

Studio di sistemi dinamici tramite FdT

Risposta transitoria e risposta a regime

- Esempio 4

$$G'(s) = \frac{50(1 + 0.4s)}{(1 + 10s)(1 + 0.2s + s^2)}$$

$$G''(s) = \frac{50(1 + 0.4s)}{(1 + 10s)(1 + 1.6s + s^2)}$$

$$g = 0$$

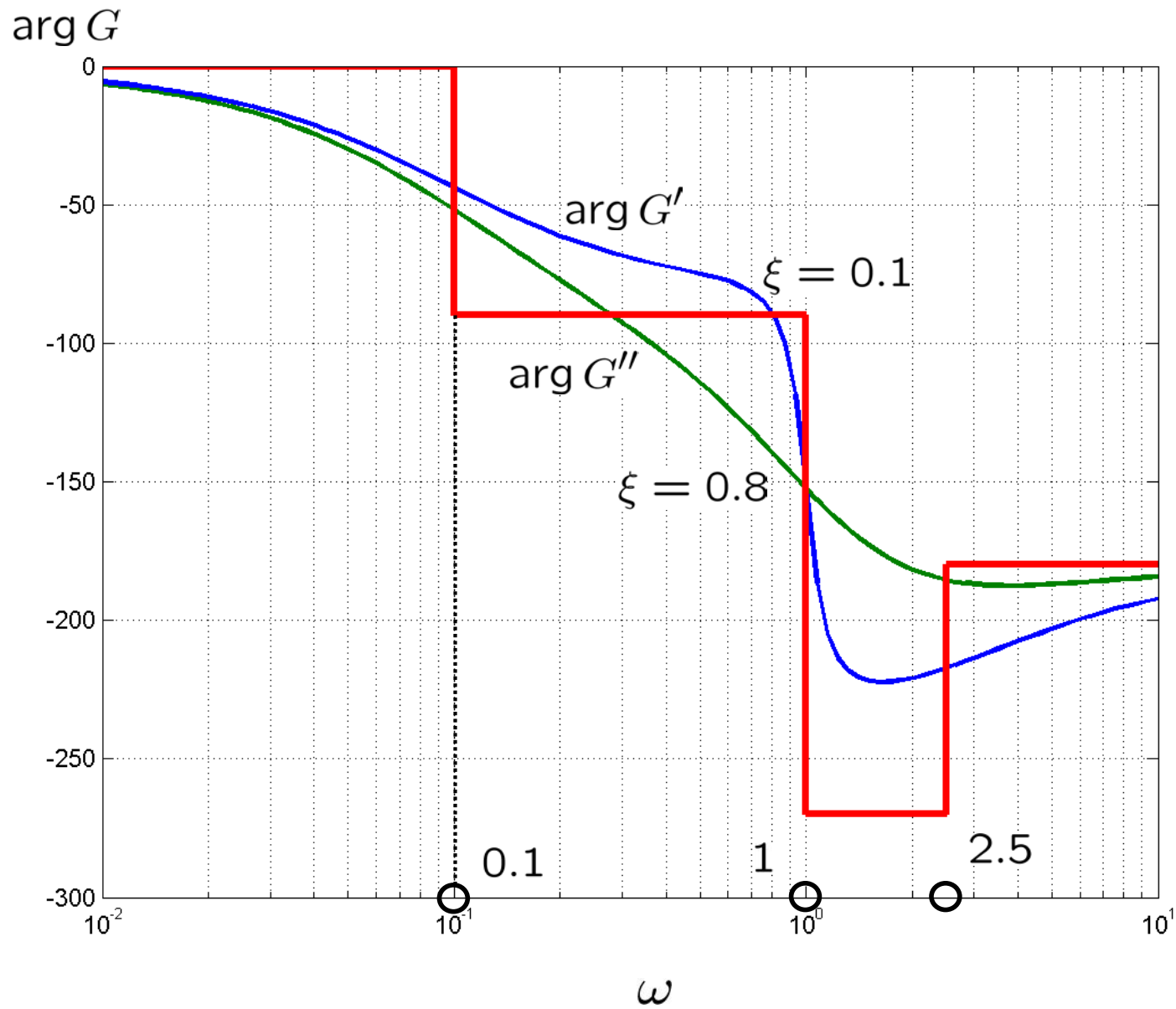
$$\mu = 50 \implies \mu_{\text{dB}} \simeq 34\text{dB}$$

$$z_1 = -2.5$$

$$p_1 = -0.1$$

$$p'_{2,3} = -0.1 \pm j\sqrt{0.99}; \omega_n = 1, \xi = 0.1$$

$$p''_{2,3} = -0.8 \pm j\sqrt{0.36}; \omega_n = 1, \xi = 0.8$$



- Legami tra $|G(j\omega)|$ e $\arg G(j\omega)$

- In generale nessuno
- Per sistemi a fase minima:

