Fondamenti di Automatica

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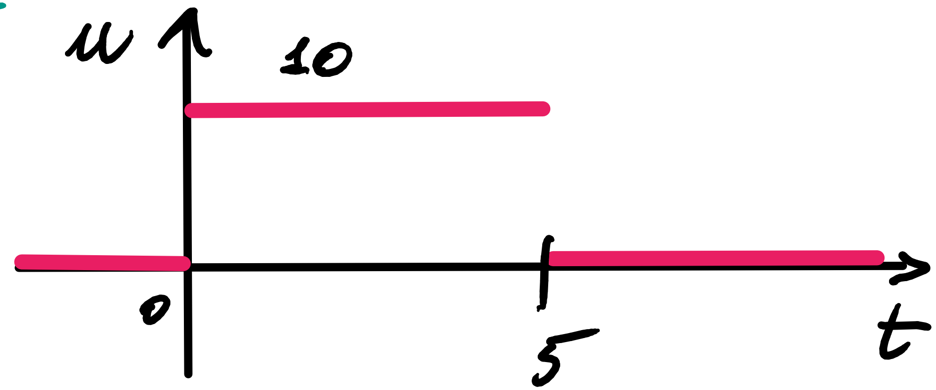
Esercizio: Trasformata di Laplace

Determinare la Trasf. di Laplace dell'ingresso $u(t)$

$$u(t) = \begin{cases} 0 & t < 0 \\ 10 & 0 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$



$u(t) \Rightarrow$ combinazione lineare di segnali elementari (scalino, rampa, sinusoidi ecc.)

$$u(t) = 10 \cdot 1(t) - 10 \cdot 1(t-5) \qquad \frac{1}{s} \cdot e^{-5s}$$

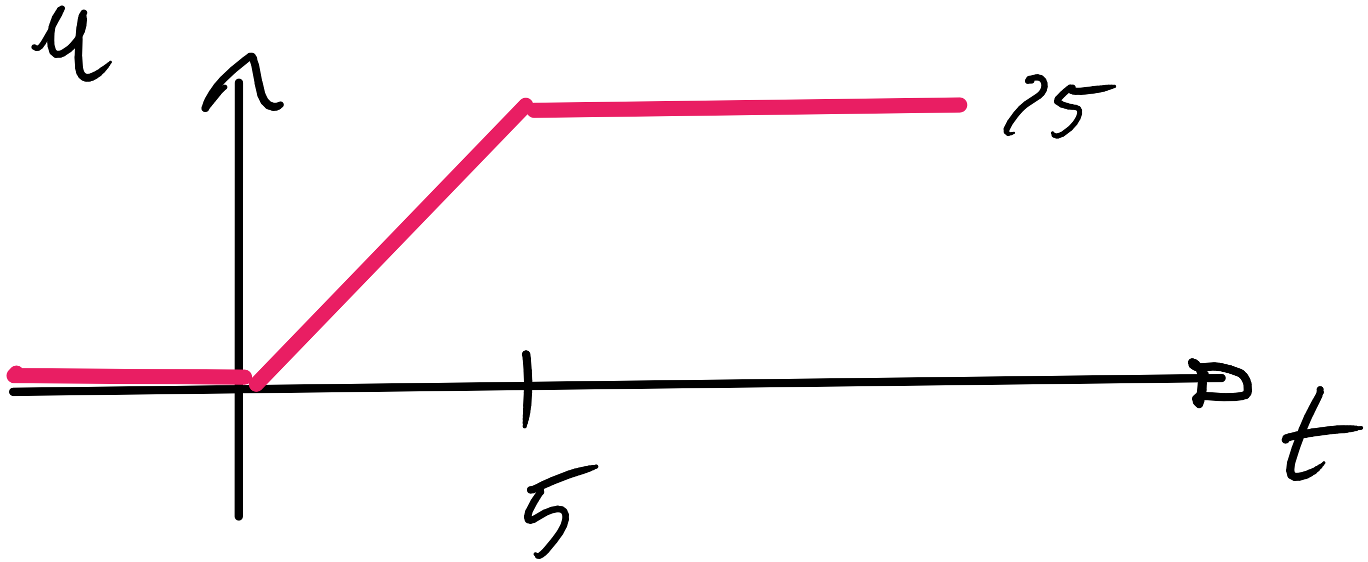
$$\mathcal{L}\{u(t)\} \underset{\text{linearità}}{=} 10\,\underbrace{\mathcal{L}\{1(t)\}}_{\frac{1}{s}} - 10\,\underbrace{\mathcal{L}\{1(t-5)\}}_{\text{trasl. nel tempo}}$$

