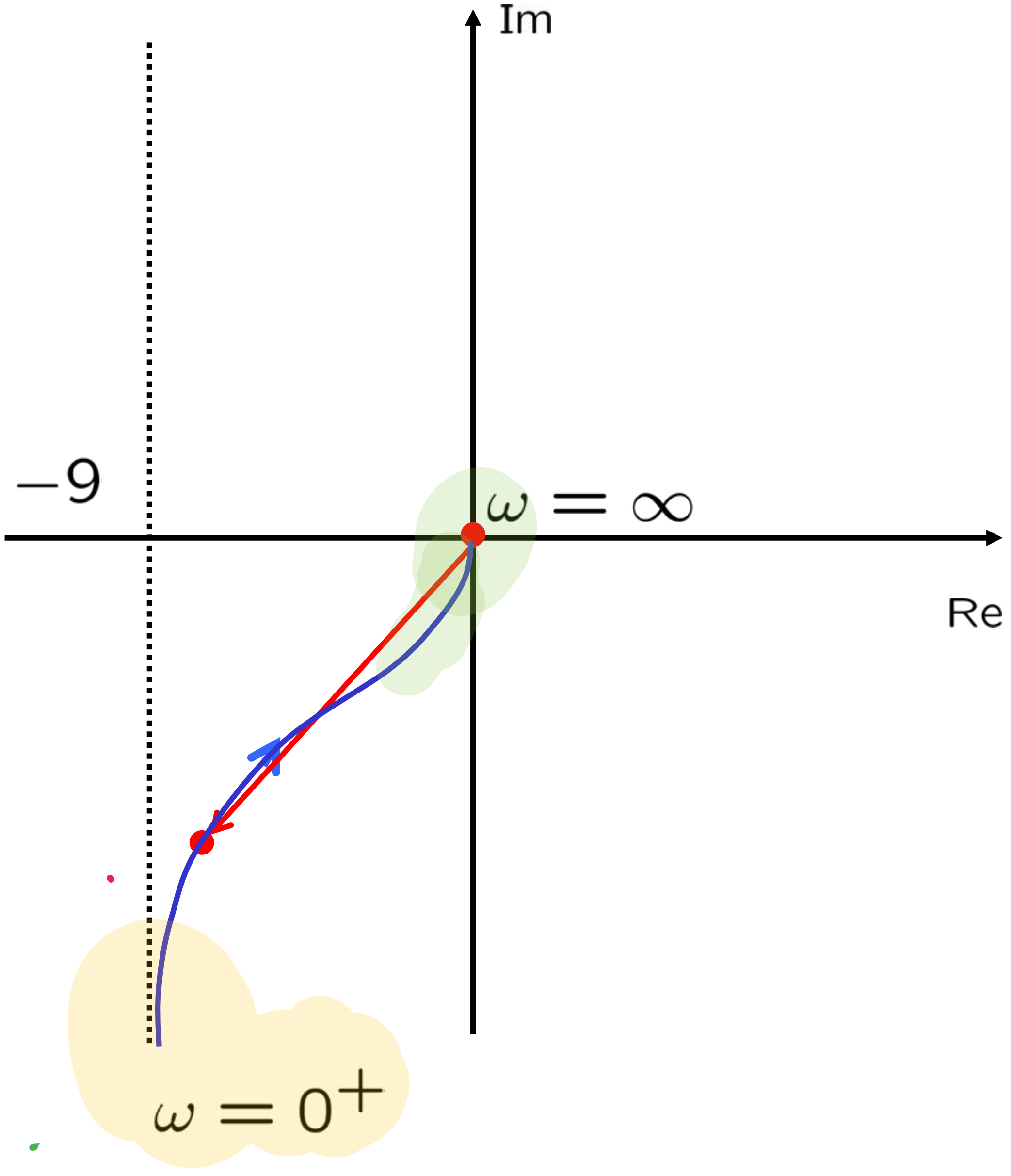
$$\omega = 1$$

$$g = 1$$

$$\mu = 10 \Longrightarrow \mu_{\text{dB}} = 20\text{dB}$$



$|G|_{\text{dB}}$
40
20
0
-20
-40
-1
-2
-1
1
10
10^{-1}
10^{0}
10^{1}
10^{2}
$\arg G$
-80
-100
-120
-140
-160
-180
10^{-1}
10^{0}
10^{1}
10^{2}
ω