- formula di Bode

$\arg G(j\omega)$ in funzione di $|G(j\omega)|$

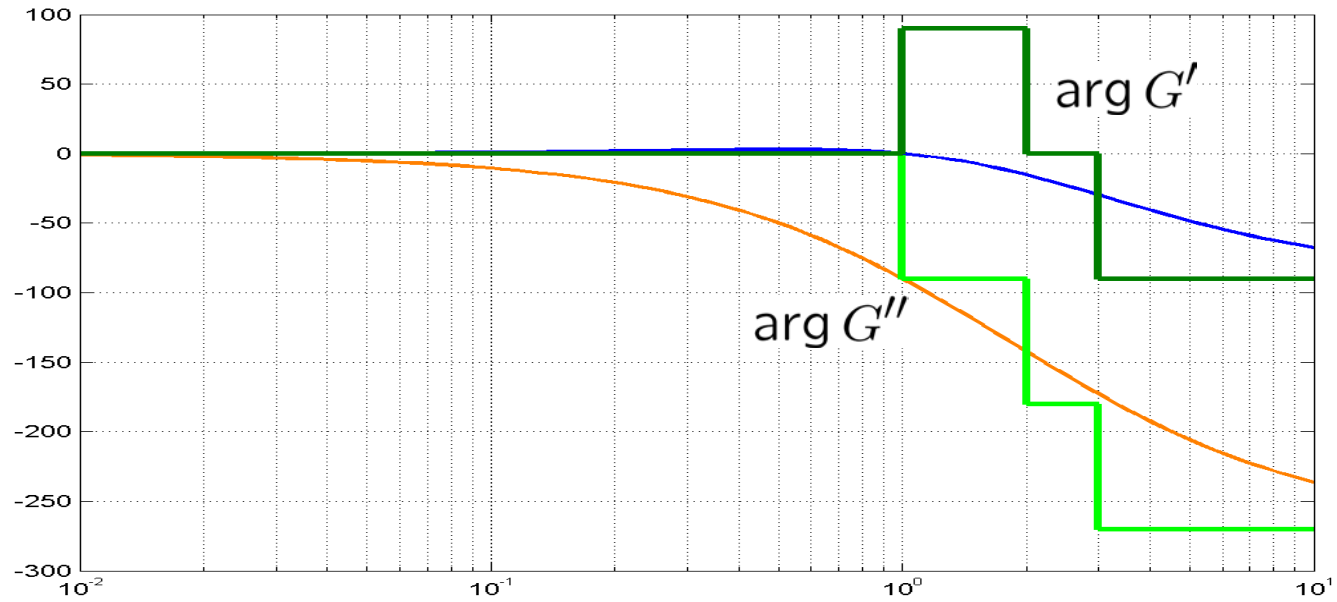
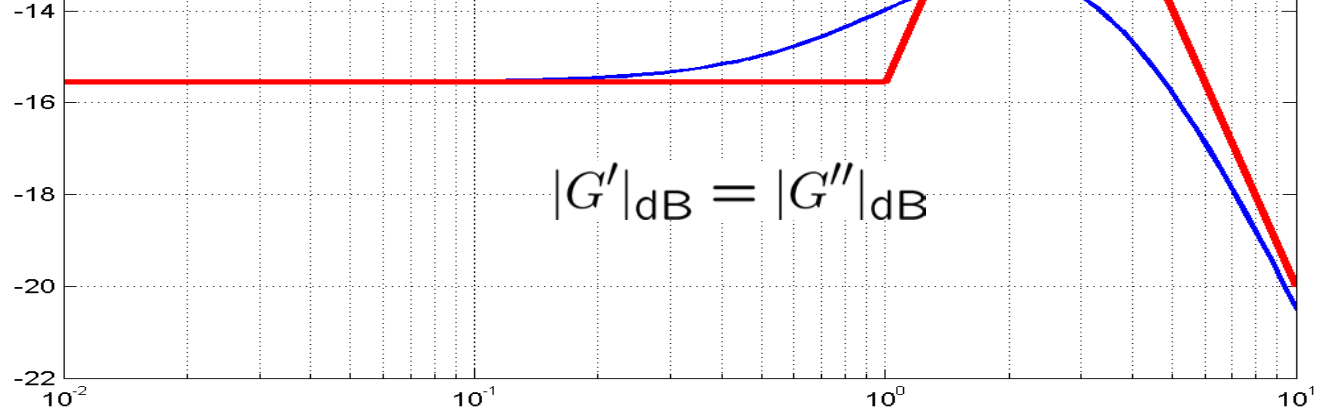
- legame tra i diagrammi asintotici

- Sistema a fase minima

- Guadagno $\mu > 0$
- Poli e zeri con $\text{Re} < 0$

$$G'(s) = \frac{1 + s}{s^2 + 5s + 6}$$

$$G''(s) = \frac{1 - s}{s^2 + 5s + 6}$$



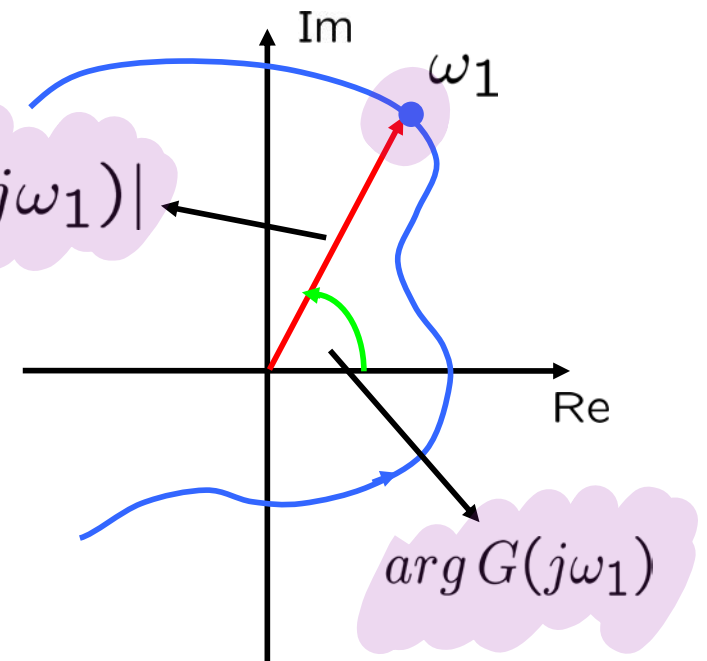
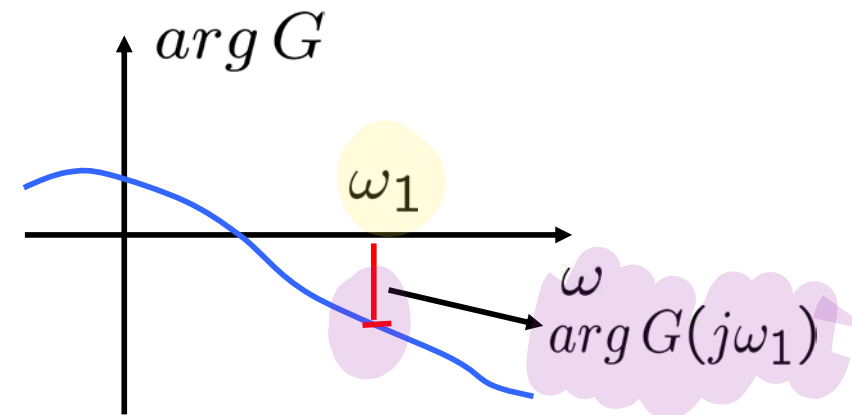
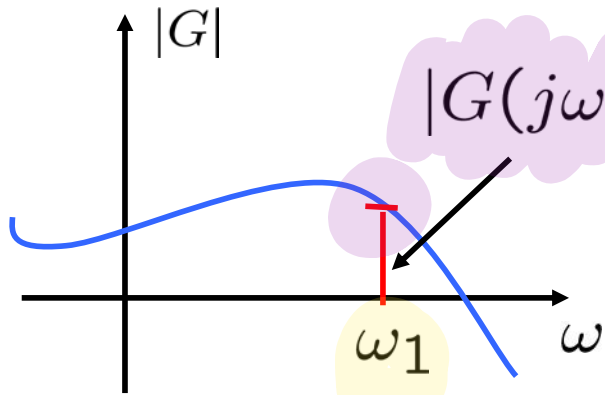
- Legami tra $|G(j\omega)|$ e $\arg G(j\omega)$

