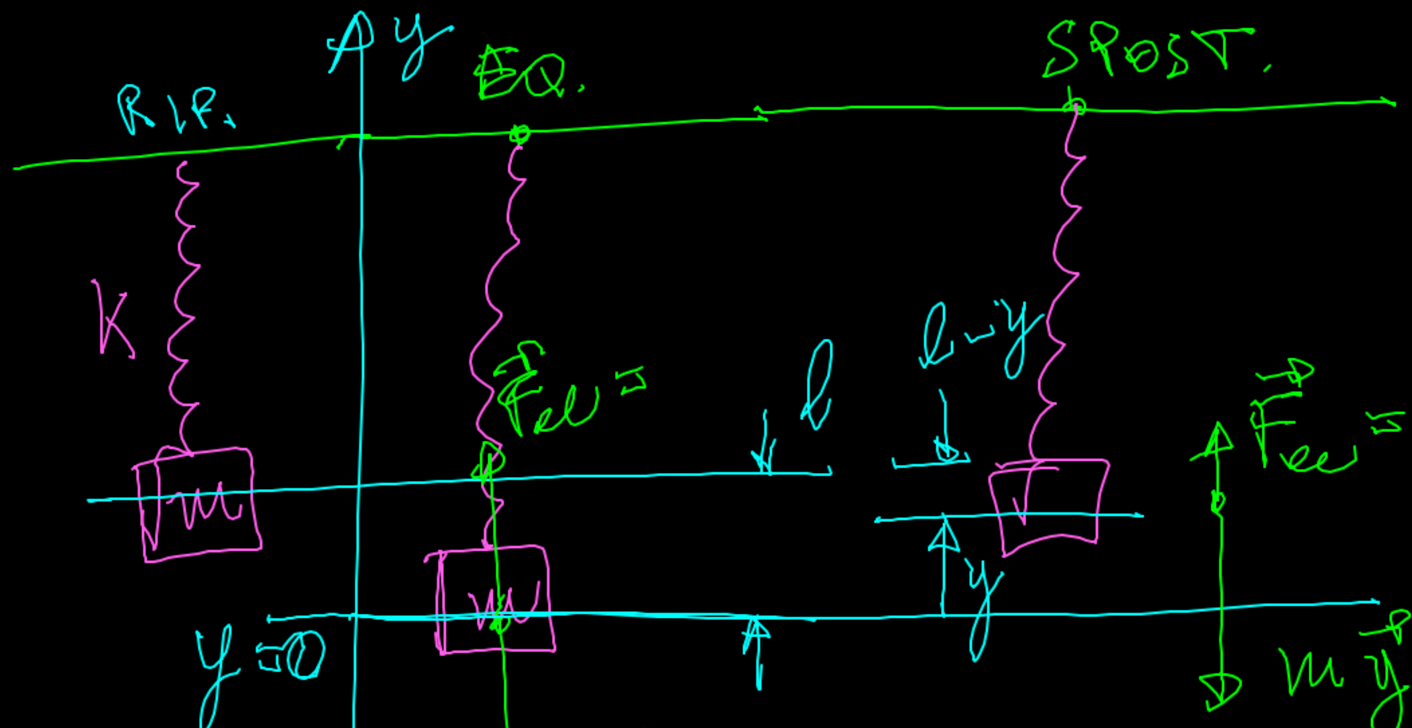


$\vec{F}_{ee} = K(l-y)\hat{j}$

$m\vec{y} = -mg\hat{j}$

$\vec{F}_{ee} + m\vec{g} = 0$

$Kl - mg = 0$

$\frac{K}{m}l - g = 0$

$K l \hat{j} - m g \hat{j} = 0$

$m a_y \hat{j} = \vec{F}_{ee} + m\vec{g} \neq 0$

(Y) $a_y = \frac{K}{m}(l - y(t)) - g$

$\frac{d^2y}{dt^2} = -\frac{K}{m}y + \frac{K}{m}l - g = 0$

$$\frac{d^2y}{dt^2} = -\frac{k}{m}y \Rightarrow y(t) = A\cos(\omega t + \phi)$$

$$\dot{y}(t) = -\omega A \sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$\ddot{x} = -\omega^2 x$$