$$U(s) = 10 \cdot \frac{1}{s} - 10 \cdot \frac{1}{s} e^{-5s}$$

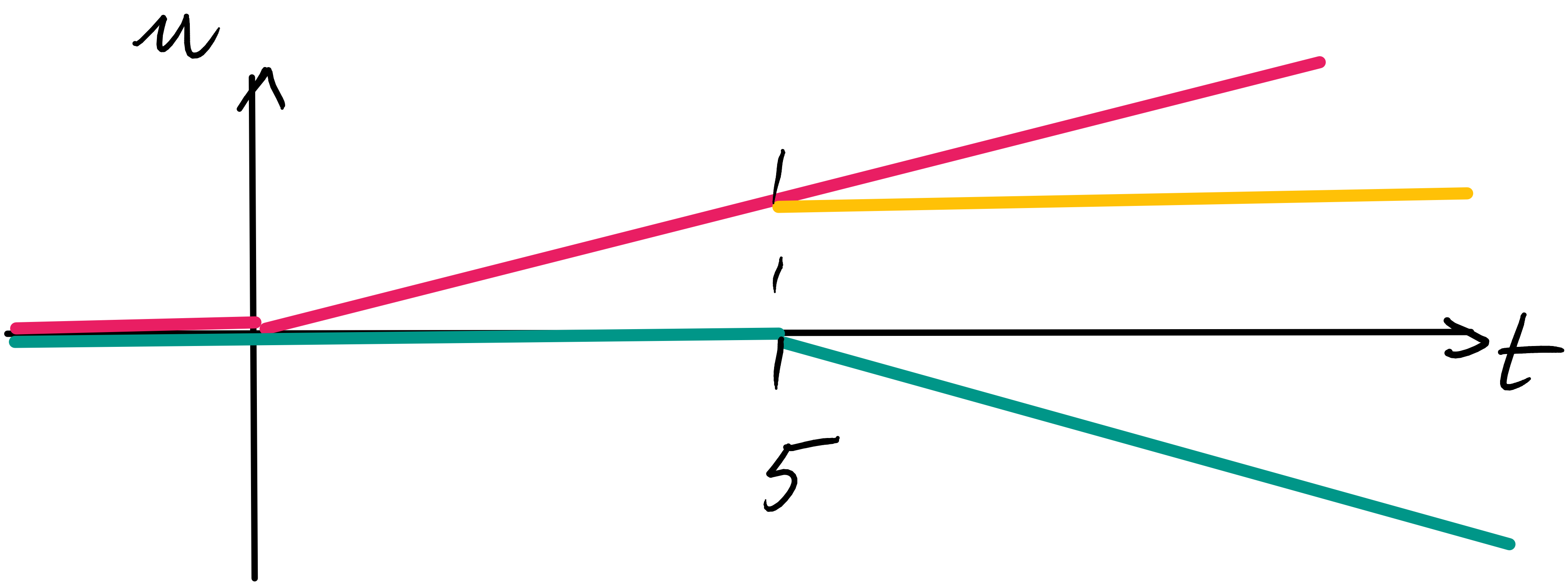
$$= 10 \, \frac{1 - e^{-5s}}{s}$$

• Determinare la Trasf. di Laplace di

$$u(t) = \begin{cases} 0 & t < 0 \\ 5t & 0 \le t < 5 \\ 25 & t \geq 5 \end{cases}$$



