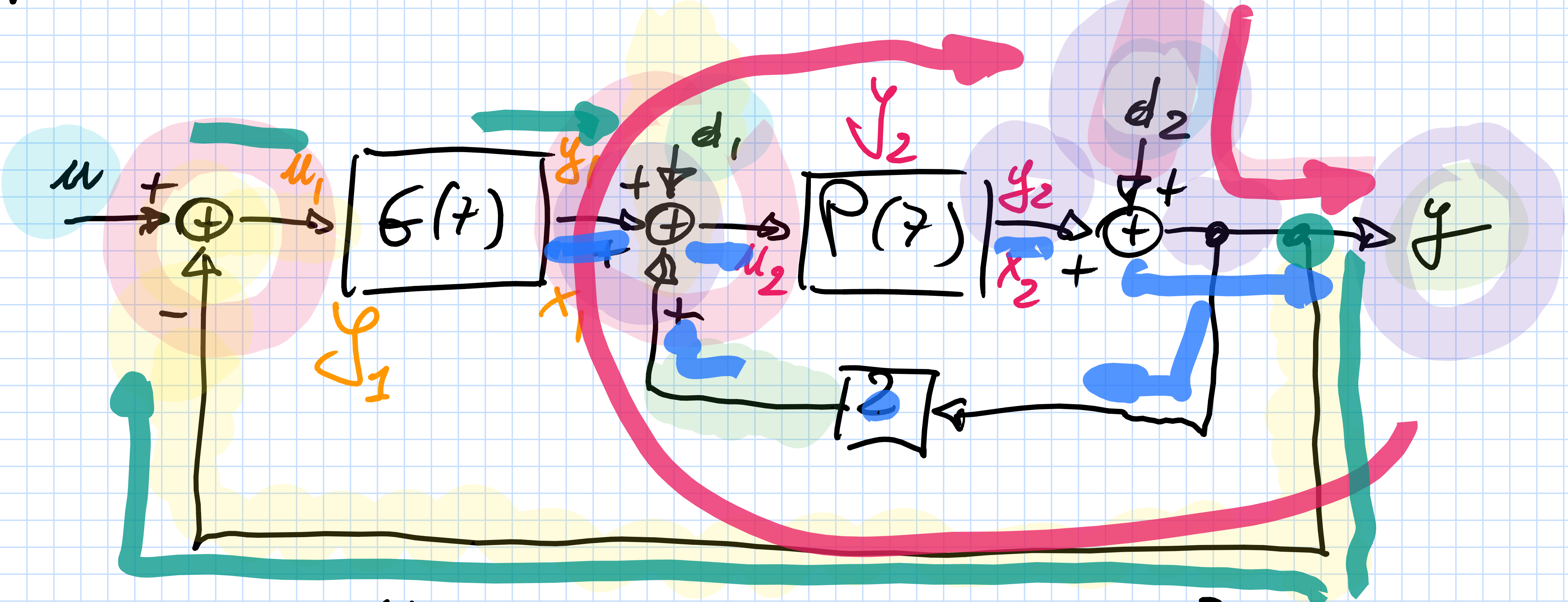
Esercizi su manipolazione di schemi a blocchi

Realizz. in equazioni di stato e manipolazione di schemi a blocchi



$$G(z) = \frac{1/2}{z - 1/10} \qquad P(z) = \frac{z + 2/5}{z + 1/2}$$

① Determinare una realizzazione in eq. di stato

$$\mathcal{S}_1 \qquad u_1 \longrightarrow \boxed{\dfrac{1/2}{z - 1/10}} \xrightarrow[x_1]{y_1}$$

FdT $\quad n = 1$

$\qquad\quad\; m = 0$

$$G(z) = \frac{1/2}{z - 1/10} \qquad\qquad T(z) = \frac{cb}{z - a}$$

$$\begin{cases} a = +\dfrac{1}{10} \\ cb = \dfrac{1}{2} \end{cases} \qquad\qquad \begin{cases} x_1(k+1) = \dfrac{1}{10}\, x_1(k) + \dfrac{1}{2}\, u_1(k) \\ y_1(k) = x_1(k) \end{cases}$$

Σ_2

$$u_2 \longrightarrow \boxed{\frac{z+2/5}{z+1/2}} \longrightarrow y_2$$

x_2

FdT $u=1$

$m=1$

$$P(z) = \frac{z+2/5}{z+1/2} = 1 - \frac{1/10}{z+1/2}$$

$$T(z) = d + \frac{cb}{z-a}$$

$$
\begin{array}{l|l}
z + \frac{2}{5} & z + \frac{1}{2} \\
\hline
-z - \frac{1}{2} & 1 \\
\hline
/ \quad -1/10 &
\end{array}
$$

$$\begin{cases} x_2(k+1) = -\frac{1}{2} x_2(k) + \frac{1}{10} u_2(k) \\ y_2(k) = -x_2(k) + u_2(k) \end{cases}$$

$$\begin{cases} d = +1 \\ a = -\frac{1}{2} \\ cb = -\frac{1}{10} \end{cases}$$

$$x_1(k+1) = -\frac{1}{10}x_1(k) + \frac{1}{2}u_1(k)$$
$$x_2(k+1) = -\frac{1}{2}x_2(k) + \frac{1}{10}u_2(k)$$

$$y_1(k) = x_1(k)$$
$$y_2(k) = -x_2(k) + u_2(k)$$
$$y(k) = d_2(k) + y_2(k)$$

$$u_1(k) = u(k) - y(k)$$
$$u_2(k) = y_1(k) + d_1(k) + 2y(k)$$

Con bici e semplici passaggi

$$
\begin{cases}
x_1(k+1) = \frac{1}{10} x_1(k) + \frac{1}{2} u(k) - \frac{1}{2} y(k) \\
x_2(k+1) = -\frac{1}{2} x_2(k) + \frac{1}{10} \left[-x_1(k) + 2x_2(k) + \right. \\
\qquad \left. - d_1(k) - 2 d_2(k) \right] \\
y(k) = -x_1(k) + x_2(k) - d_1(k) - d_2(k)
\end{cases}
$$

Per casa → terminare le sostituzioni ...

2 variabili di stato → ordine 2

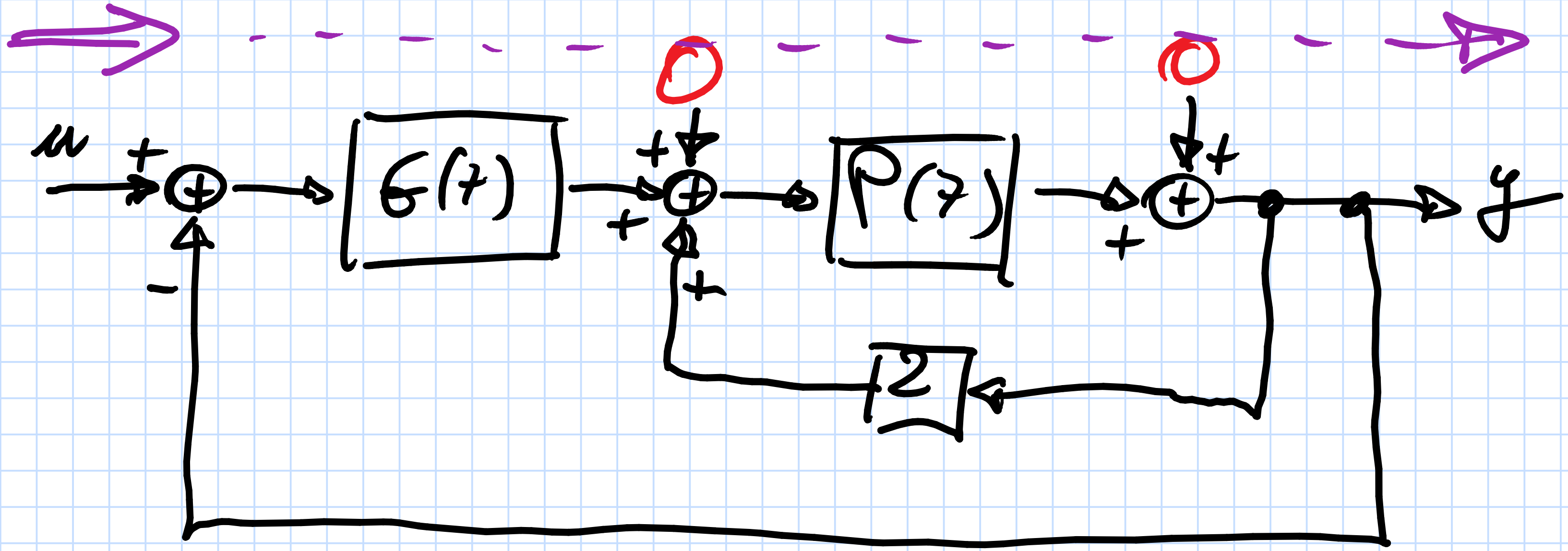
3 ingressi (u, d_1, d_2)

