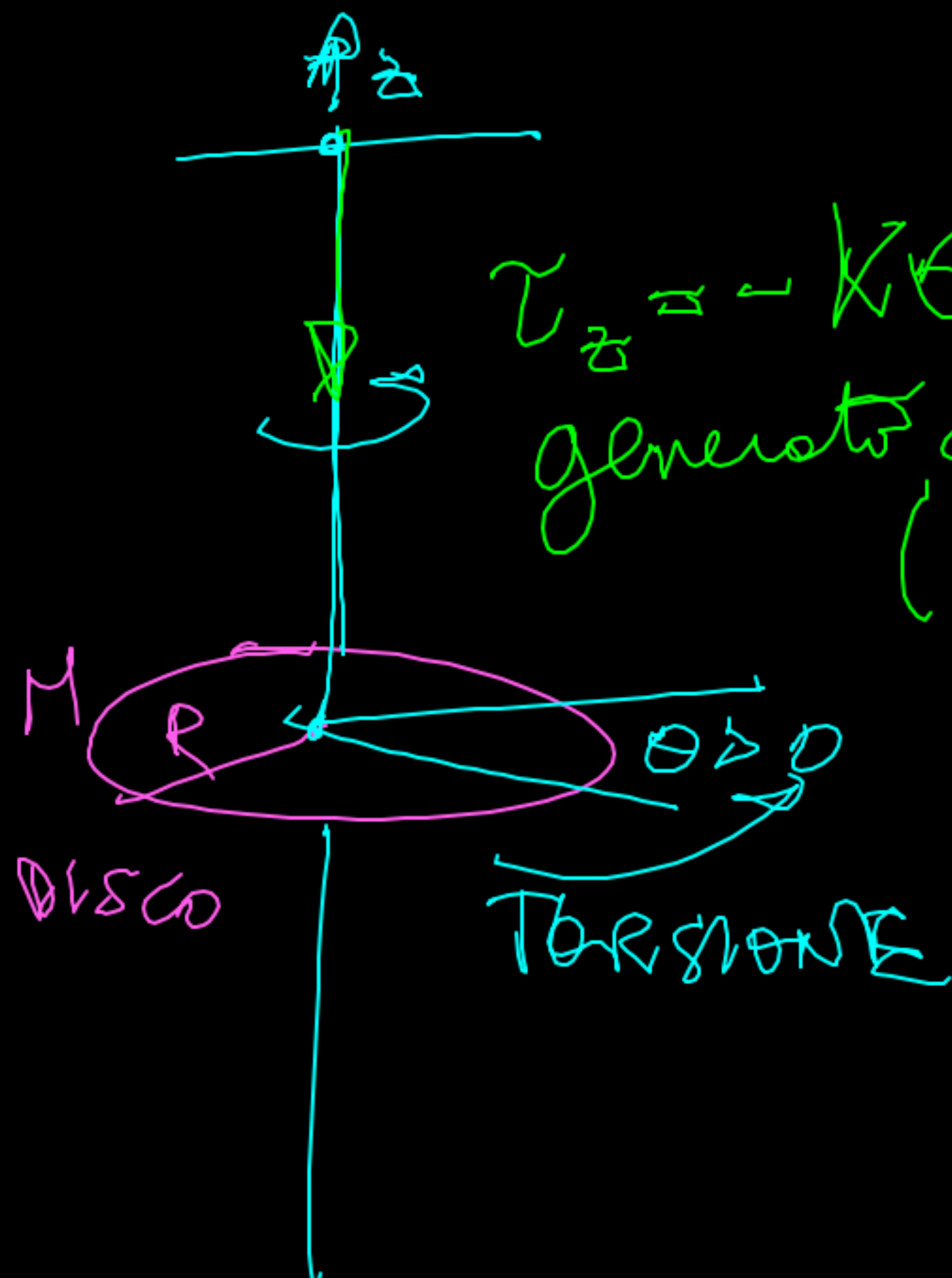
PENDOLO DI TORSIONE



$\tau_z = -K\theta$ generato dal filo (di acciaio)