Im
-9
$\omega = \infty$
Re
$\omega = 0^+$

- Compito a casa:

valutare la posizione dell' asintoto

$$\lim_{\omega \longrightarrow 0} Re(G(j\omega)) = ... = -9$$

$$\lim_{\omega \to 0} Im(G(j\omega)) = ?$$

$$\lim_{\omega \to 0} Re(G(j\omega)) = ...$$

$$(G(j\omega)) = \frac{j\omega + 10}{j\omega(j\omega + 1)} = \frac{-\omega + 10j}{-\omega(j\omega + 1)} =$$

$\Im m < 0 \quad \omega > 0$

$\Im m \xrightarrow[\omega \to 0^+]{} -\infty$

$$= \frac{(-\omega + 10j)(1 - j\omega)}{-\omega(1 + \omega^2)} = \frac{-\omega + 10\omega + j(10 + \omega^2)}{-\omega(1 + \omega^2)}$$

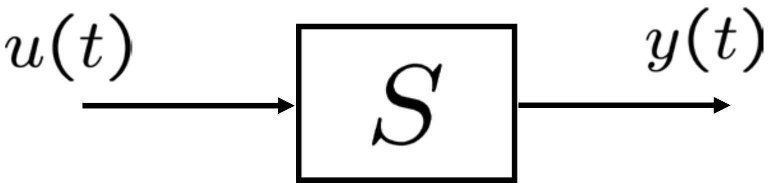
< 0

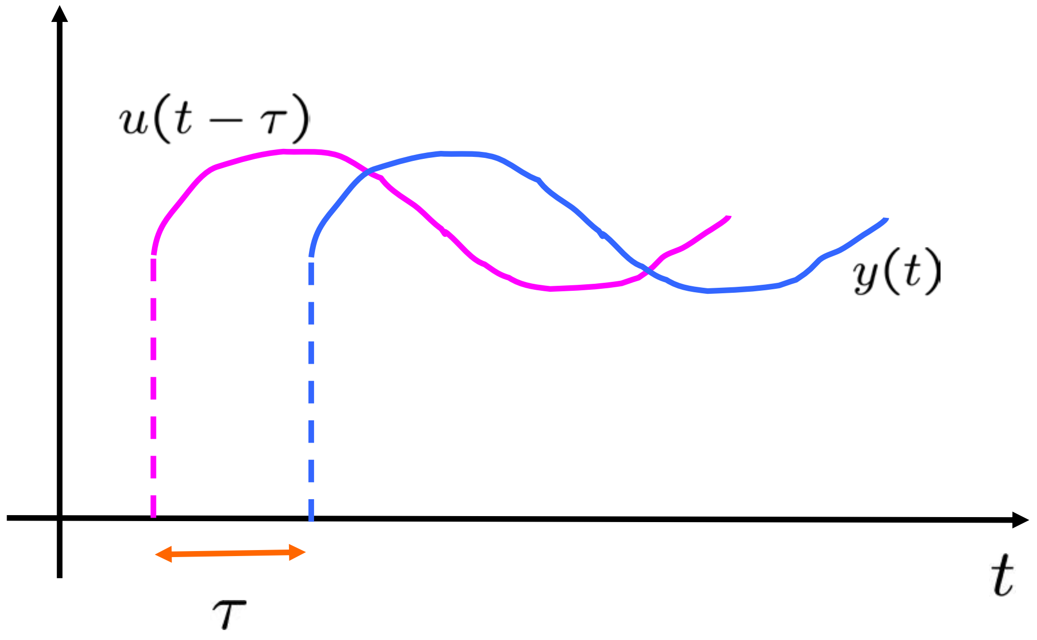
$\forall \omega \geq 0$

$\omega \to 0$

-9

- Ritardo di tempo





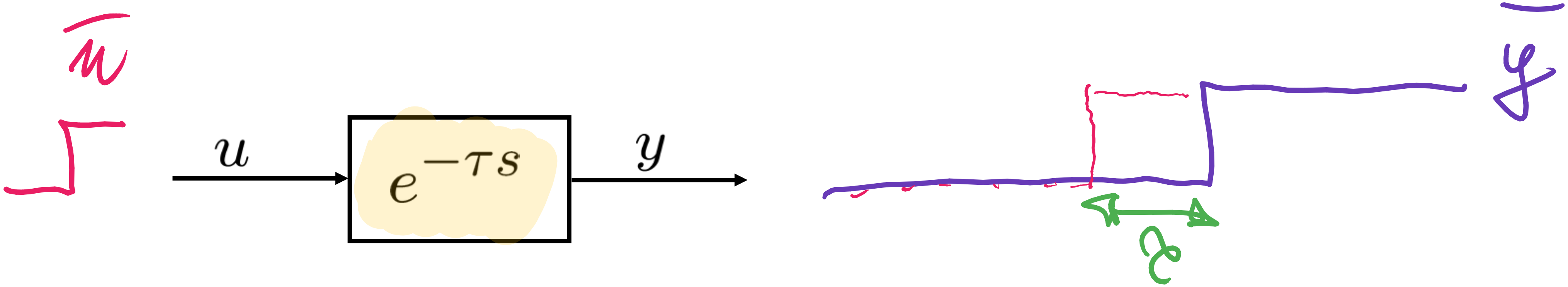
$$y(t) = u(t - \tau)$$

Ritardo

- Funzione di trasferimento

$$Y(s) = \mathcal{L}\{u(t-\tau)\} = \underbrace{e^{-\tau s}}_{} \cdot U(s)$$

G(s) → Non è razionale



Guadagno statico:

$$G(0) = 1 = \frac{\bar{y}}{\bar{u}}$$

- Risposta alla sinusoide

$$u(t) = A \sin(\omega t)$$

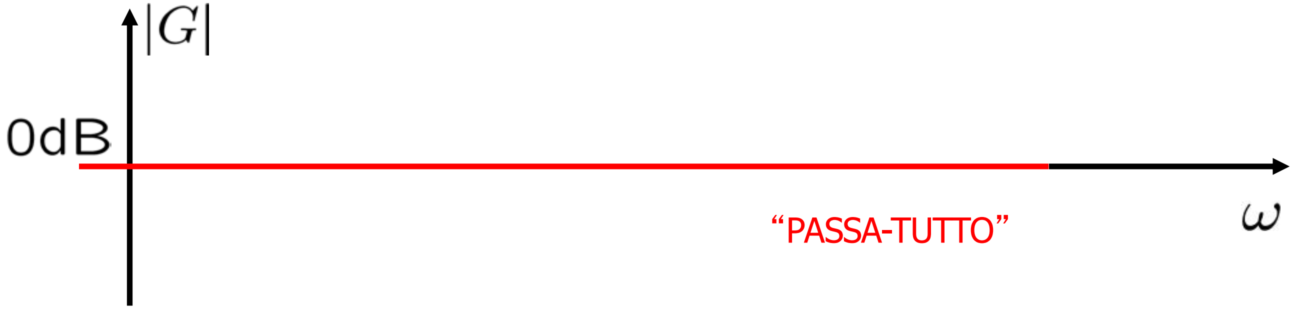
↳

$$y(t) = A \sin[\omega(t - \tau)]$$

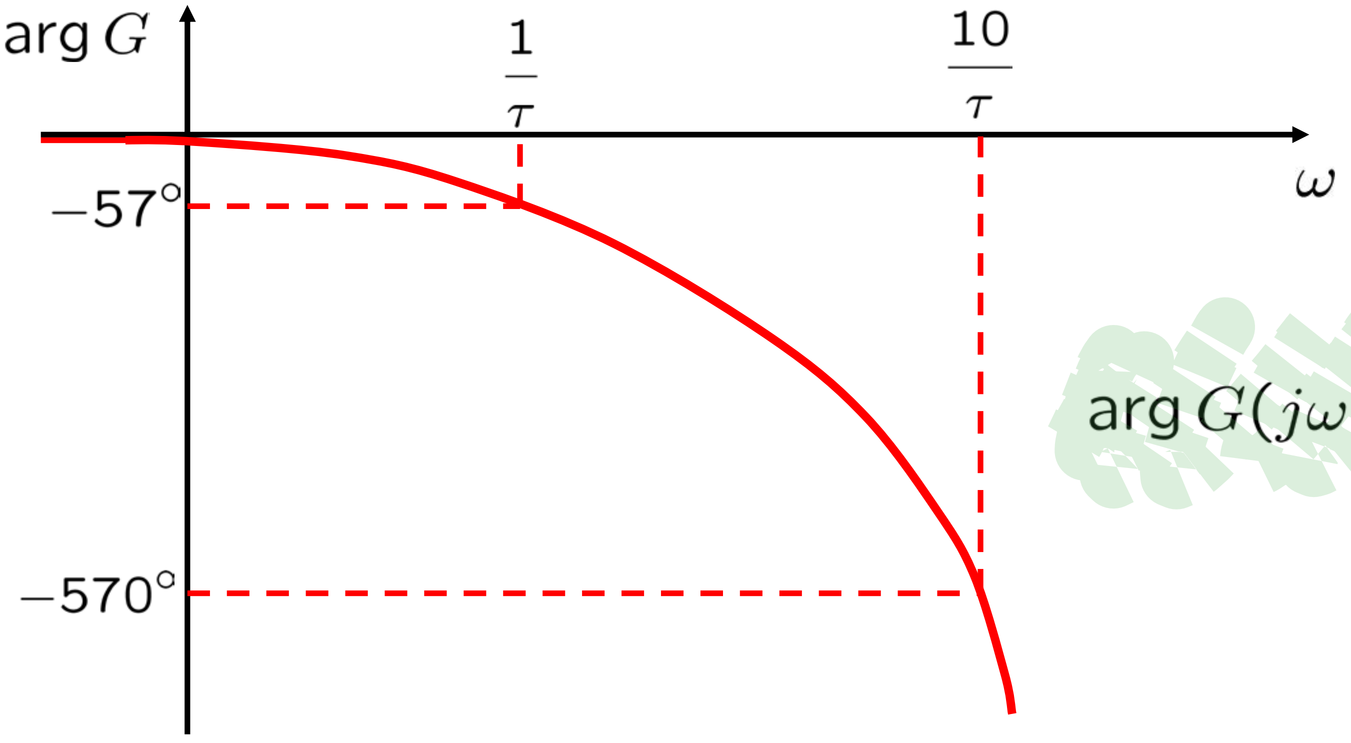
$$= \underset{\displaystyle |G(j\omega)|}{\underset{\downarrow}{\boxed{1}}} \cdot A \sin(\omega t \underbrace{- \omega\tau}_{\displaystyle \arg G(j\omega)})$$