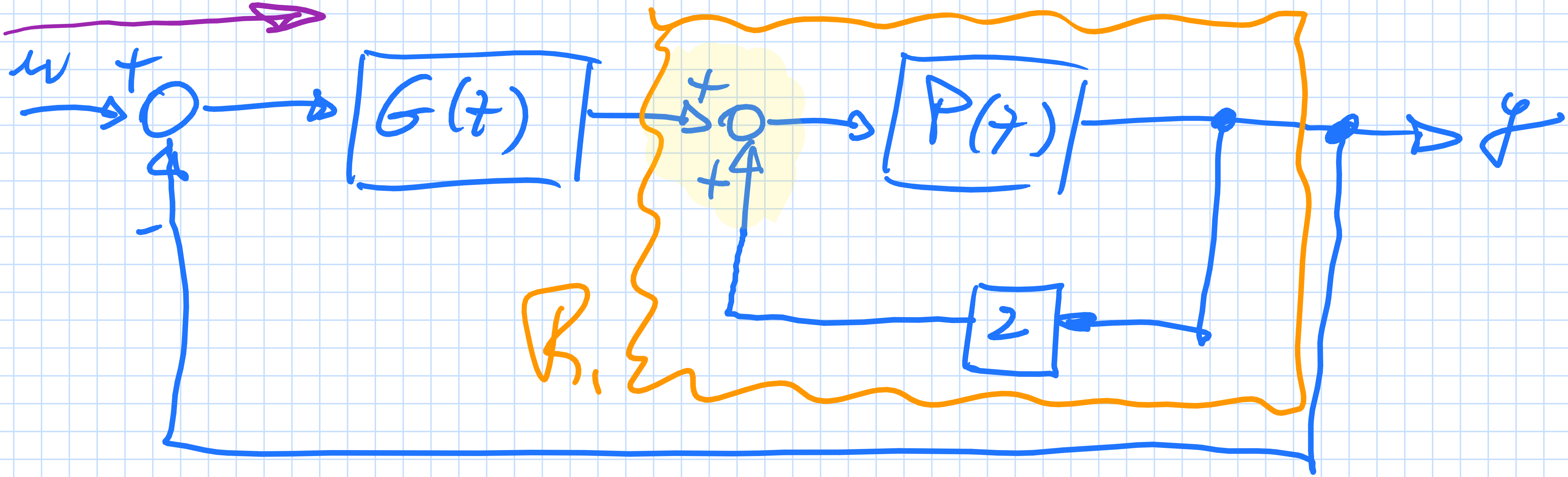
1 uscita y

① Determinare le FdT tra $u(k)$, $d_1(k)$, $d_2(k)$ e l'uscita $y(k)$

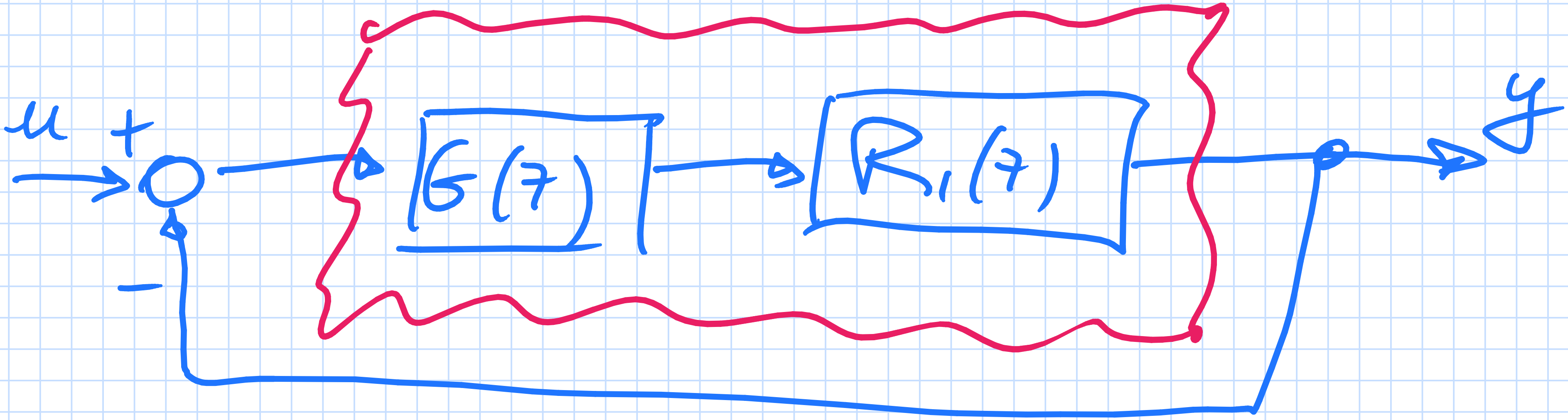
Sovrapposizione degli effetti $\Longleftrightarrow$
- pongo 2 ingressi a zero
- ridisegno lo schema
- cerco le FdT tra l'ingresso rimasto ed y

1° caso FdT tra $u(k)$ e $y(k)$





$$R_1(z) = \frac{P(z)}{1 - 2P(z)}$$



$$T_{u,y}(z) = \frac{G(z) \cdot R_1(z)}{1 + G(z) R_1(z)}$$

$$R \longrightarrow R_1(z) = \frac{P(z)}{1 - 2P(z)}$$

$$= \frac{G(z) P(z)}{1 - 2P(z) + G(z) P(z)} \Leftarrow \begin{cases} G(z) = \dfrac{1/2}{z - 1/10} \\ P(z) = \dfrac{z + 2/5}{z + 1/2} \end{cases}$$

$$1 - P(z)\left[2 - G(z)\right]$$

$$= \frac{-0{,}5\,(z + 0{,}4)}{z^2 - 0{,}3z - 0{,}23}$$

$n = 2$

$m = 1$

$$T_{u,y}(z) = \frac{-0,5(z+0,4)}{z^2-0,3z-0,23}$$

$$u \rightarrow \boxed{T_{u,y}} \rightarrow y$$

$\delta(t)$?

$1(t)$?

$$\mathcal{Z}\{\delta(t)\} = 1$$

risposta all'impulso

$$y(0) = 0$$

$$y(1) = -\frac{1}{2}$$

$$Y(z) = T_{u,y}(z)\, U(z)$$

per $\delta(t) \rightarrow U(z) = 1$

per $1(t) \rightarrow U(z) = \frac{z}{z-1}$

$$Y(z) = T_{u,y}(z) \cdot 1 = \frac{-0{,}5\,(z+0{,}4)}{z^2 - 0{,}3z - 0{,}23}$$

Prof. teorema val iniziale?

$$y(0) = \lim_{z\to+\infty} Y(z) = 0$$

$$y(1) = \lim_{z\to+\infty} \mathcal{Z}\left\{y(k+1)\right\} = \lim_{z\to+\infty} z \cdot Y(z) = -\frac{1}{2}$$

