Fondamenti di Automatica

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Esercizi

Uso della trasformata di Laplace

parte 2

Trasf. Laplace - poli multipli

È nota la trasf. di Laplace della risposta $y(t)$ di un sistema dinamico

$$Y(s) = \frac{2}{s^2(s^2+5s+6)}$$

Determinare l'espressione di $y(t)$

$Y(s)$ zeri $\rightarrow$ NO

poli $\rightarrow$ $p_1 = 0$ doppio

$s^2+5s+6=0 \Rightarrow \begin{array}{l} p_2 = -2 \\ p_3 = -3 \end{array}$ semplici

$$Y(s) = \frac{C_{1,1}}{s} + \frac{C_{1,2}}{s^2} + \frac{C_2}{s+2} + \frac{C_3}{s+3}$$

- formula dei residui

$$C_2 = \lim_{s\to -2} Y(s)(s+2) = \lim_{s\to -2} \frac{2}{s^2\cancel{(s+2)}(s+3)}\cancel{(s+2)} =$$

$$= \frac{2}{4(-2+3)} = \frac{1}{2}$$

$$C_3 = \lim_{s\to -3} Y(s)(s+3) = \lim_{s\to -3} \frac{2}{s^2(s+2)\cancel{(s+3)}}\cancel{(s+3)} =$$

$$= \frac{2}{9(-3+2)} = -\frac{2}{9}$$

$$C_{1,2} = \lim_{s \to 0} Y(s) \cdot s^2 = \lim_{s \to 0} \frac{2}{\cancel{s^2}(s+2)(s+3)} \cdot \cancel{s^2}$$

$$= \frac{2}{6} = 1/3$$

$$C_{1,1} = \lim_{s \to 0} \frac{d}{ds}\left[Y(s)\, s^2\right] =$$

$$\lim_{s \to 0} \frac{d}{ds}\left[\frac{2}{(s+2)(s+3)}\right] =$$

$$C_{1,1} = \lim_{s \to 0} \frac{d}{ds}\left[\frac{2}{(s^2+5s+6)}\right] = \lim_{s \to 0} \frac{-2(2s+5)}{(s+2)^2(s+3)^2}$$

$$= \frac{(\overset{-1}{\cancel{-2}}) \cdot 5}{\underset{2}{\cancel{4}} \cdot 9} = -\frac{5}{18}$$

$$Y(s) = -\frac{5}{18} \cdot \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{s^2} + \frac{1}{2} \cdot \frac{1}{s+2} + \left(-\frac{2}{9}\right) \frac{1}{s+3}$$

$$\frac{1}{s} \to 1(t) \qquad \frac{1}{s^2} \to t \cdot 1(t) \qquad \frac{1}{s+2} \to e^{-2t} \cdot 1(t) \qquad \frac{1}{s+3} \to e^{-3t} \cdot 1(t)$$

$$Y(s) = -\frac{5}{18}\cdot \underset{\underset{1(t)}{\downarrow}}{\frac{1}{s}} + \frac{1}{3}\cdot \underset{\underset{t\cdot 1(t)}{\downarrow}}{\frac{1}{s^2}} + \frac{1}{2}\cdot \underset{\underset{e^{-2t}\cdot 1(t)}{\downarrow}}{\frac{1}{s+2}} + \left(-\frac{2}{9}\right) \underset{\underset{e^{-3t}\cdot 1(t)}{\downarrow}}{\frac{1}{s+3}}$$

$$y(t) = \left[-\frac{5}{18} + \frac{1}{3}t + \frac{1}{2}e^{-2t} - \frac{2}{9}e^{-3t}\right]\cdot 1(t)$$

II modo: principio di identità dei polinomi

$$Y(s) = \frac{C_{1,1}}{s} + \frac{C_{1,2}}{s^2} + \frac{C_2}{s+2} + \frac{C_3}{s+3}$$

$$= \frac{N(s)}{s^2(s^2+5s+6)}$$

4 incognite $\begin{cases} C_{11} \\ C_{12} \\ C_2 \\ C_3 \end{cases}$

$N(s) = 2 \Rightarrow$ 4 equazioni
4 incognite

Soluzione di una eq. differenziale lineare a coeff. costanti utilizzando trasf. Laplace