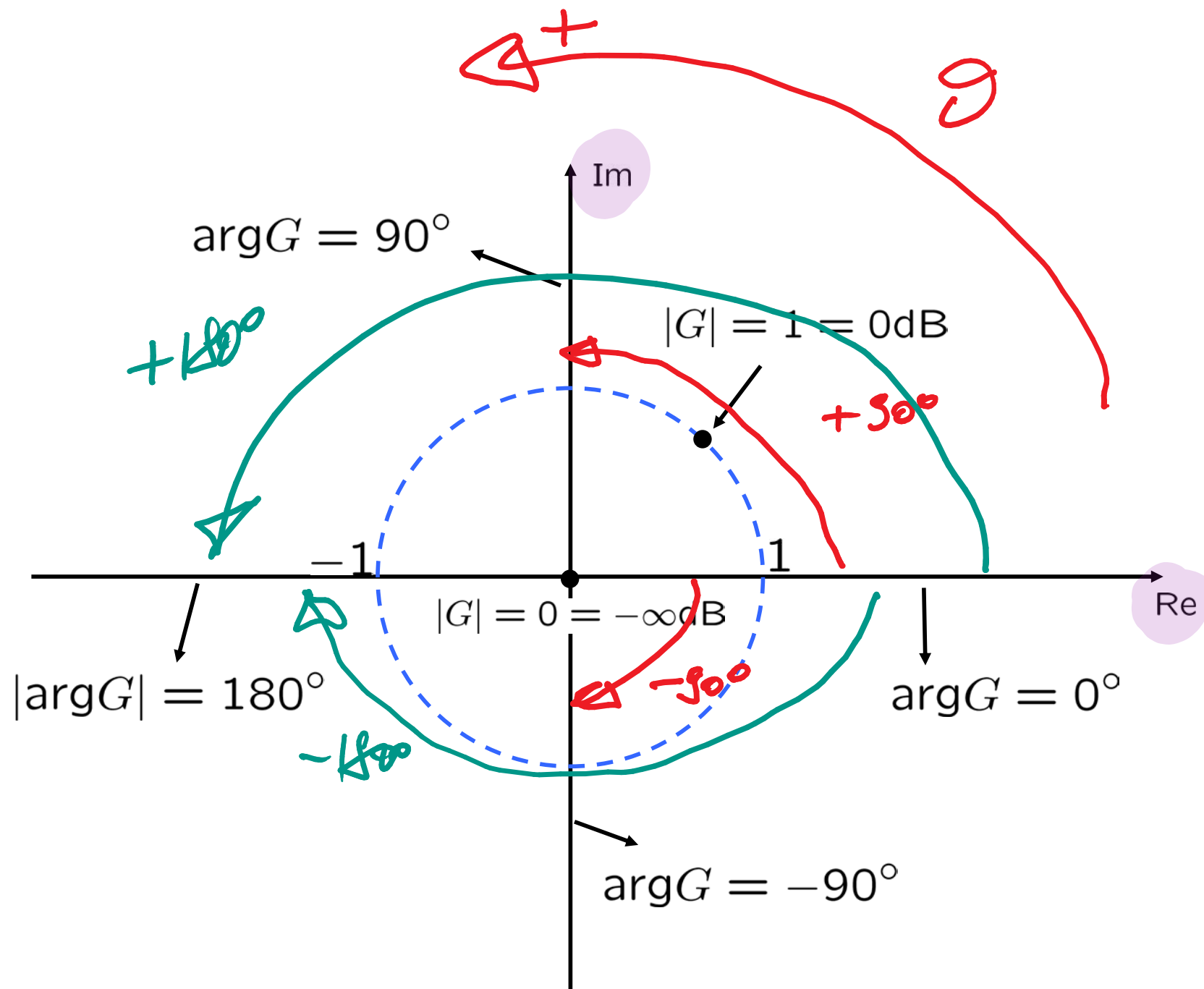
Per sistemi a fase minima:

	Pendenza $ G(j\omega) $	Valore $\arg G(j\omega)$
Polo	-1	-90°
Zero	+1	+90°

- Diagrammi polari

$$G(j\omega), \quad \omega \geq 0$$





- Esempio 1

$$G(s) = \frac{10}{(1 + 10s)(1 + 2s)}$$

(*) esprimere le
FdT in termini
di K , φ , T

$$\mu = 10 \implies \mu_{dB} = 20dB$$

$$g = 0$$

$$\tau_1 = 10$$

$$\omega_1 = 0.1$$

$$\tau_2 = 2$$

$$\omega_2 = 0.5$$

Inciamento di frequenza fase $L(j\omega)$

① per $\omega = 0 \Rightarrow |L(j0)| \neq L(j0)$

② per $\omega \rightarrow +\infty \Rightarrow |L(j\omega)| \rightarrow ?$
 $\neq L(j\omega) \xrightarrow{\omega \rightarrow +\infty} ?$