COND. INIZ.

$$y(0) = y_0 > 0$$

$$\dot{y} = \frac{dy}{dt} = 0$$

$$y_0 = A\cos\phi$$

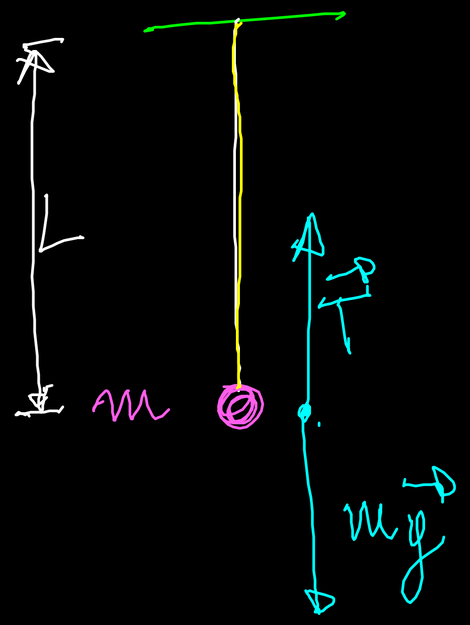
$$0 = -\omega A \sin\phi \Rightarrow \phi = 0$$

$$y_0 = A$$

$$y(t) = y_0 \cos \omega t$$

PENDOLO SEMPLICE

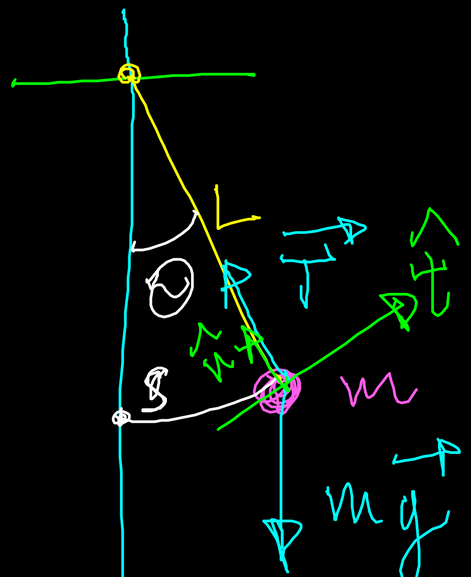
ROS. DI EQ.

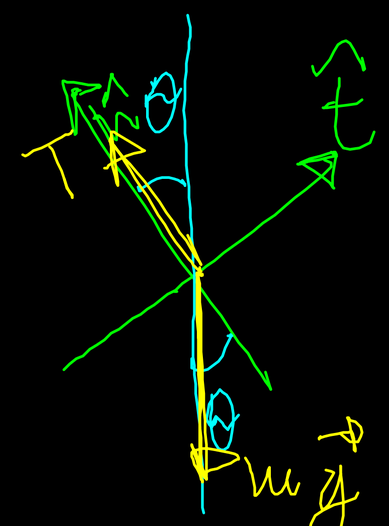


$mg \approx T$

$\theta \approx 0$

FUORI EQ.