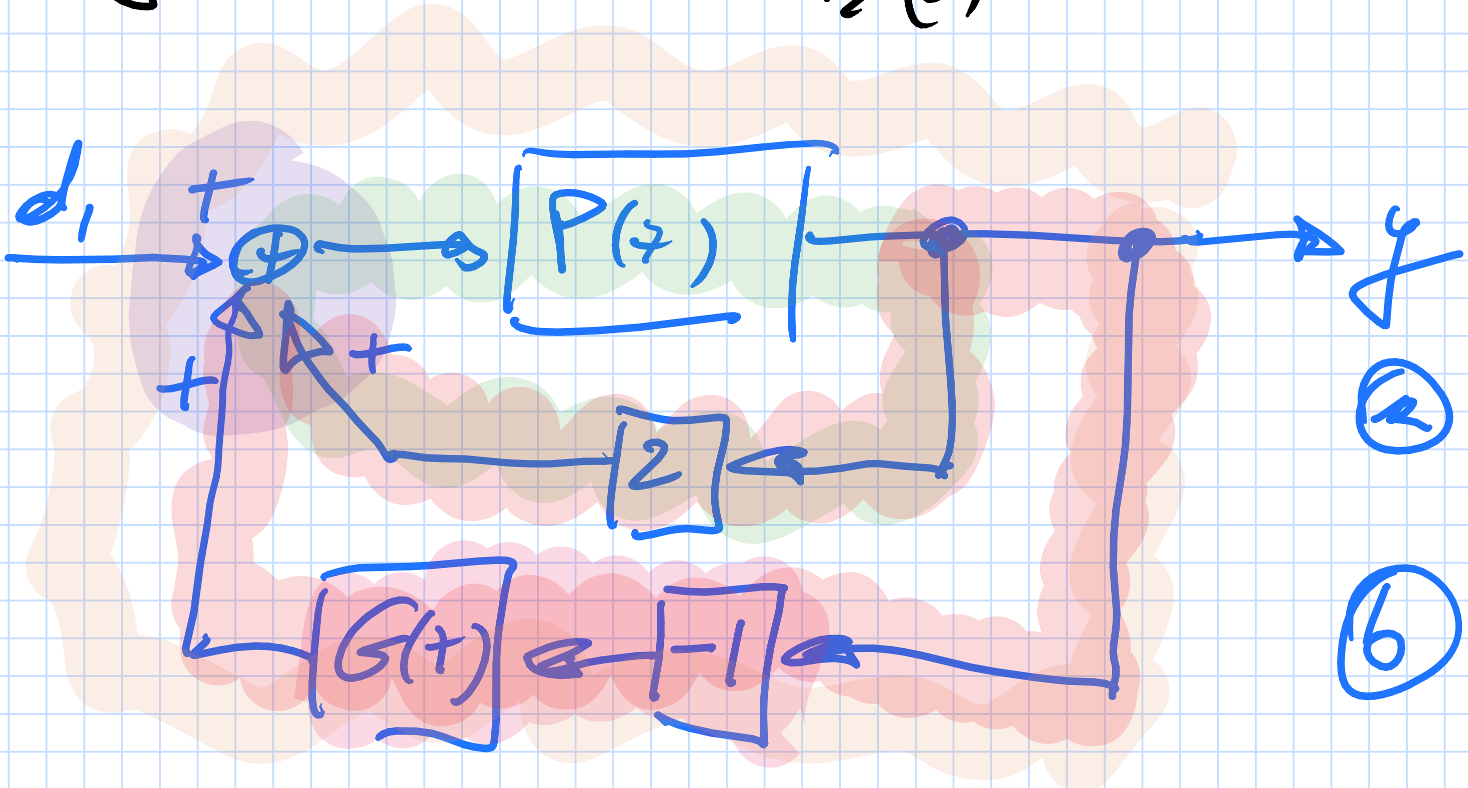
Risposta allo scalino

$y_s(k) \qquad y_s(0) = 0 \qquad y_s(1) \neq 0$

$T_{d_1,y}(z)$

$u(t) \equiv 0$

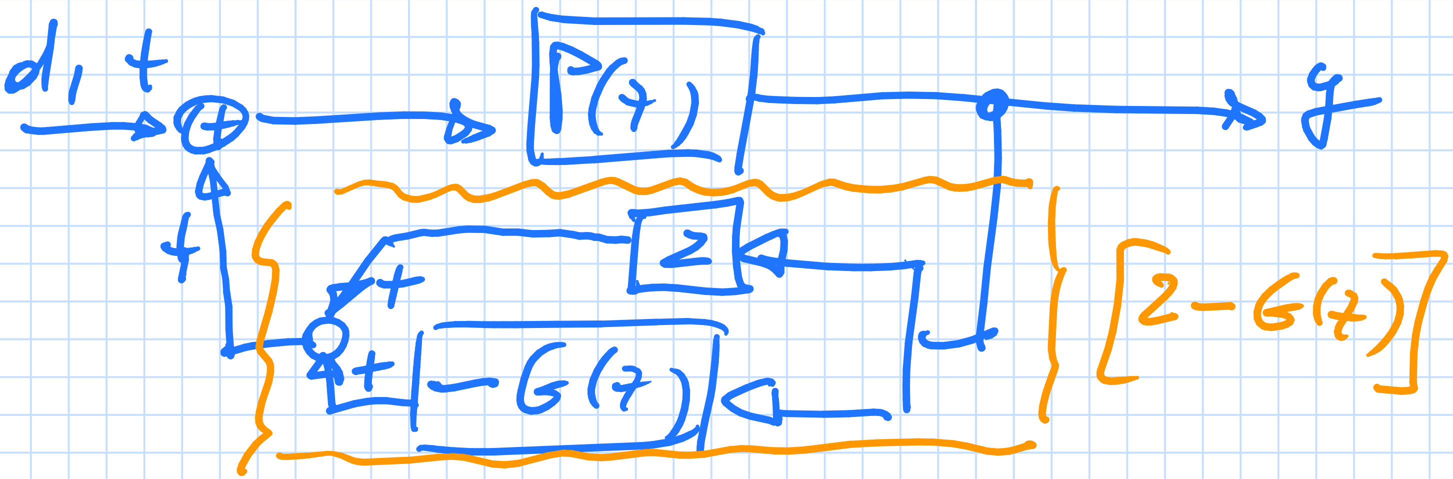
$d_2(t) \equiv 0$

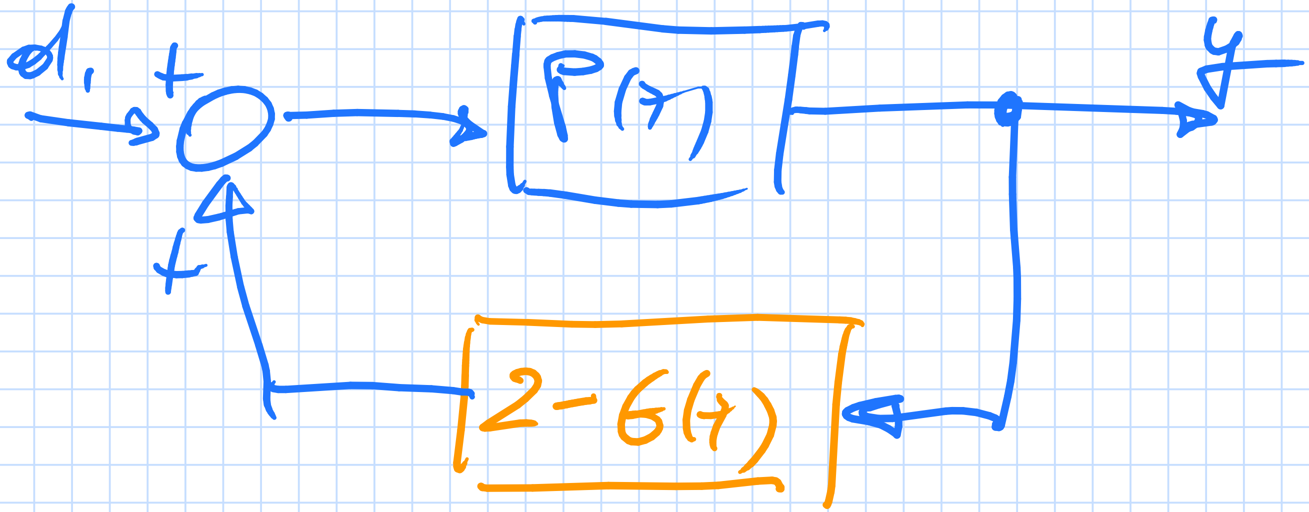


ⓐ 2 retroazioni