③ quanto gradiente occorre il diagramma
fase? $\leftarrow \neq L(j\omega)$

④ intersezioni con gli assi del diagramma fase

⑤ gli asintoti (esistono, valori scelti per ω)

$$L(j\omega) = \frac{a + jb}{c + jd} = \frac{a(\omega) + j b(\omega)}{c(\omega) + j d(\omega)}$$

$$= \frac{a + jb}{c + jd} \cdot \frac{c - jd}{c - jd} = \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + j \frac{bc - ad}{c^2 + d^2}$$

Re

Im

$$L(s) = \frac{10}{(1+10s)(1+7s)}$$

$$= \frac{10}{s^2 + 17s + 70}$$

$$\Rightarrow L(j\omega) = \frac{10}{(1-70\omega^2) + 12j\omega}$$

$$= \frac{a + jb}{c + jd}$$

Re = 0

$$\frac{ac + bd}{c^2 + d^2} = 0$$

$$10(1-70\omega^2) + 0 = 0$$

$$20\omega^2 = 1$$

$$\omega = \pm \frac{\sqrt{20}}{20}$$

non rec. ($\angle \theta$)

$$L\left(j \frac{\sqrt{20}}{20}\right) = \frac{10}{+12j \frac{\sqrt{20}}{20}} = -j \frac{10}{12 \frac{\sqrt{20}}{20}}$$