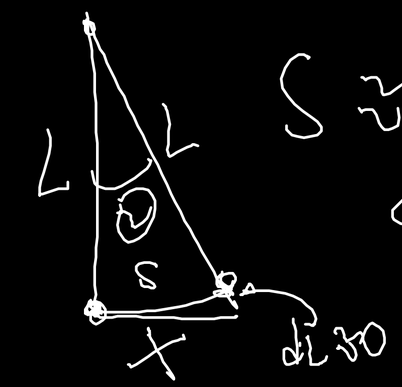




(t) $-mg\sin\theta = ma_t$

(n) $T - mg\cos\theta = 0$

θ "piccolo", $\theta \ll 1$ rad



$s \approx x$

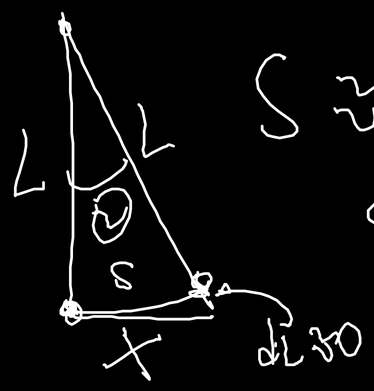
$\sin\theta = \frac{x}{L}$

$a_t \approx a_x$

$$-mg\sin\theta = ma_t$$

$$T - mg\cos\theta = 0$$

θ "piccolo", $\theta \ll 1\,rad$



$s \simeq x$

$\sin\theta = \frac{x}{L}$

$a_t \simeq a_x$

$$ma_x = m\frac{d^2x}{dt^2} = -mg\frac{x}{L}$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$T_{PS} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$\Delta a_t = \alpha L$

$$-mg\sin\theta = m\alpha L = m\ddot{\theta}L$$

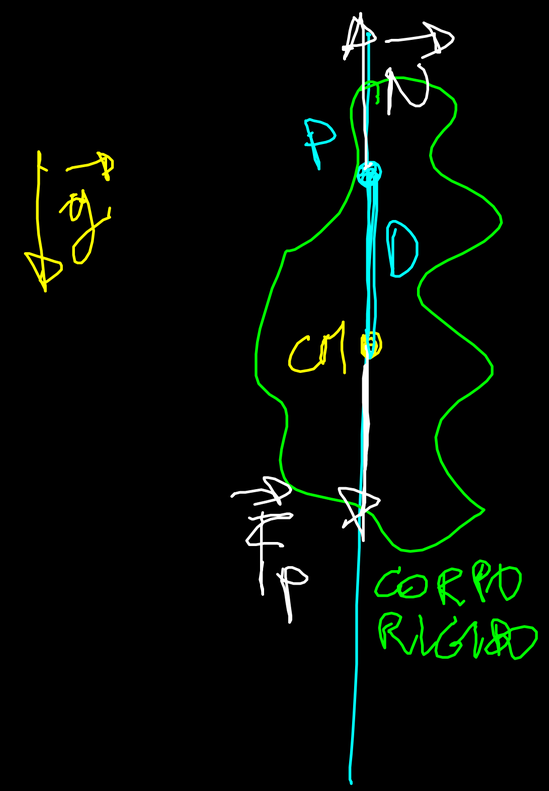
$\theta \ll 1 \Rightarrow$ $\ddot{\theta} = -\frac{g}{L}\theta(t)$

$$\theta(t) = A\cos(\omega t + \phi)$$

$\omega = \sqrt{\frac{g}{L}}$

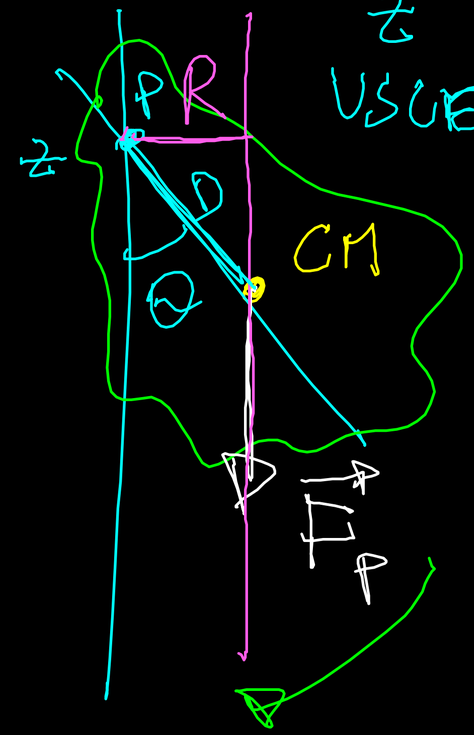
$$g = \frac{4\pi^2}{T_{PS}^2}L$$

PENDOLO FISICO



EQUIL.