ⓑ parallelo + retroazione

ⓑ



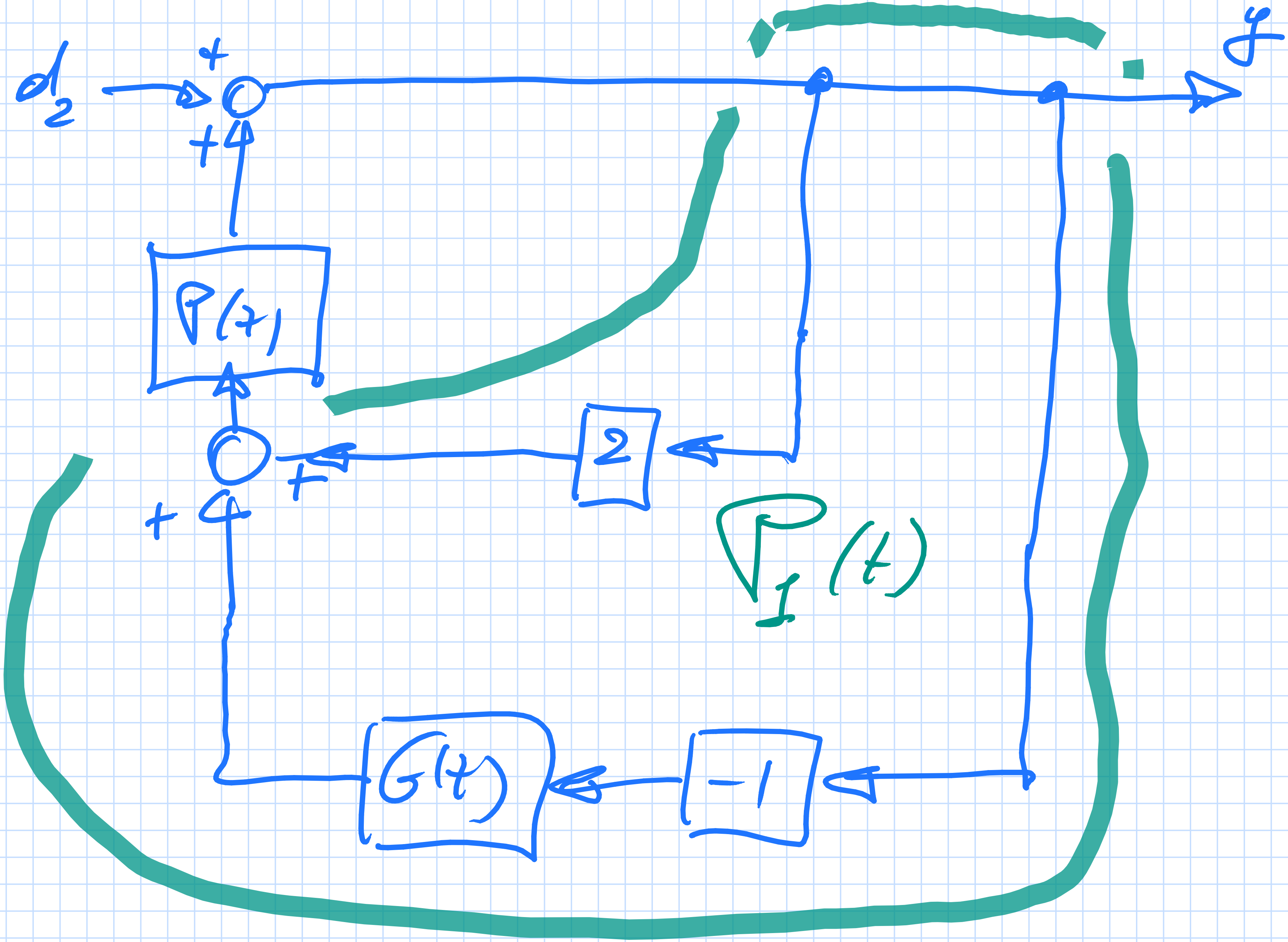


$$T_{d_1 y}(z) = \frac{P(z)}{1 - [2 - G(z)] \cdot P(z)}$$

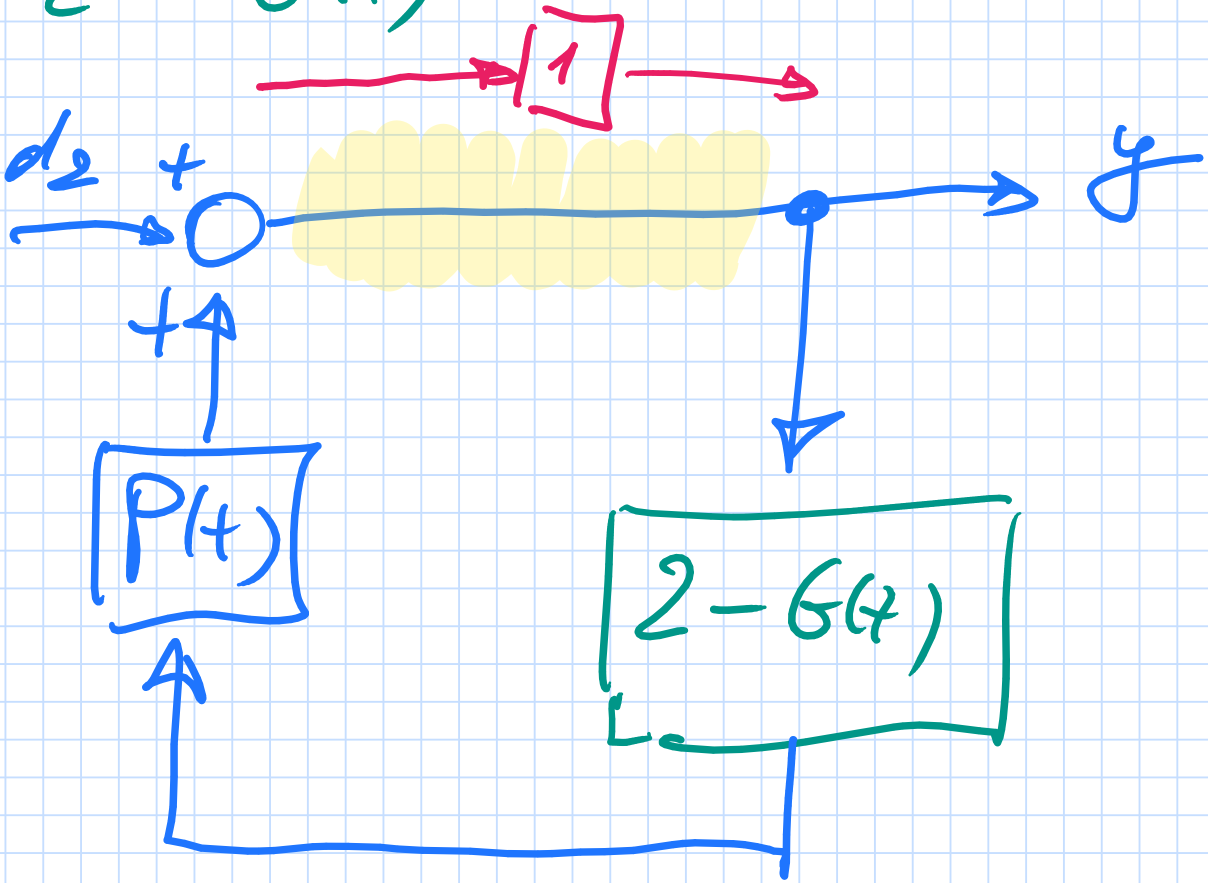
per ottenere l'espressione vanno sostituite le espressioni di $P(z)$ e $G(z)$