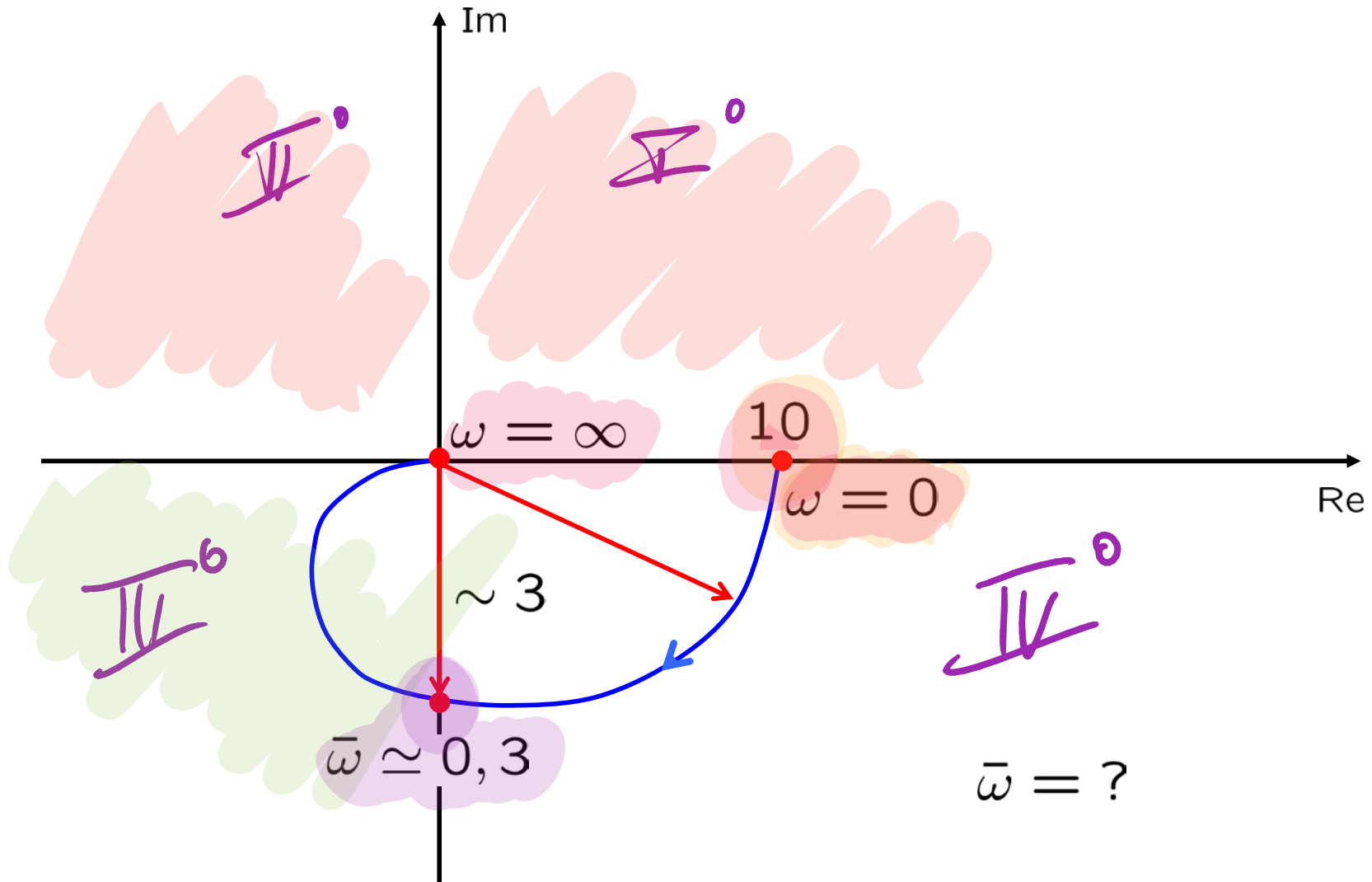
$$\frac{bc - ad}{c^2 + d^2} = 0$$

$$L(j\omega) = \frac{10}{(1 - 70\omega^2) + 12j\omega}$$

$$-10 \cdot 12\omega = 0 \Rightarrow \omega = 0$$

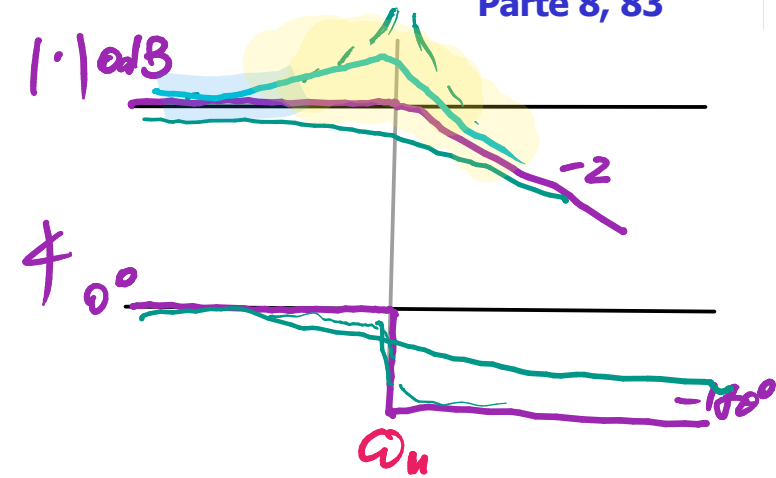
↓

$$L(j0) = \frac{10}{1 + j0} = +10$$



- Esempio 2

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



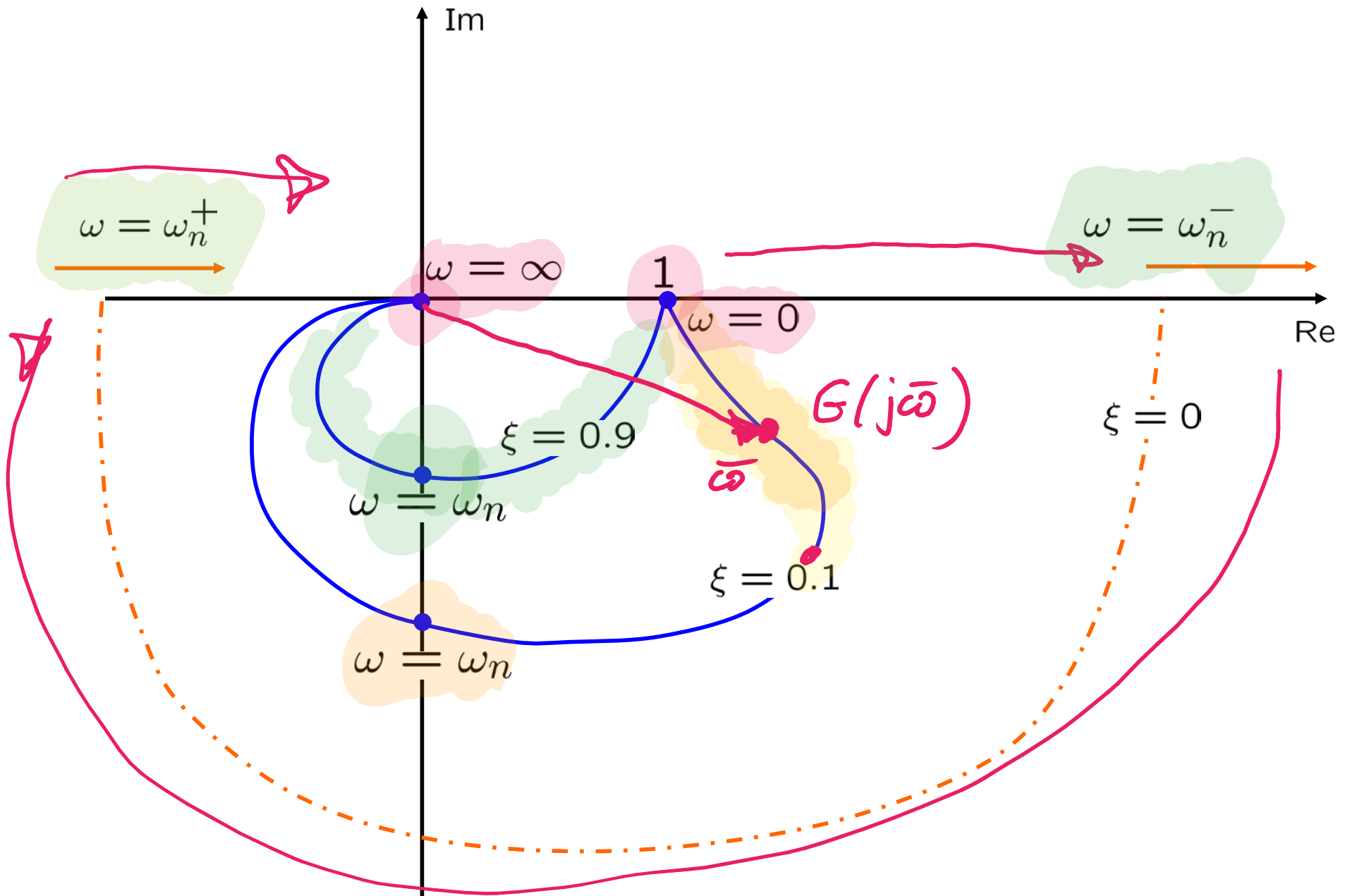
$$\mu = G(0) = 1$$

$$g = 0$$

$$\xi = 0.9$$

$$\xi = 0.1$$