$\vec{\tau}_P = 0$



FUORI EQ.

$\vec{\tau}_P \neq 0$

z ASSE FISSO ORIZZ. PER P USCENTE

$\tau_z \neq 0$

$|\tau_z| = MgR = MgD \sin\theta$

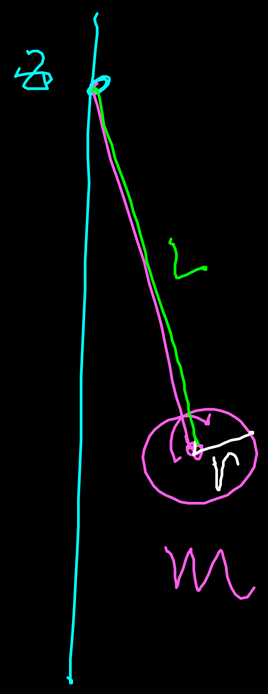
$\tau_z = I_z \alpha \qquad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$

$-MgD \sin\theta = I_z \ddot{\theta}$

per $\theta \ll 1 \Rightarrow \ddot{\theta} = -\frac{MgD}{I_z}\theta$

$\omega = \sqrt{\frac{MgD}{I_z}}$

$$\omega_{PF} = \sqrt{\frac{Mgl}{I_z}}$$



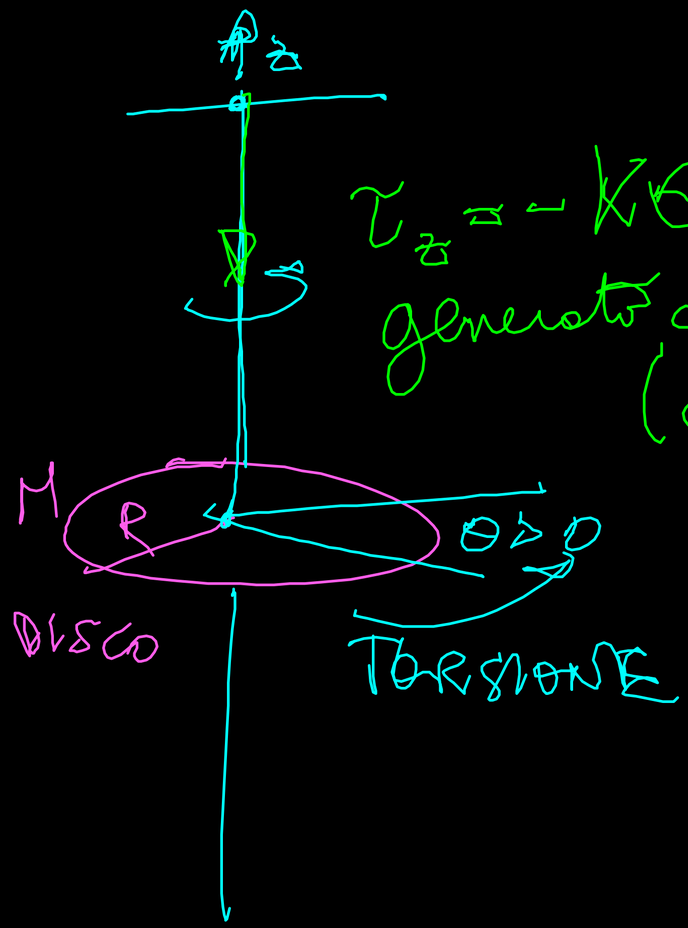
$$I_z = I_{cm} + ML^2 = \frac{2}{5}Mr^2 + ML^2$$

$$\omega_p = \sqrt{\frac{MgL}{\frac{2}{5}Mr^2 + ML^2}} = \sqrt{\frac{MgL}{ML^2\left(1+\frac{2r^2}{5L^2}\right)}}$$

$$= \sqrt{\frac{g}{L\left(1+\frac{2r^2}{5L^2}\right)}} \xrightarrow{r \ll L} \sqrt{\frac{g}{L}}$$

$$\omega_{PS} = \sqrt{\frac{g}{L}}$$

PENDOLO DI TORSIONE



$\tau_z = -K\theta$
generato dal filo
(di acciaio)