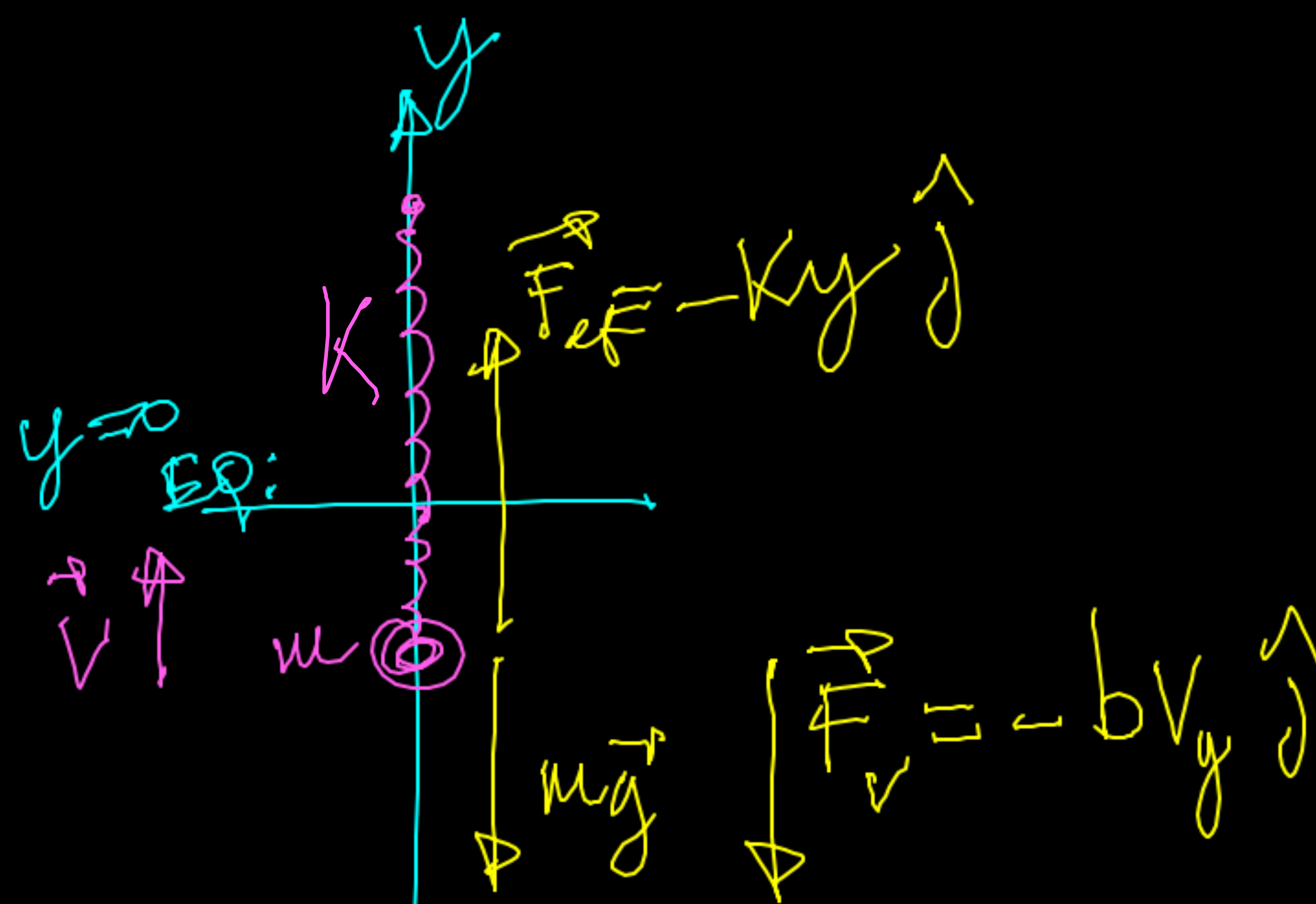
$$\tau_z = I_z \alpha = I_z \ddot{\theta}$$

$$-K\theta = \frac{1}{2} M R^2 \ddot{\theta}$$

$$\boxed{\ddot{\theta} = -\frac{2K}{MR^2}\theta}$$

$$\omega_{PT} = \sqrt{\frac{2K}{MR^2}}$$

OSCILLAZIONI SMORZATE



ATTRITO VISCOSO

PROP. ALLA $\vec{V}$

$$m a_y = -ky - b\frac{dy}{dt}$$

$$\ddot{y} = -\frac{K}{m}y - \frac{b}{m}\dot{y}$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{K}{m}y = 0$$