$T_{d_2,y}(t)$

$u(t) \equiv 0 \quad d_1 \equiv 0 \quad d_2$

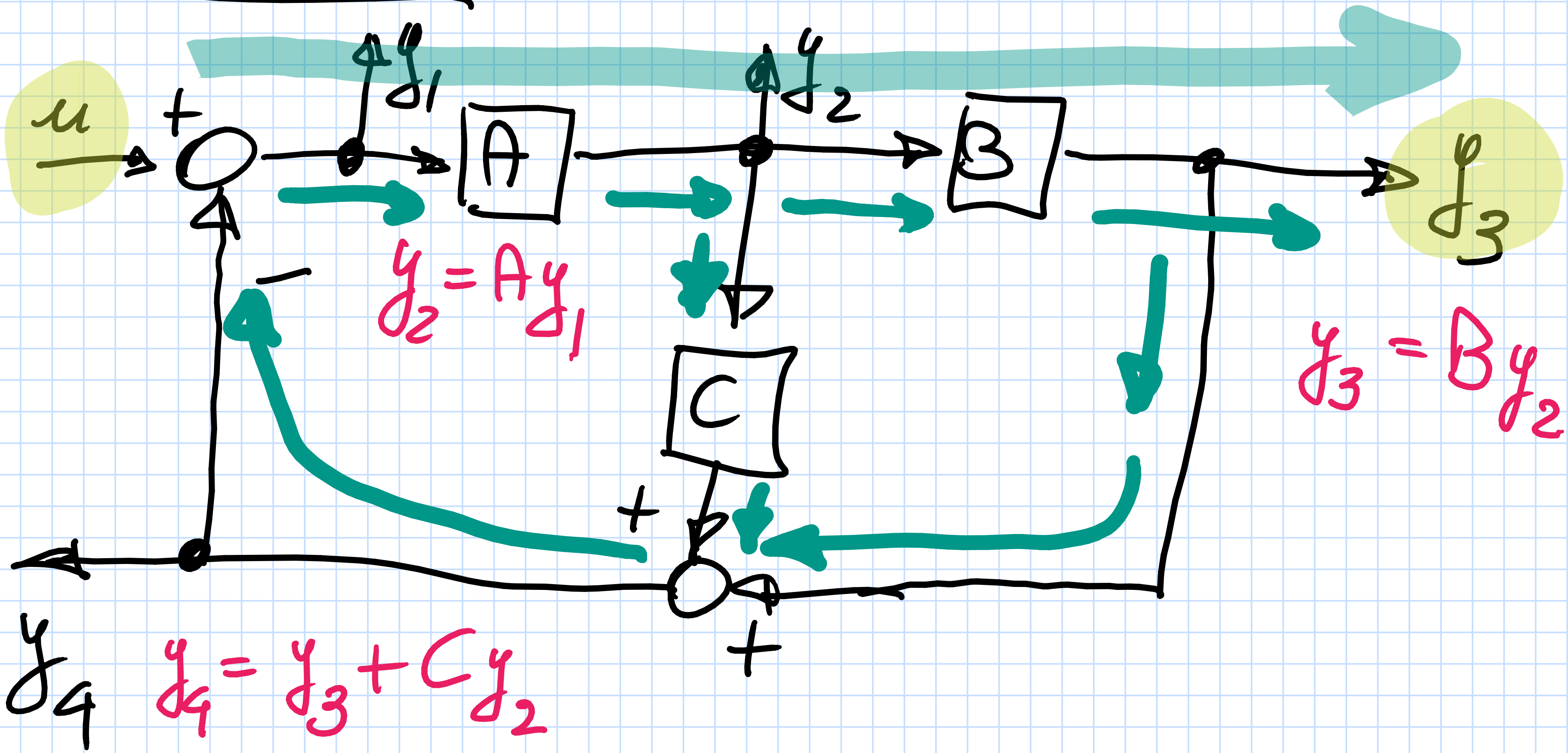


$$\Gamma_1(t) = 2 - G(t)$$



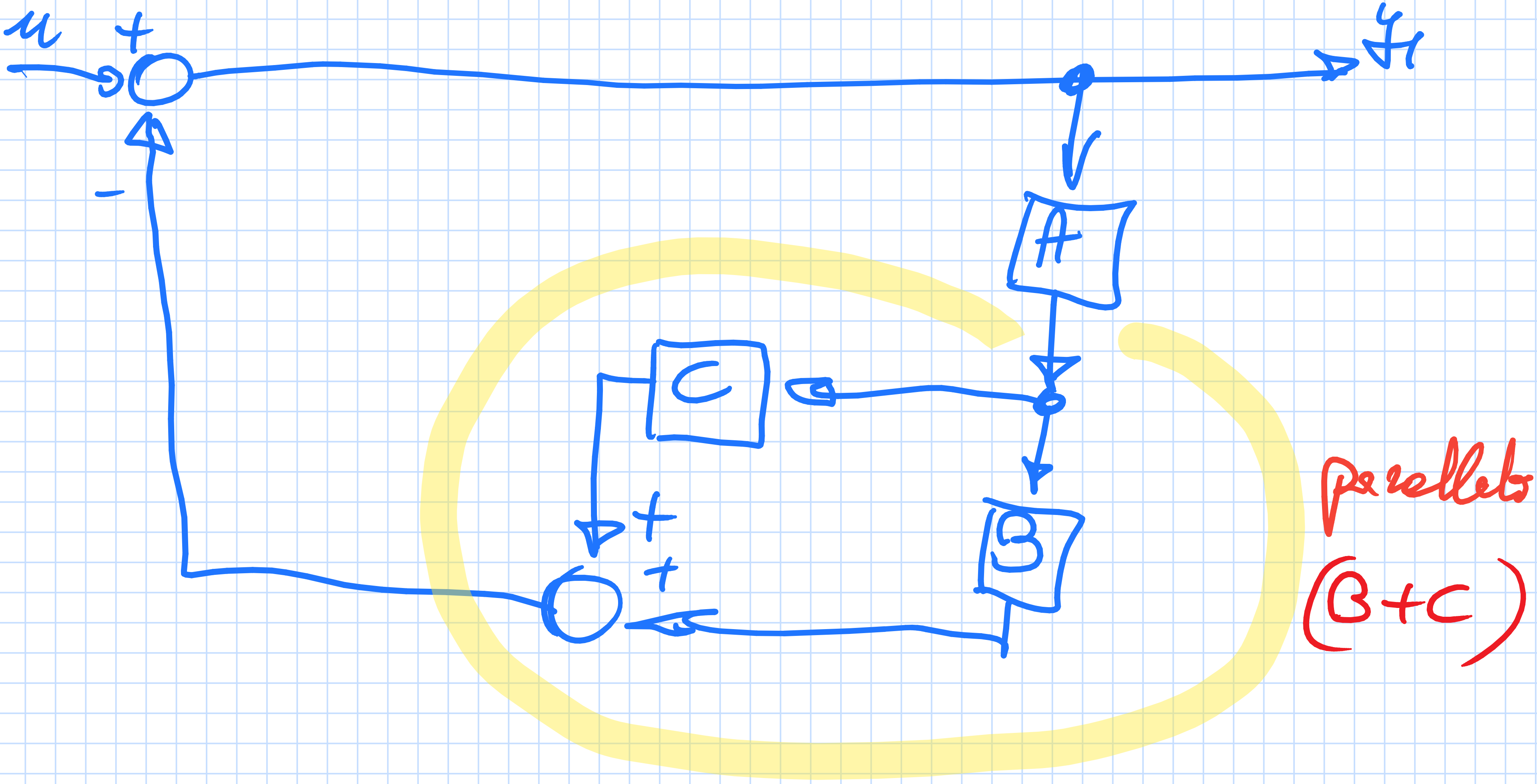
$$T_{d_2 y}(t) = \frac{1}{1 - P(t)\left[2 - G(t)\right]}$$

Es. 1 1/6/2005 | Dato lo schema a blocchi



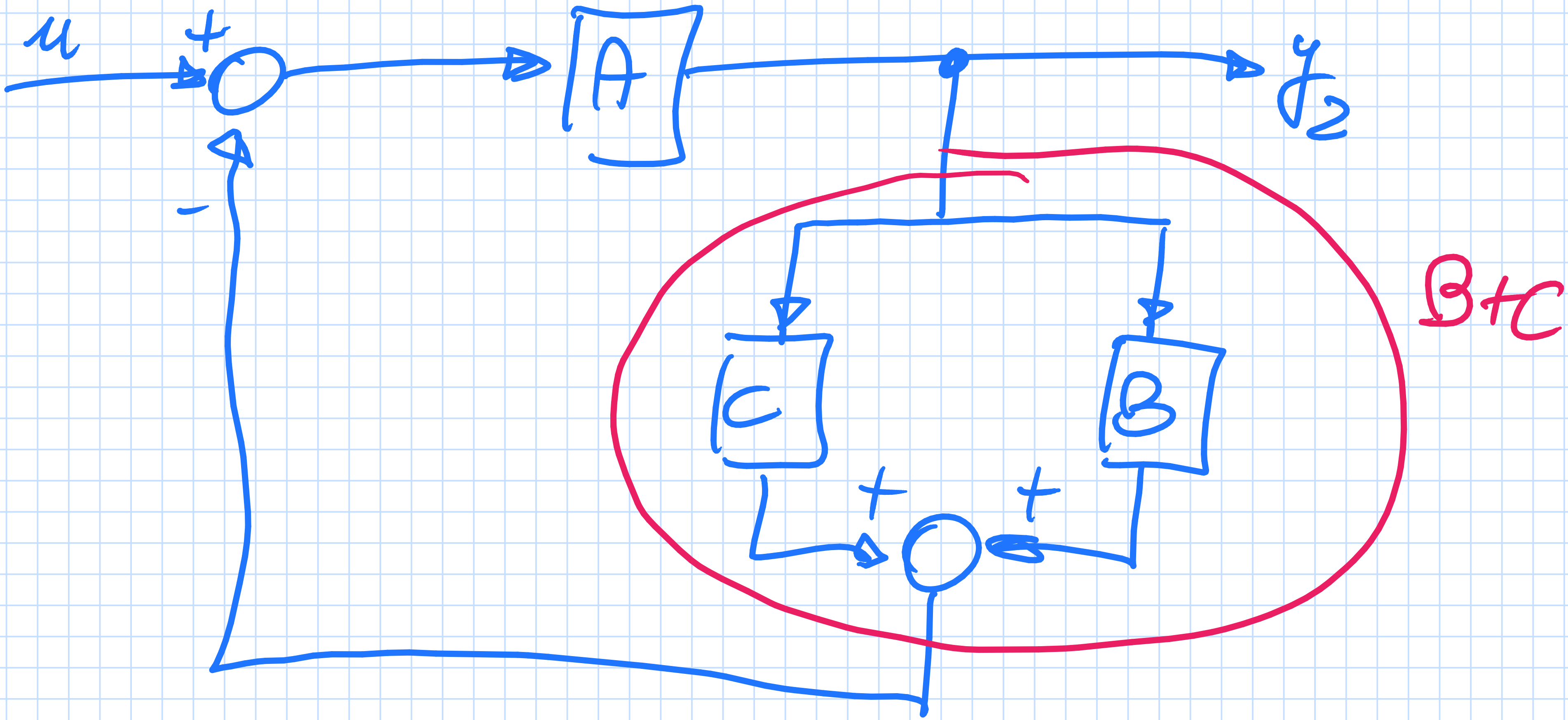
$T_{u,y_1} = ?$ $T_{u,y_2} = ?$ $T_{u,y_3} = ?$ $T_{u,y_4} = ?$