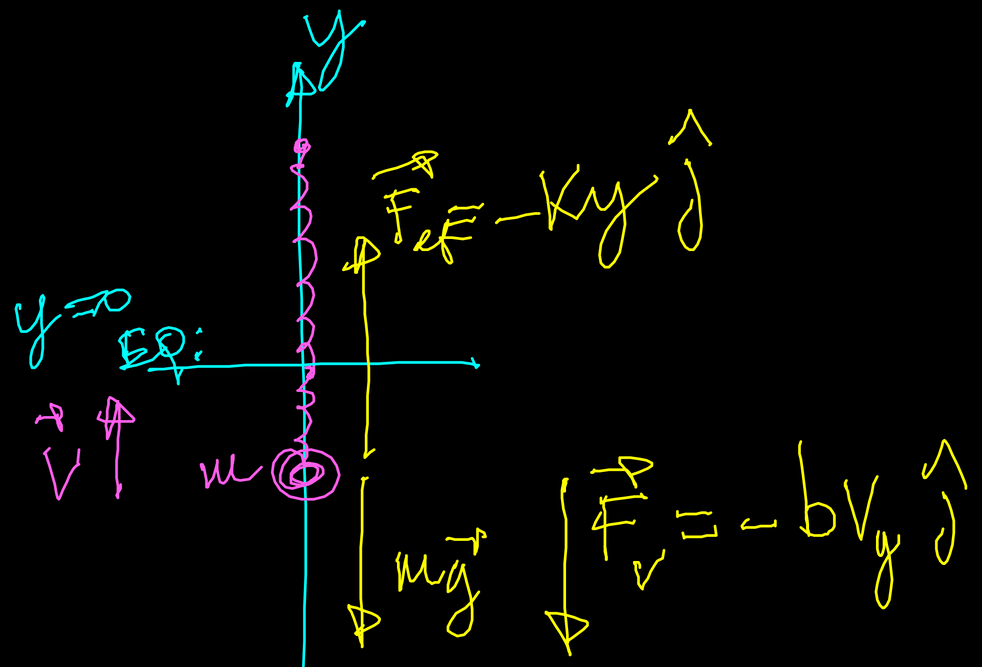
$$\tau_z = I_z \alpha = I_z \ddot{\theta}$$

$$-K\theta = \frac{1}{2} M R^2 \ddot{\theta}$$

$$\ddot{\theta} = -\frac{2K}{MR^2}\theta$$

$$\omega_{PT} = \sqrt{\frac{2K}{MR^2}}$$

OSCILLAZIONI SMORZATE



ATTRITO VISCOSO
PROP. ALLA $\vec{v}$

$$m a_y = -ky - b\frac{dy}{dt}$$

$$\ddot{y} = -\frac{K}{m}y - \frac{b}{m}\dot{y}$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{K}{m}y = 0$$