$$\boxed{\ddot{y} + \gamma\dot{y} + \omega^2 y = 0}$$

$$\gamma = \frac{b}{m} \qquad \omega = \sqrt{\frac{K}{m}}$$

OSC. SMORZATO

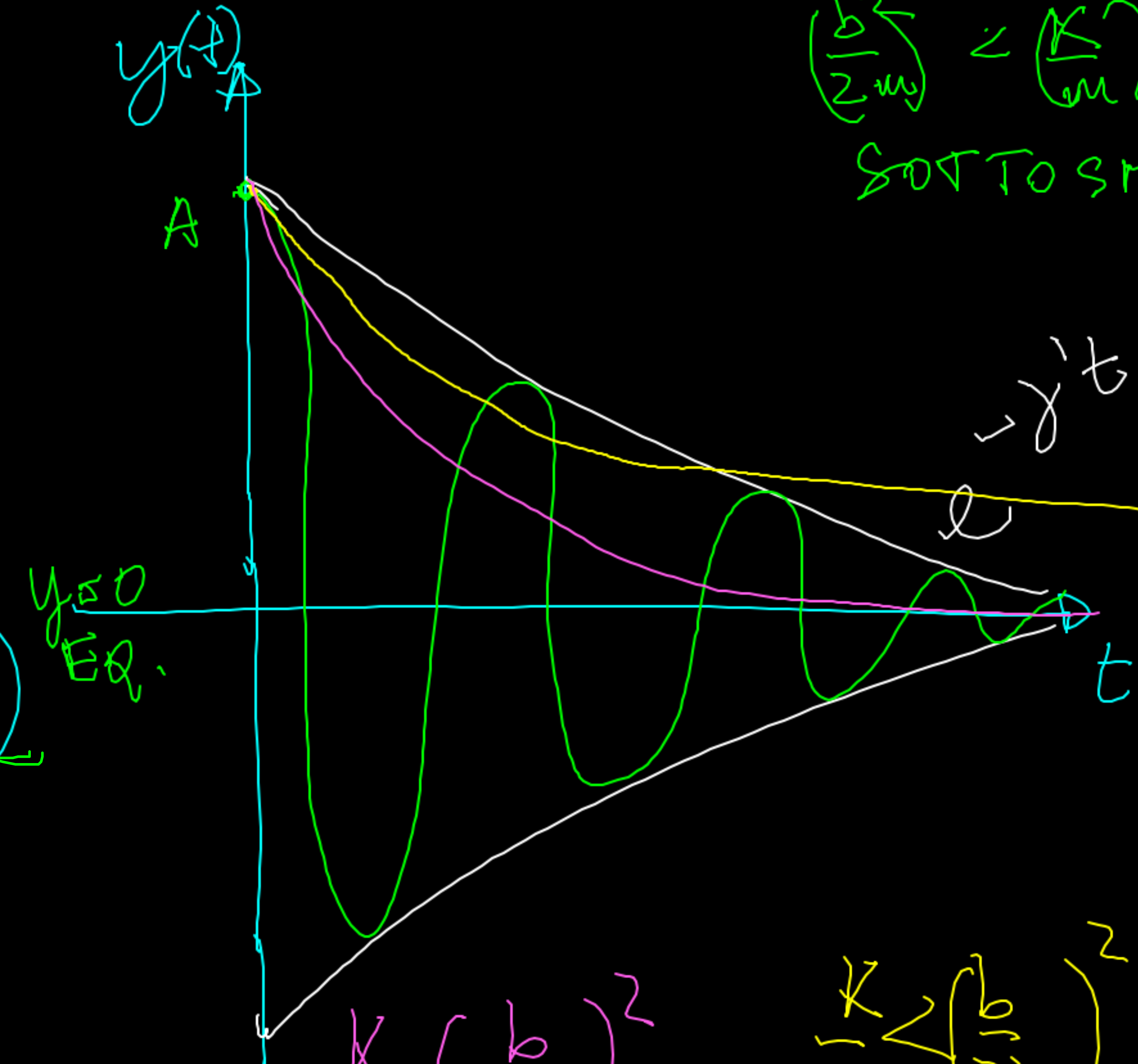
$$\boxed{\ddot{y} + \gamma \dot{y} + \omega^2 y = 0 \qquad \gamma = \frac{b}{m} \qquad \omega = \sqrt{\frac{K}{m}}}$$

OSC. SMORZATO

$$y(t) = \underbrace{e^{-\gamma' t} A}_{\text{ampiezza}} \cos(\omega_s t + \phi)$$

$$\gamma' = \frac{\gamma}{2} = \frac{b}{2m}$$

$$\omega_s = \sqrt{\underbrace{\frac{K}{m}}_{\text{"puls. naturale"}} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega^2 - \gamma'^2}$$



$$\left(\frac{b}{2m}\right)^2 < \left(\frac{K}{m}\right)$$

SOTTOSM.

$$\frac{K}{m} = \left(\frac{b}{2m}\right)^2$$

SMORZ. CRITICO

$$\frac{K}{m} < \left(\frac{b}{2m}\right)^2$$

SOVRASM.

OSCILLAZIONI FORZATE

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{K}{m}y = 0$$

OSC. SMORZATO

$$\underbrace{m\ddot{y}}_{a_y} = \underbrace{-b\dot{y}}_{\text{attrito viscoso}} \underbrace{-Ky}_{\text{forza di "richiamo"}} + F_0\cos(\omega_E t)$$

Termine "forzante"

$$\ddot{y} + \gamma\dot{y} + \omega^2 y = \left(\frac{F_0}{m}\right)\cos(\omega_E t)$$

$y(t) =$ "TRANSIENTE" + "SOLUZ. STAZIONARIA"

$$\xrightarrow{\text{per } t \to \infty} \quad y(t) \simeq \text{"SOL. STAZ."}$$

OSC. FORZATE — SOL. STAZIONARIA

$$\ddot{y} + \gamma\dot{y} + \omega^2 y = \left(\frac{F_0}{m}\right)\cos(\omega_E t) \qquad y(t) = \underbrace{A_0}\cos(\omega_E t - \underbrace{\phi_E})$$

$\gamma = \frac{b}{m}$ $\omega = \sqrt{\frac{k}{m}}$

$$A_0 = \frac{(F_0/m)}{\sqrt{(\omega_E^2 - \omega^2)^2 + \gamma^2\omega_E^2}}$$

FORZ. NAT.

$$\tan\phi_E = \frac{\gamma\omega_E}{\omega^2 - \omega_E^2}$$