T_1



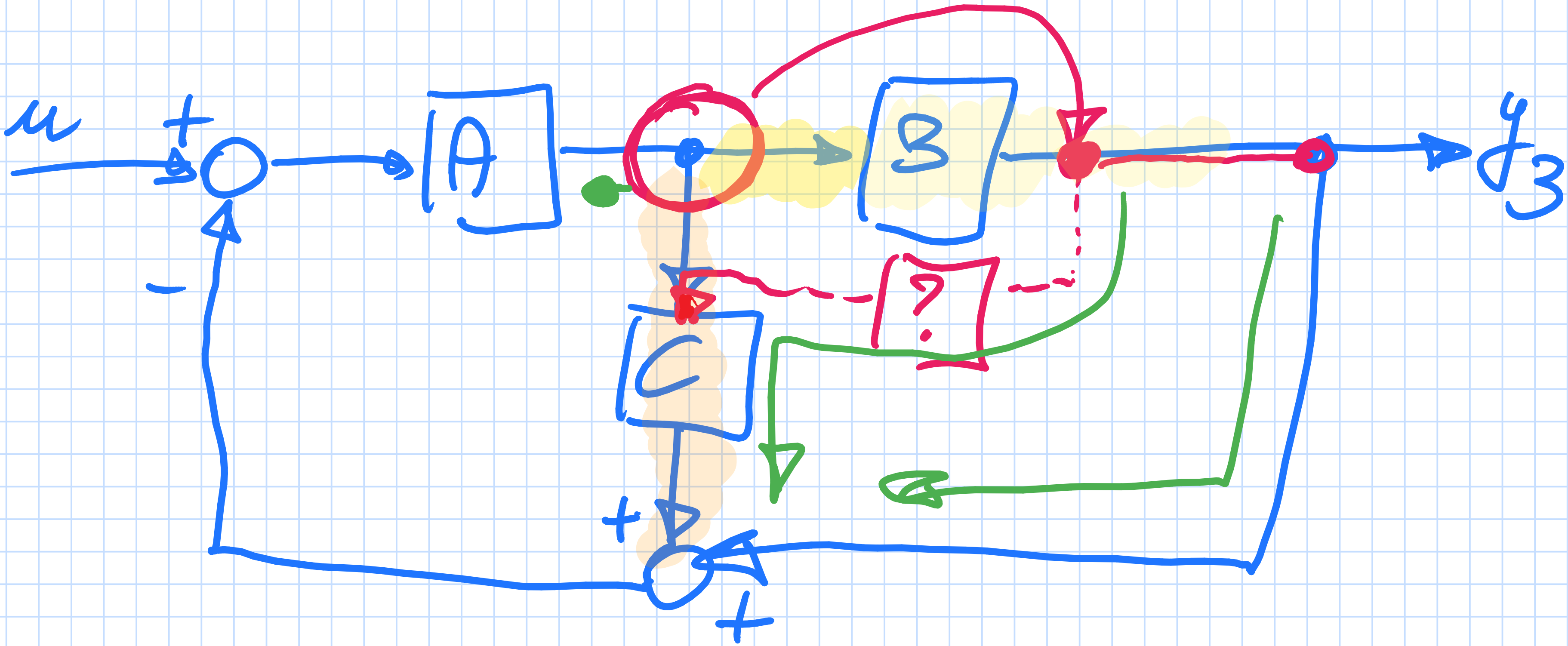
$$T_1 = \frac{1}{1 + A(B+C)}$$

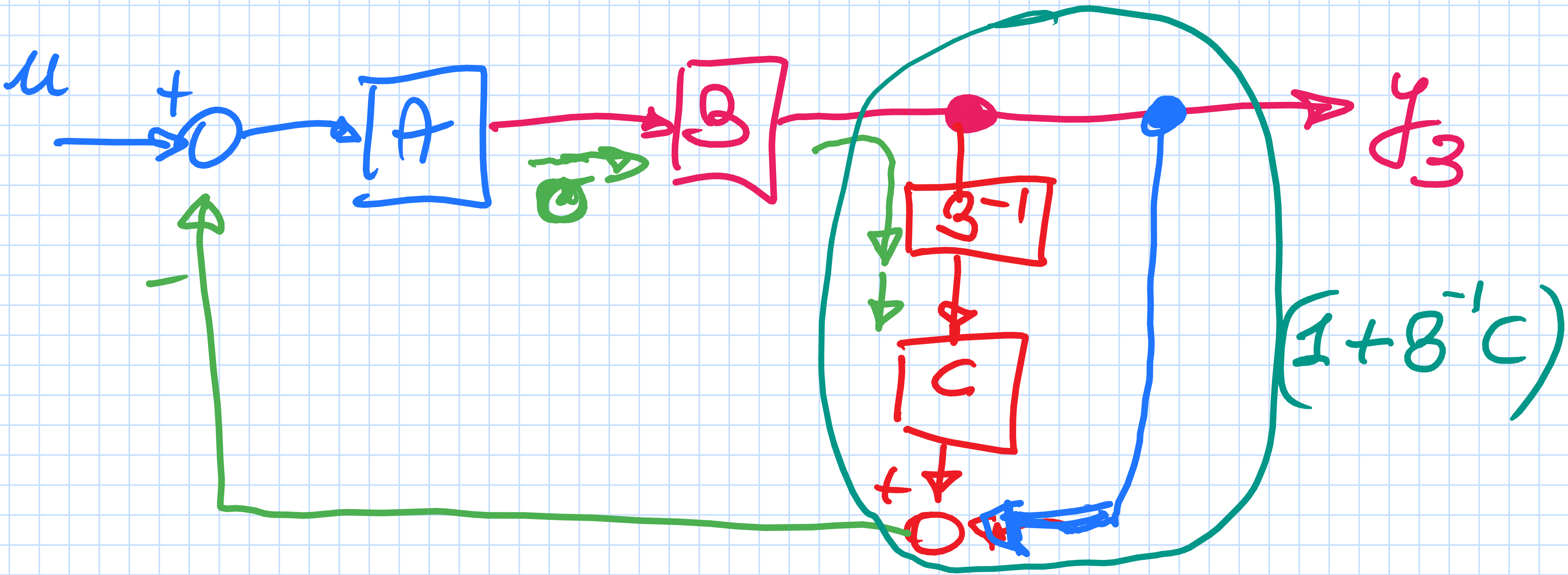
T_2

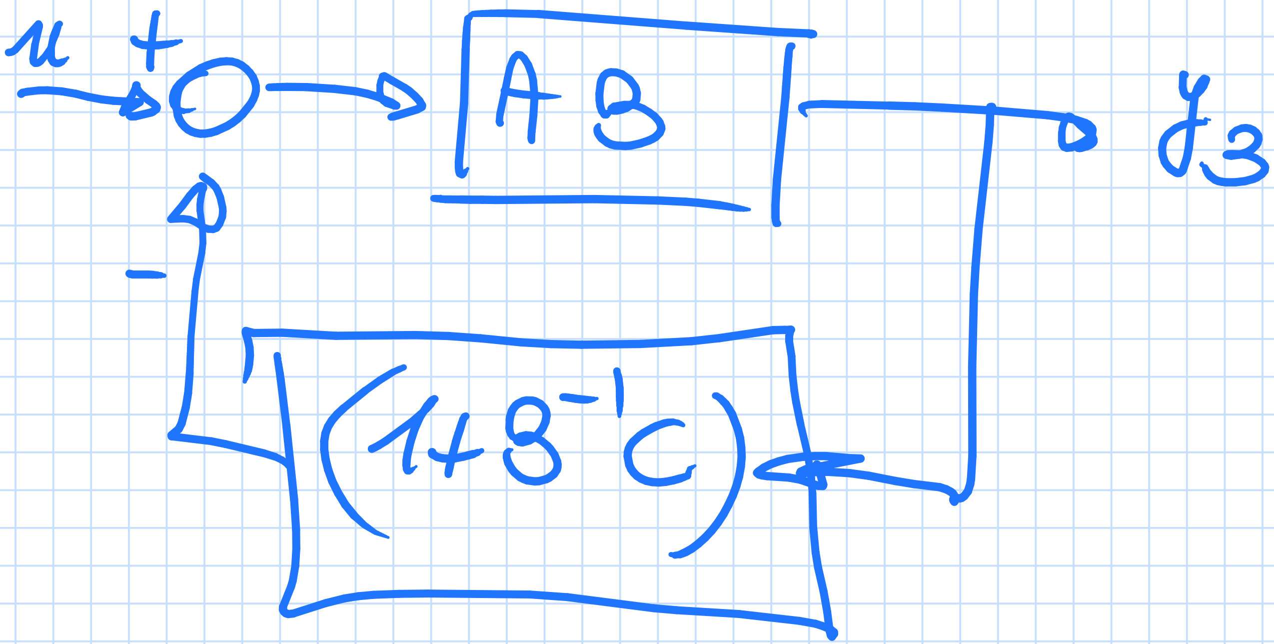


$$T_2 = \frac{A}{1 + A\left[B + C\right]} = A \cdot T_1$$

T_3







$$T_3 = \frac{AB}{1+AB(1+B^{-1}C)} = \frac{AB}{1+A\cancel{B}\cancel{B^{-1}}(B+C)}$$