VALE IL TEOREMA R.I.F.!!!!!

- Diagramma di Bode del modulo

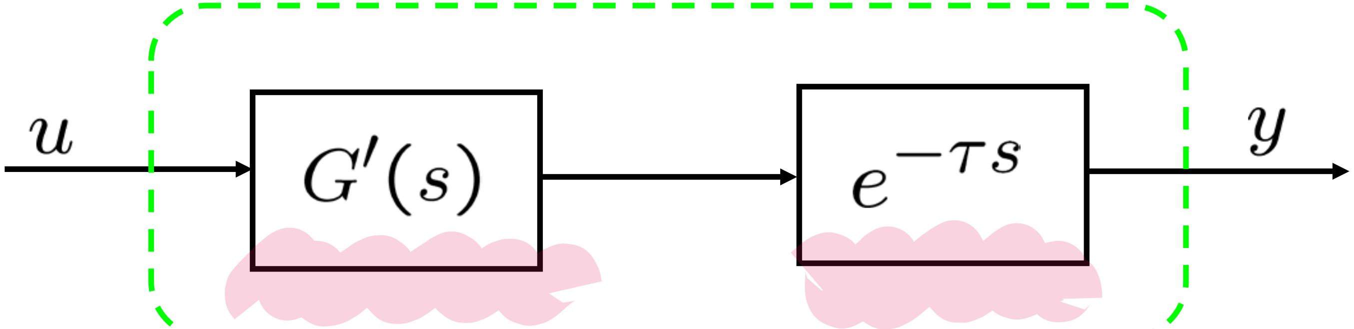


- Diagramma di Bode della fase



$$\arg G(j\omega) = -\omega\tau\frac{180}{\pi}$$

- Sistemi con ritardo



$$G(s) = G'(s) \cdot e^{-\tau s}$$

$$|G(j\omega)| = |G'(j\omega)| \cdot \underbrace{|e^{j\omega\tau}|}_{1} = |G'(j\omega)|$$

$$\arg G(j\omega) = \arg G'(j\omega) - \omega\tau\frac{180}{\pi}$$