sistema massa - molla - smorzatore

$$M\ddot{x}(t) + b\dot{x}(t) + k\,x(t) = u(t)$$

$x(t)$ spostamento del carrello

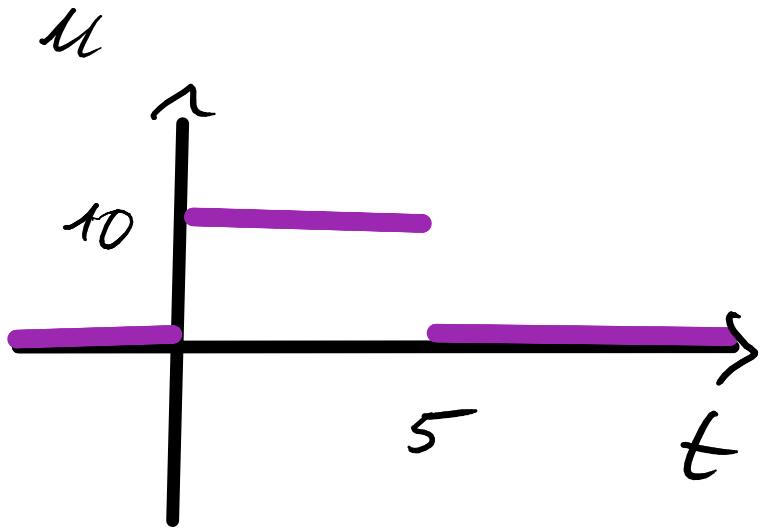
$u(t)$ forza applicata

$x(0) = 0$

$\dot{x}(0) = 0$

$M = 1\,kg$ $\quad b = 2\,Ns/m$ $\quad k = 10\,N/m$

$$u(t) = \begin{cases} 0 & t < 0 \\ 10 & 0 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$



$$\cancel{U(s) = 10\,\frac{1-e^{-5s}}{s}}$$

$$M\left[s^2X(s) - s\cancel{x(0)}^{\,0} - \cancel{\dot{x}(0)}^{\,0}\right] +$$

$$+ b\left[sX(s) - \cancel{x(0)}^{\,0}\right] +$$

$$+ kX(s) = U(s)$$

$$(s^2+2s+10)X(s) = 10\,\frac{1-e^{-5s}}{s}$$

$$X(s)=\frac{10}{s(s^2+2s+10)}\left(1-e^{-5s}\right)$$

$$X(s)=\boxed{\frac{10}{s(s^2+2s+10)}}-\boxed{\frac{10}{s(s^2+2s+10)}\cdot\boxed{e^{-5s}}}$$

$$\Big\downarrow \mathcal{L}^{-1} \qquad\qquad\qquad \Big\downarrow \mathcal{L}^{-1}$$

$$x_1(t)\cdot 1(t) \qquad\qquad\qquad x_1(t-5)\cdot 1(t-5)$$

$$X_1(s) = \frac{10}{s(s^2+2s+10)} = \frac{10}{s\left[(s+1)^2+3^2\right]}$$

$$= \frac{C_1}{s} + \frac{As+B}{(s+1)^2+3^2}$$

$$s^2+2s+10=0$$

$$p_{1,2} = -1 \pm j3$$

$$C_1 = \lim_{s\to 0} X_1(s)\cdot s = 1$$

$$\frac{10}{s(s^2+2s+10)} = \frac{1}{s} + \frac{As+B}{s^2+2s+10}$$

$$= \frac{(s^2+2s+10) + As^2 + Bs}{s(s^2+2s+10)}$$

$$A = -1$$

$$B = -2$$

$$X_1(s) = \frac{1}{s} - \frac{s+2}{(s+1)^2 + 3^2}$$

$$e^{\sigma t}\operatorname{sen}\omega t\cdot 1(t) \longleftrightarrow \frac{\omega}{(s-\sigma)^2+\omega^2}$$

$$e^{\sigma t}\cos\omega t\cdot 1(t) \longleftrightarrow \frac{s-\sigma}{(s-\sigma)^2+\omega^2}$$

$$X_1(s) = \frac{1}{s} - \frac{s+2}{(s+1)^2+3^2}$$

$$s+2 = D(s+1) + E \cdot 3$$

$$= Ds + (D + 3E)$$

$$\begin{cases} D = 1 \\ D + 3E = 2 \end{cases}$$

$$D = 1$$

$$E = 1/3$$

$$X_1(s) = \frac{1}{s} - \left[\frac{s+1}{(s+1)^2+3^2} + \frac{1}{3}\cdot\frac{3}{(s+1)^2+3^2}\right]$$

$$\frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} 1(t)$$

$$\frac{s+1}{(s+1)^2+3^2} \xrightarrow{\mathcal{L}^{-1}} e^{-t}\cos 3t \cdot 1(t)$$

$$\frac{3}{(s+1)^2+3^2} \xrightarrow{\mathcal{L}^{-1}} e^{-t}\,\mathrm{sen}\,3t \cdot 1(t)$$

$$x_1(t) = \left\{1 - e^{-t}\left[\cos 3t + \frac{1}{3}\,\mathrm{sen}\,3t\right]\right\}\cdot 1(t)$$

$$x(t) = x_1(t)\cdot 1(t) - x_1(t-5)\cdot 1(t-5)$$

$$x(t) = \left[1 - e^{-t}\left(\cos 3t + \frac{1}{3}\operatorname{sen} 3t\right)\right]\cdot 1(t) +$$

$$- \left[1 - e^{-(t-5)}\left(\cos 3(t-5) + \frac{1}{3}\operatorname{sen} 3(t-5)\right)\right]\cdot 1(t-5)$$