$$\ddot{y} + \gamma\dot{y} + \omega^2 y = 0$$

$\gamma = \frac{b}{m}$

$\omega = \frac{K}{m}$

OSC. SMORZATO

$$\ddot{y} + \gamma \dot{y} + w^2 y = 0$$

$\gamma = \frac{b}{m}$

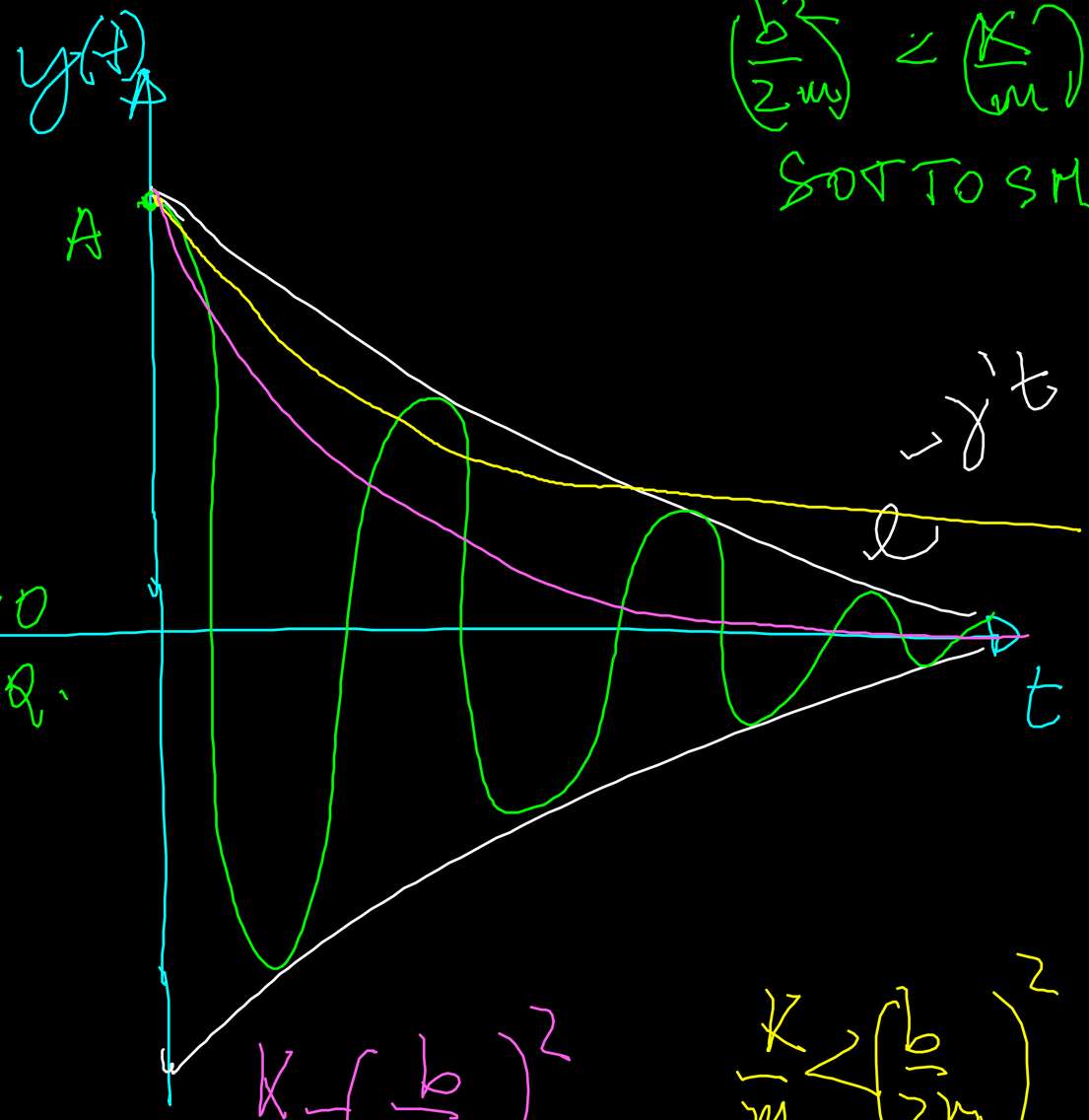
$w = \frac{K}{m}$

OSC. SMORZATO

$$y(t) = e^{-\gamma' t} A \cos(w_s t + \phi)$$ EQ.

$$\gamma' = \frac{\gamma}{2} = \frac{b}{2m}$$

$$w_s = \sqrt{\frac{K}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{w^2 - \gamma'^2}$$



$\left(\frac{b}{2m}\right)^2 < \left(\frac{K}{m}\right)$

SOTTOSM.

$\frac{K}{m} = \left(\frac{b}{2m}\right)^2$

SMORZ. CRITICO

$\frac{K}{m} < \left(\frac{b}{2m}\right)^2$

SOVRASM.