$$u(t) = 5t \cdot 1(t) - 5(t-5) \cdot 1(t-5)$$



linearità

$$\mathcal{L}\{u(t)\} = 5\mathcal{L}\{t \cdot 1(t)\} - 5\mathcal{L}\{\underbrace{(t-5) \cdot 1(t-5)}_{\text{trasl. nel tempo}}\}$$

$$\mathcal{L}\{u(t)\} = 5 \cdot \frac{1}{s^2} - 5\,\mathcal{L}\{(t-5)\cdot 1(t-5)\}$$

trasl. nel tempo

$$\frac{1}{s^2} \cdot e^{-5s}$$

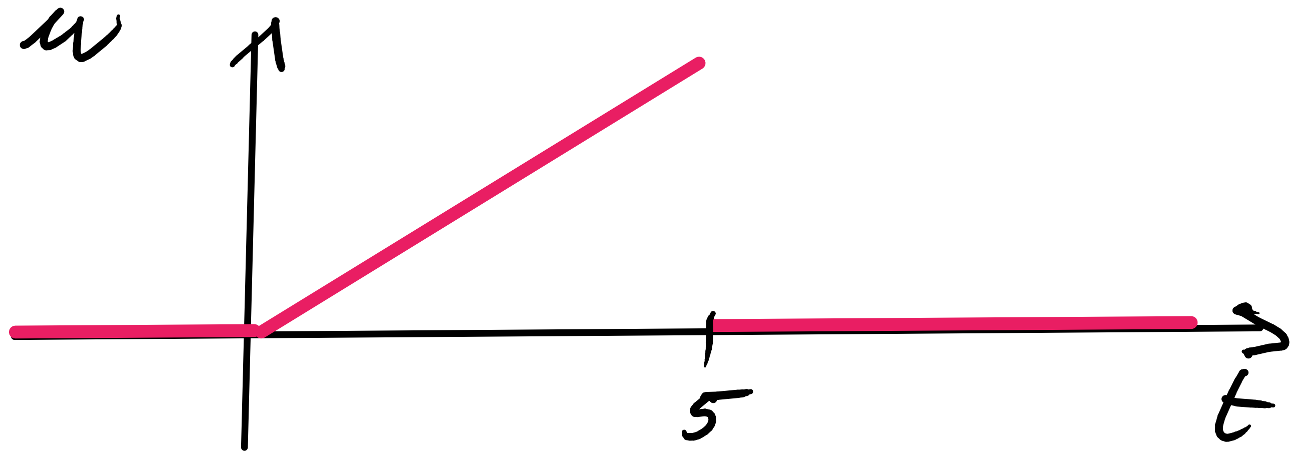
$$U(s) = 5 \cdot \frac{1-e^{-5s}}{s^2}$$

Homework

Determinare la trasf. di Laplace di

$$u(t) = \begin{cases} 0 & t < 0 \\ 5t & 0 \le t < 5 \\ 0 & t \geq 5 \end{cases}$$

$U(s) = ?$



Trasf. di Laplace: completamento dei quadrati

Data

$$G(s) = \frac{1}{s^2+2s+10}$$

determinare $g(t) = \mathcal{L}^{-1}\{G(s)\}$

(1) poli di $G(s)$ $\qquad s^2+2s+10=0$

$$p_{1,2} = \frac{-1 \pm \sqrt{1-10}}{1} = -1 \pm 3j$$

$$P_1 = -1 - 3j \qquad \sigma \pm j\omega$$

$$P_2 = -1 + 3j \qquad \sigma = -1 \qquad \omega = 3$$

$$s^2 + 2s + 10 = (\underbrace{s+1} + \overbrace{3j})(\underbrace{s+1} - \overbrace{3j})$$