$$= \frac{AB}{1+A(B+C)} = A \cdot B \cdot T_1$$

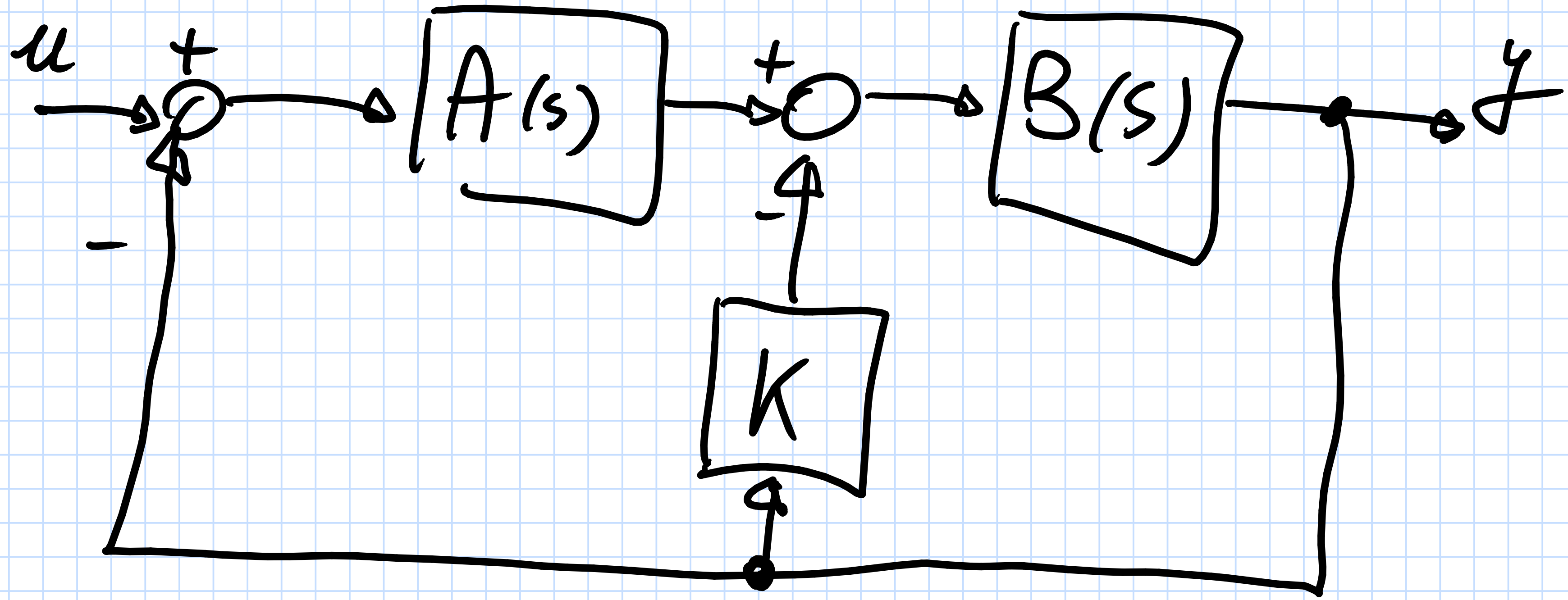
T_4

per caso

$$T_4 = \frac{A(8+C)}{1+A(8+C)}$$

$$y_4 = y_3 + C y_2$$

Per casa



$$A(s) = \frac{2}{s+1} \qquad B(s) = \frac{1}{s-2} \qquad K \in \mathbb{R}$$

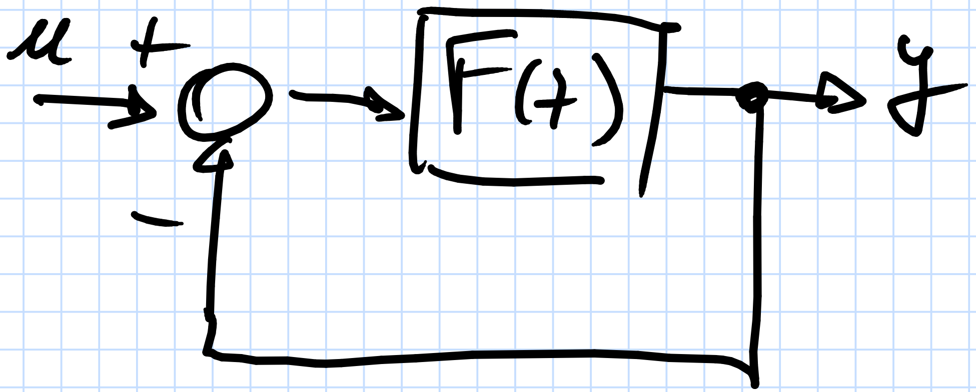
(•) realizz. in sp di ~~stato~~

(•) $T_{u,y}$

$$T_{u,y} = \frac{2}{s^2 + (K-1)s + K}$$

Analisi di stabilità

$$F(z) = \frac{z-a}{(z-1)\left(z-\frac{1}{2}\right)}$$



$a \in \mathbb{R}$

(a) stabilità a ciclo chiuso al variare di $a \in \mathbb{R}$

(b) per $a = +\frac{1}{2}$ $\quad y(k) \xrightarrow[k \to +\infty]{} \ ?$

$$u(k) = 1(k) - 1(k-1) + 1(k-2) - 1(k-3)$$

Sol. (a) $\quad W(z) = \dfrac{F(z)}{1+F(z)}$

$$W(z) = \frac{\dfrac{(z-a)}{(z-1)\left(z-\frac{1}{2}\right)}}{1+\dfrac{z-a}{(z-1)\left(z-\frac{1}{2}\right)}} = \frac{z-a}{z^2-\frac{1}{2}z+\left(\frac{1}{2}-a\right)}$$

$$p(z) = N(z)+D(z) \qquad F(z) = \frac{N(z)}{D(z)}$$

$$p(z) = z^2 - \frac{1}{2}z + \left(\frac{1}{2}-a\right) = 0 \qquad a \in \mathbb{R}$$

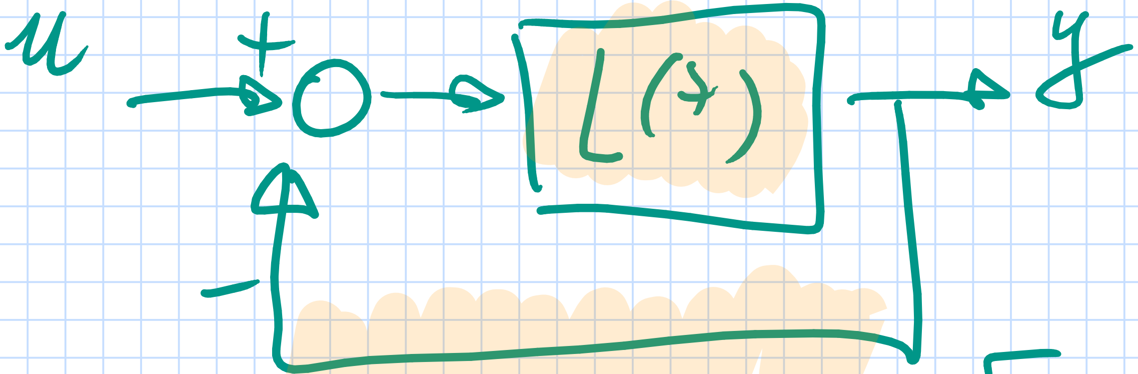
$$|\cdot| \lesseqgtr 1$$

$$z = \frac{w+1}{w-1} \Longrightarrow \left(\frac{w+1}{w-1}\right)^2 - \frac{1}{2}\left(\frac{w+1}{w-1}\right) + \left(\frac{1}{2}-a\right) = 0 \;/ (w-1)^2$$