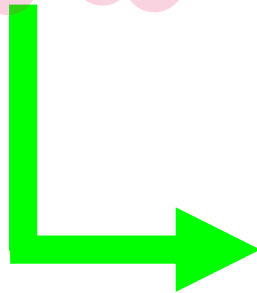
$$\xi = 0$$



$$\xi = 0$$

$$G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2}$$



E' reale

per $\omega < \omega_n$

per $\omega > \omega_n$

- Esempio 3

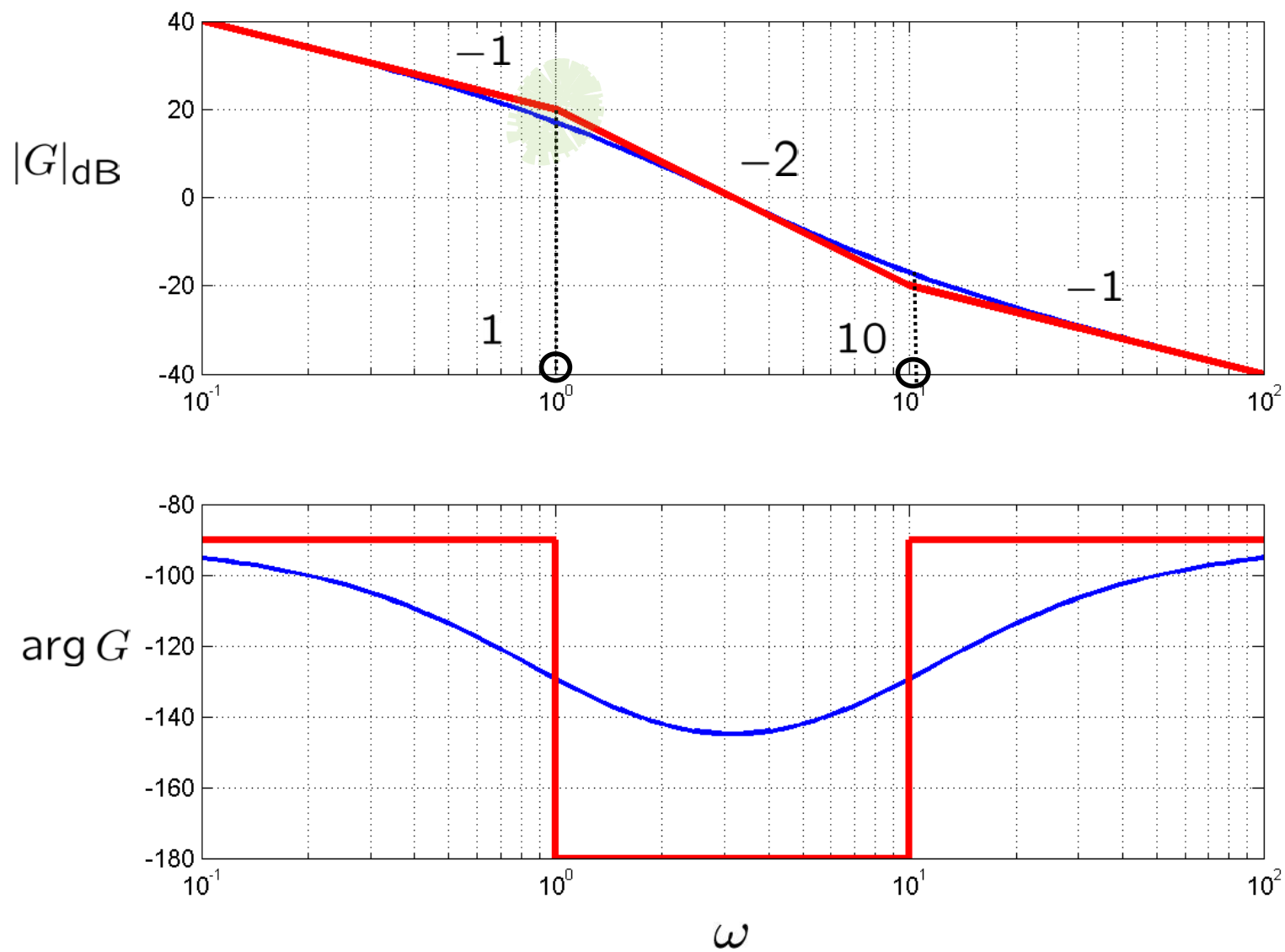
$$G(s) = \frac{s + 10}{s(s + 1)} = \frac{10(1 + \frac{s}{10})}{s(1 + s)}$$

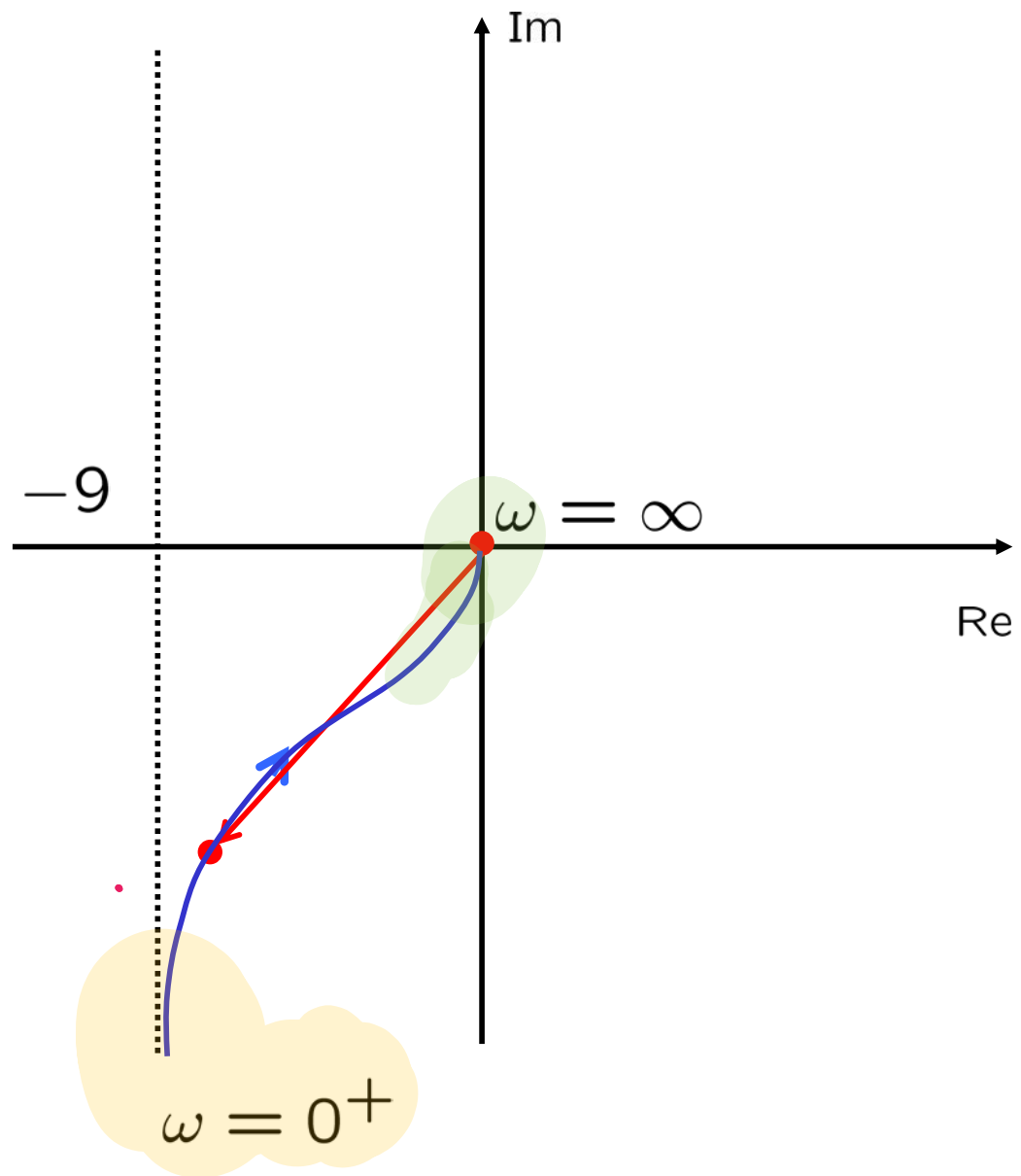
$\omega = 10$

$\omega = 1$

$$g = 1$$

$$\mu = 10 \implies \mu_{dB} = 20dB$$





- Compito a casa:

valutare la posizione dell' asintoto

$$\lim_{\omega \rightarrow 0} \operatorname{Re}(G(j\omega)) = \dots = -9$$

$$\lim_{\omega \rightarrow 0} \operatorname{Im}(G(j\omega)) = ?$$

$$\lim_{\omega \rightarrow 0} \operatorname{Re}(G(j\omega)) = \dots$$

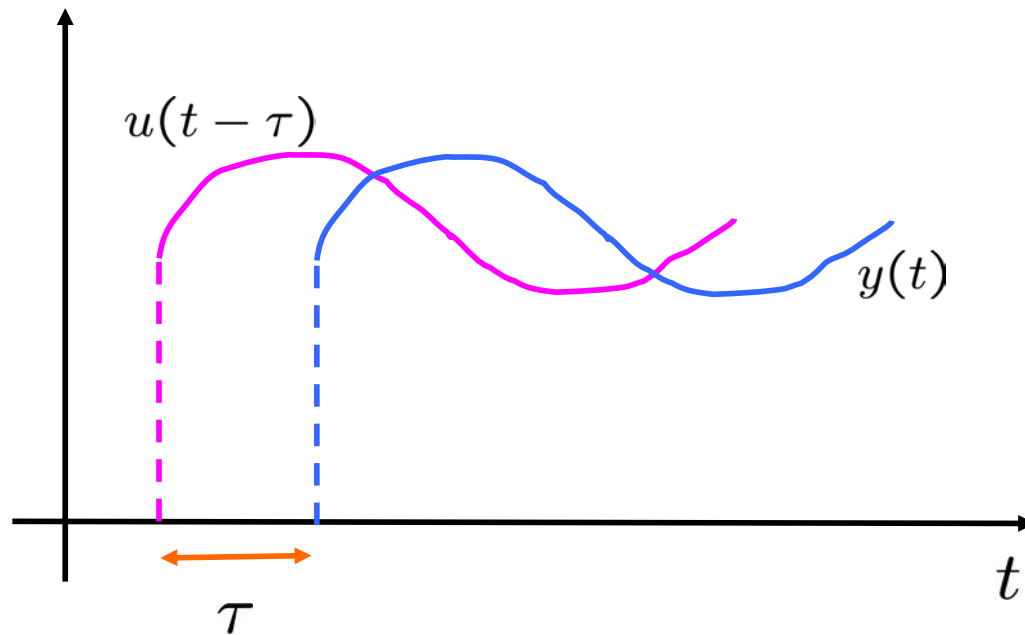
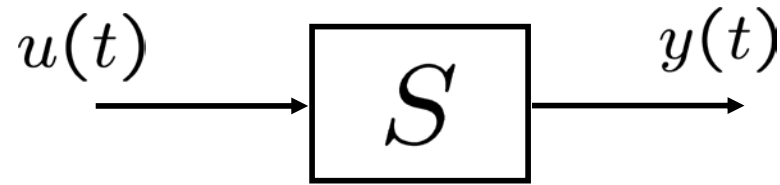
$$(G(j\omega)) = \frac{j\omega + 10}{j\omega(j\omega + 1)} = \frac{-\omega + 10j}{-\omega(j\omega + 1)} =$$

$$= \frac{(-\omega + 10j)(1 - j\omega)}{-\omega(1 + \omega^2)} = \frac{-\omega + 10\omega + j(10 + \omega^2)}{-\omega(1 + \omega^2)}$$

$$\omega \rightarrow 0$$

$$-9$$

- Ritardo di tempo



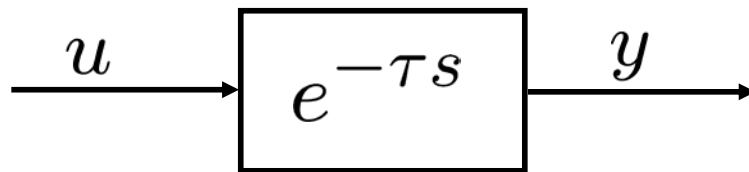
$$y(t) = u(t - \tau)$$

↑
Ritardo

- Funzione di trasferimento

$$Y(s) = \mathcal{L}\{u(t - \tau)\} = \underbrace{e^{-\tau s}}_{G(s)} \cdot U(s)$$

$G(s)$ \longrightarrow Non è
razionale



Guadagno statico:

$$G(0) = 1 = \frac{\bar{y}}{\bar{u}}$$

- Risposta alla sinusoidale

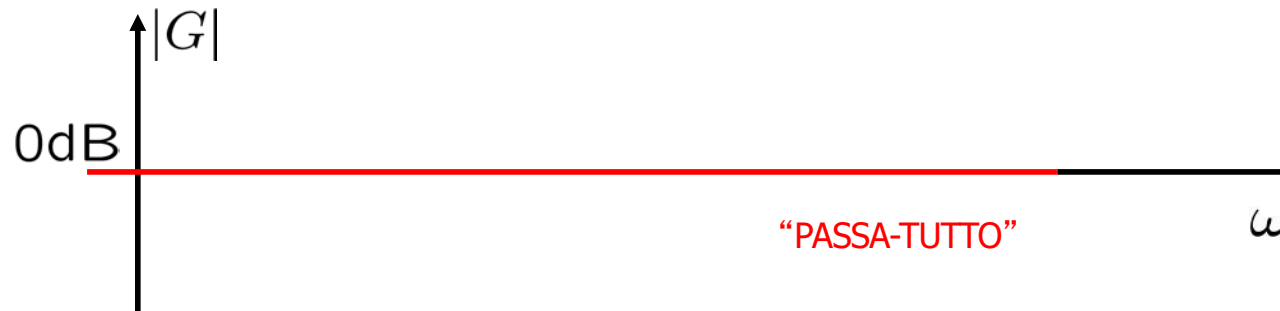
$$u(t) = A \sin(\omega t)$$

$$\downarrow y(t) = A \sin[\omega(t - \tau)]$$

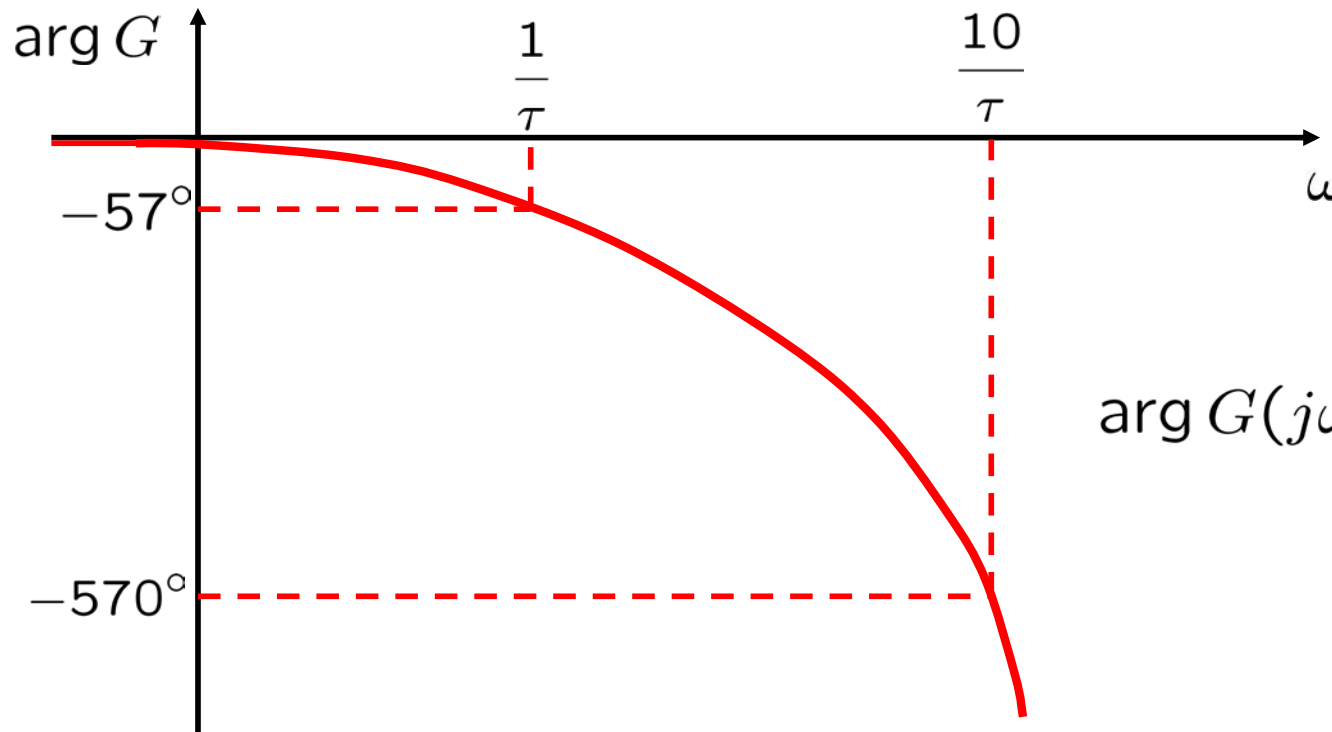
$$= \underbrace{1}_{|G(j\omega)|} \cdot A \sin(\omega t - \underbrace{\omega\tau}_{\arg G(j\omega)})$$

VALE IL TEOREMA R.I.F.!!!!!!

- Diagramma di Bode del modulo

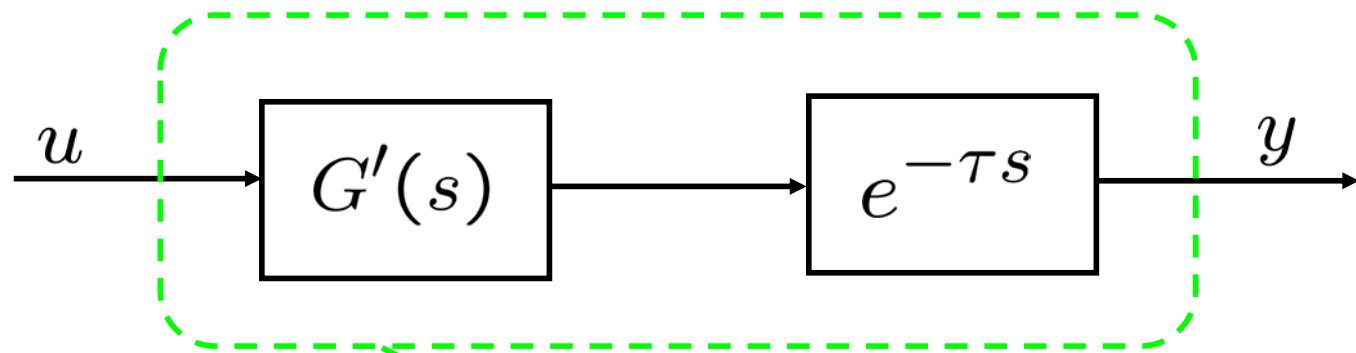


- Diagramma di Bode della fase



$$\arg G(j\omega) = -\omega\tau \frac{180}{\pi}$$

- Sistemi con ritardo



$$G(s) = G'(s) \cdot e^{-\tau s}$$

$$|G(j\omega)| = |G'(j\omega)| \cdot \underbrace{|e^{j\omega\tau}|}_{1} = |G'(j\omega)|$$

$$\arg G(j\omega) = \arg G'(j\omega) - \omega\tau \frac{180}{\pi}$$