

# Analisi di root locus



$$G(s) = K \frac{s+4}{s(s+3)}$$

$$K \in \mathbb{R} \quad K > 0$$

Determinare

- (a) i valori di  $K$  per cui il sistema a ciclo chiuso sia es. stabile
- (b) i valori di  $K$  per cui il sistema a ciclo chiuso (sia es. stabile) abbia una coppia di radici complesse coniugate sovracritiche
- (c) il valore di  $K$  per cui la sovracriticità è massima

Problema 2

$$F(s) = \frac{G(s)}{1 + G(s)}$$

$$G(s) = \frac{N(s)}{D(s)}$$

$$= \frac{N(s)}{N(s) + D(s)}$$

$$\downarrow D_F(s) = N(s) + D(s) = s^2 + (3+k)s + 4k$$

Criterio di R.H.

$$\begin{array}{c|c} 2 & 1 & 4k \\ 1 & 3+k & \\ 0 & 4k & \end{array}$$

$$\left\{ \begin{array}{l} 3+k > 0 \\ 4k > 0 \\ k > 0 \end{array} \right.$$

stabile  
asintotica  
 $\forall k > 0$

Übung 6

$$F(s) = \frac{K(s+4)}{s^2 + (3+K)s + 4K}$$

$K > 0$

FA Parte 7, #72

$$\mu \frac{(1+sT)}{1 + \frac{2\xi}{\omega_n} s + \frac{s^2}{\omega_n^2}}$$

$$F(s) = \frac{\cancel{4K} (1 + s/4)}{\cancel{4K} \left[ 1 + \frac{3+K}{4K} s + \frac{s^2}{4K} \right]}$$

$K > 0$

$$= \frac{1 + s/4}{1 + \frac{3+K}{4K} s + \frac{s^2}{4K}} \quad \mu = 1$$

$$F(s) = 1 \cdot \frac{1 + \frac{s}{4}}{1 + \frac{3+K}{4K}s + \frac{s^2}{4K}}$$

$$T = \frac{1}{4}$$

$$2\frac{\xi}{\omega_n} = \frac{3+K}{4K}$$

$$\omega_n^2 = 4K$$

$$\omega_n = 2\sqrt{K}$$

$$\xi = \frac{\sqrt{K}}{4} \left( 1 + \frac{3}{K} \right)$$

für eine Sprungantwort  $\Rightarrow 0 < \xi < 1$

$$K > 0 \Rightarrow 0 < \frac{\sqrt{K}}{4} \left( 1 + \frac{3}{K} \right) < 1$$

$$\frac{\sqrt{k}}{4} \left(1 + \frac{3}{k}\right) < 1 \quad / \cdot 4k \quad k > 0$$

$$\sqrt{k(k+3)} < 4k \quad / \left(\quad\right)^2$$

$$\cancel{k}^1 (k^2 + 6k + 9) < 16\cancel{k}^1 \quad / : k (> 0)$$

$$k^2 - 10k + 9 < 0$$

$$1 < k < 9$$

per  $K=1$ ? o per  $K=9$ ? e per  $K < 1$  o  $K > 9$ ?

$$D_F(s) = s^2 + (3+K)s + 4K$$

il valore di  $K$  sono zero:

- $\geq$  poli reali distinti
- $\geq$  poli reali coincidenti
- $\geq$  poli compl. con.

$$s^2 + (3+K)s + 4K = 0$$

$$\Delta = (3+K)^2 - 16K$$

$$\Delta > 0 \quad K^2 - 10K + 9 > 0$$

( $K > 0$ )  $K < 1, K > 9$

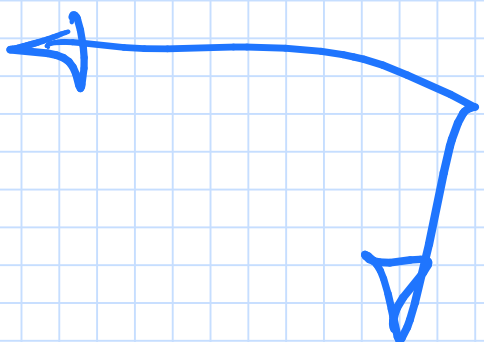
$$\Delta = 0 \quad K = 1, K = 9$$

$$\Delta < 0 \quad 1 < K < 9$$

Pirjoba ② |  $1 < K < 9$

sovadanjetbuic  $\Delta = e^{\left( \frac{-\xi \pi}{\sqrt{1-\xi^2}} \right)}$

max  $\Delta$   
 $\xi \leftarrow \xi(K)$



min  $\xi$   $\frac{\xi \pi}{\sqrt{1-\xi^2}}$

$\xi = \frac{\sqrt{K}}{4} \left( 1 + \frac{3}{K} \right) \Rightarrow$

min  
K

$$\frac{\pi^2 \frac{K}{16} \left(1 + \frac{3}{K}\right)^2}{1 - \frac{K}{16} \left(1 + \frac{3}{K}\right)^2}$$

min  
K

$$\frac{\pi^2 (k^2 + 6k + 9)}{-k^2 + 10k - 9}$$

$$\frac{d}{dk} = 0 \Rightarrow$$

$$\cancel{-2(k+3)}(k-1)(k-3) + (-2)(k+3)\cancel{(k-5)} = 0$$

$$K > 0$$



$$\frac{d(\ )}{dk} = 0 \implies 8k - 24 = 0$$

$$k = 3$$

$$k=3 \implies \xi = \frac{\sqrt{k}}{1+k} \left(1 + \frac{3}{k}\right) = \frac{\sqrt{3}}{2} \approx 0,867$$

foli molto numerosi  
c.c.

$$\Delta_H = \Delta(k=3) = e^{\frac{-3\pi}{\sqrt{1-\xi^2}}} \approx 4,23 \cdot 10^{-3}$$

Per quale valore di  $K$  ottengo  $t_{0.1\%} = 1s$  ?

$$t_{0.1\%} \stackrel{?}{=} \frac{5}{\xi \omega_n}$$