$$\underbrace{(1-a)w^2+(2a+1)w+(2-a)}_{q(w)}=0$$

Criterio di Routh - Hurwitz

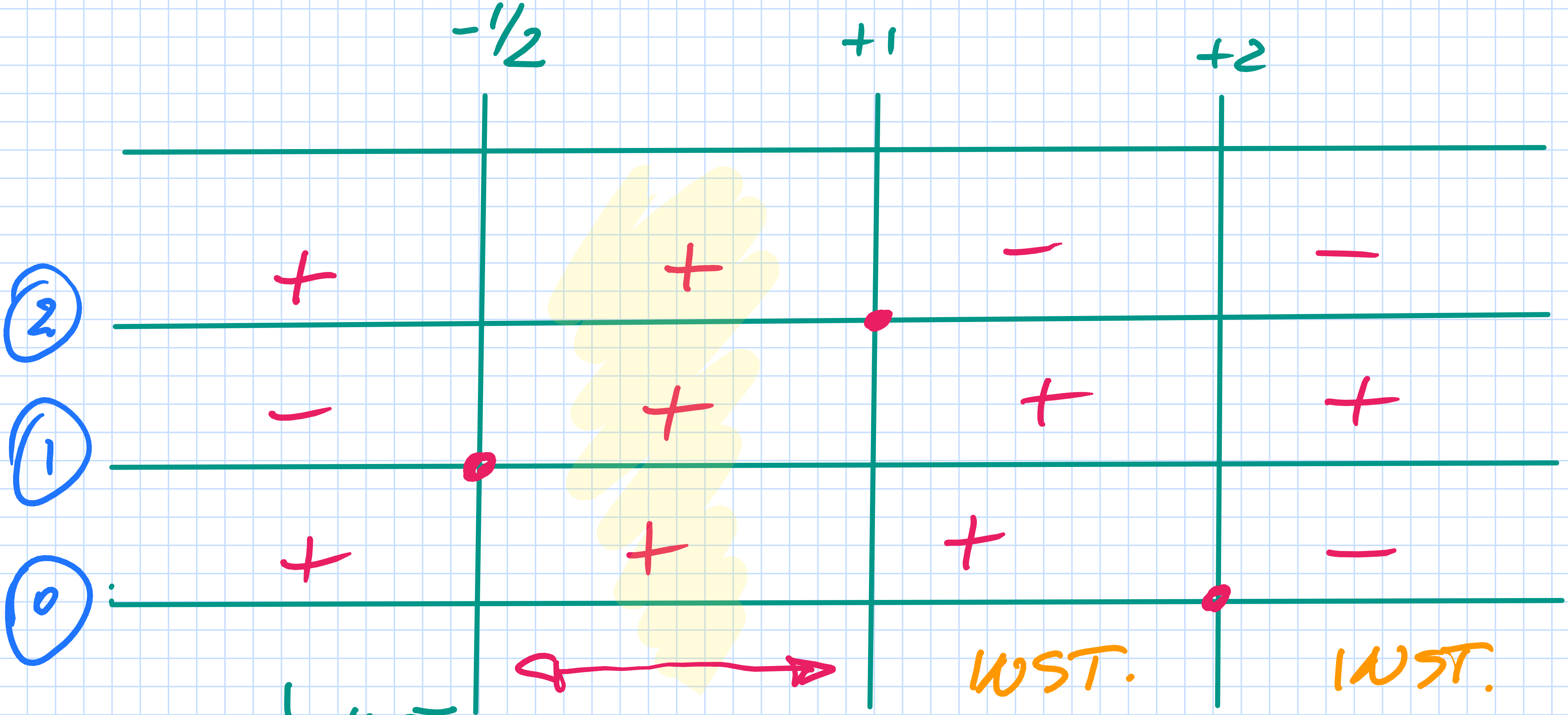
2	$(1-a)$	$(2-a)$	$1-a \geq 0$	$a \leq 1$
1	$(2a+1)$		$2a+1 \geq 0$	$a \geq -\frac{1}{2}$
0	$(2-a)$		$2-a \geq 0$	$a \leq 2$



$$L(z) = \frac{N(z)}{D(z)}$$

$$F(z) = \frac{L(z)}{1 + L(z)}$$

$$F(z) = \frac{\frac{N(z)}{D(z)}}{1 + \frac{N(z)}{D(z)}} = \frac{N(z)}{N(z) + D(z)}$$



$a < -\frac{1}{2}$ INST

$a = -\frac{1}{2}$

2 radici imm. in ω

2 radici $|z_{1,2}| = 1$ STAB semp.

$-\frac{1}{2} < a < +1$

2 permanenze

$\Downarrow$

AS. STAB.

$-\frac{1}{2} < \alpha < 1 \longleftrightarrow$ as. stabile

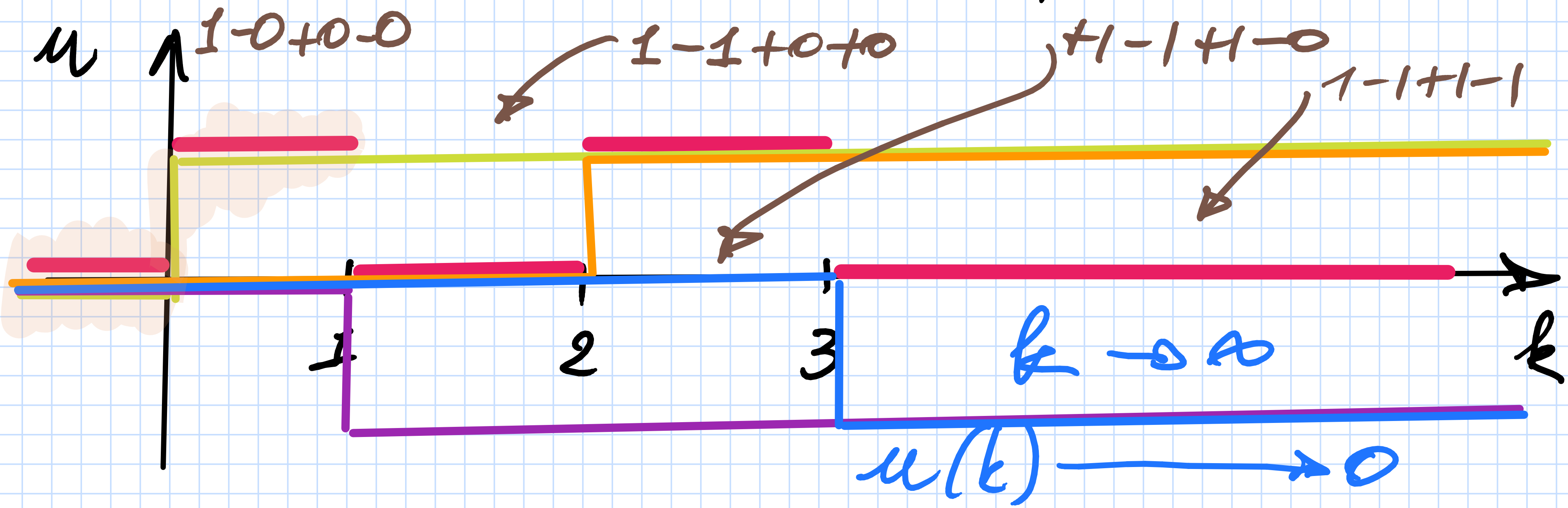
$\alpha = -\frac{1}{2} \longleftrightarrow$ stab. semplice

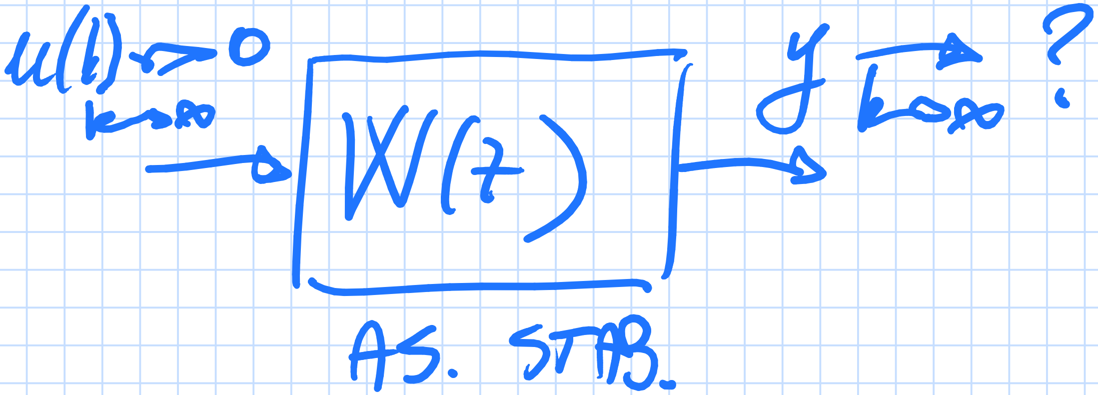
$$\left.\begin{array}{l} \alpha < -\frac{1}{2} \\ \alpha > 1 \end{array}\right\} \longleftrightarrow \text{instabile}$$

(b) $\alpha = \frac{1}{2} \Longrightarrow$ as. stabile

$$u(k) = 1(k) - 1(k-1) + 1(k-2) - 1(k-3)$$

$$u(k) = 1(k) - 1(k-1) + 1(k-2) - 1(k-3)$$





$$0 = y_\infty$$