$$a + b \qquad a - b \qquad a^2 - b^2$$

$$= (s+1)^2 - (3j)^2 = (s+1)^2 + 3^2$$

$$G(s) = \frac{1}{(s+1)^2 + 3^2}$$

$$\frac{\omega}{(s-\sigma)^2 + \omega^2}$$

$$\mathcal{L}\left\{ e^{\sigma t} \operatorname{sen} \omega t \cdot 1(t) \right\}$$

$$= \frac{1}{3} \cdot \frac{3}{(s+1)^2 + 3^2}$$

$$g(t) = \frac{1}{3} e^{-t} \operatorname{sen} 3t \cdot 1(t)$$

$$G(s) = \frac{c_1}{s-p_1} + \frac{c_1^*}{s-p_1^*}$$

$$p_1 = -1-3j$$

$$p_2 = -1+3j = p_1^*$$

$$c_1 = \lim_{s \to p_1} G(s)(s-p_1)$$

$$= \lim_{s \to -1-3j} \frac{1}{\cancel{(s+1+3j)}(s+1-3j)} \cancel{(s+1+3j)}$$

$$= j\frac{1}{6}$$

$$c_1^* = -j\frac{1}{6}$$

$$G(s) = \frac{j/6}{s-p_1} - \frac{j/6}{s-p_1^*}$$

$$\frac{1}{s-p}$$

$$\updownarrow$$

$$e^{pt} \cdot 1(t)$$

$$g(t) = \dots$$

$$= \frac{1}{3} e^{-t} \operatorname{sen} 3t \cdot 1(t)$$

$$X(s) = \frac{10}{s(s^2+2s+10)} = \frac{C_1}{s} + \frac{C_2}{s-p_1} + \frac{C_2^*}{s-p_1^*}$$

$$[(s+1)^2+3^2]$$

$$C_1 = \lim_{s \to 0} X(s) \cdot s = \lim_{s \to 0} \frac{10}{\cancel{s}(s^2+2s+10)} \cdot \cancel{s} \overset{!}{=} 1$$

$$X(s) = \frac{1}{s} + \frac{As+B}{(s+1)^2+3^2} = \frac{10}{s\left[(s+1)^2+3^2\right]}$$

$$\frac{s^2+2s+10+As^2+Bs}{s\left[(s+1)^2+3^2\right]} = \frac{10}{s\left[(s+1)^2+3^2\right]}$$

$$\begin{cases} 1+A=0 \\ 2+B=0 \end{cases} \qquad 10=10$$

$$\begin{cases} 1+A=0 & A=-1 \\ 2+B=0 & B=-2 \end{cases}$$

$$X(s)=\frac{1}{s}-\frac{s+2}{(s+1)^2+3^2}$$

$$\frac{\omega}{(s-\sigma)^2+\omega^2} \longleftrightarrow e^{\sigma t}\,\mathrm{sen}\,\omega t\cdot \mathbb{1}(t)$$

$$\frac{(s-\sigma)}{(s-\sigma)^2+\omega^2} \longleftrightarrow e^{\sigma t}\cos\omega t\cdot \mathbb{1}(t)$$

$$s+2 = K_1(s+1) + K_2\,3$$
$$= K_1 s + (K_1 + 3K_2)$$

$$\begin{cases} K_1 = 1 \\ K_1 + 3K_2 = 2 \end{cases} \qquad \begin{aligned} K_1 &= 1 \\ K_2 &= 1/3 \end{aligned}$$

$$\frac{s+2}{(s+1)^2+3^2} = \frac{(s+1) + 1/3 \cdot 3}{(s+1)^2 + 3^2}$$

$$X(s) = \frac{1}{s} - \left[\frac{s+1}{(s+1)^2 + 3^2} + \frac{1}{3} \cdot \frac{3}{(s+1)^2 + 3^2} \right]$$

$$1(t) \qquad e^{-t}\cos 3t \cdot 1(t) \qquad e^{-t}\operatorname{sen} 3t \cdot 1(t)$$

$$x(t) = \left[1 - e^{-t}\left(\cos 3t + \frac{1}{3}\operatorname{sen} 3t \right) \right] \cdot 1(t)$$