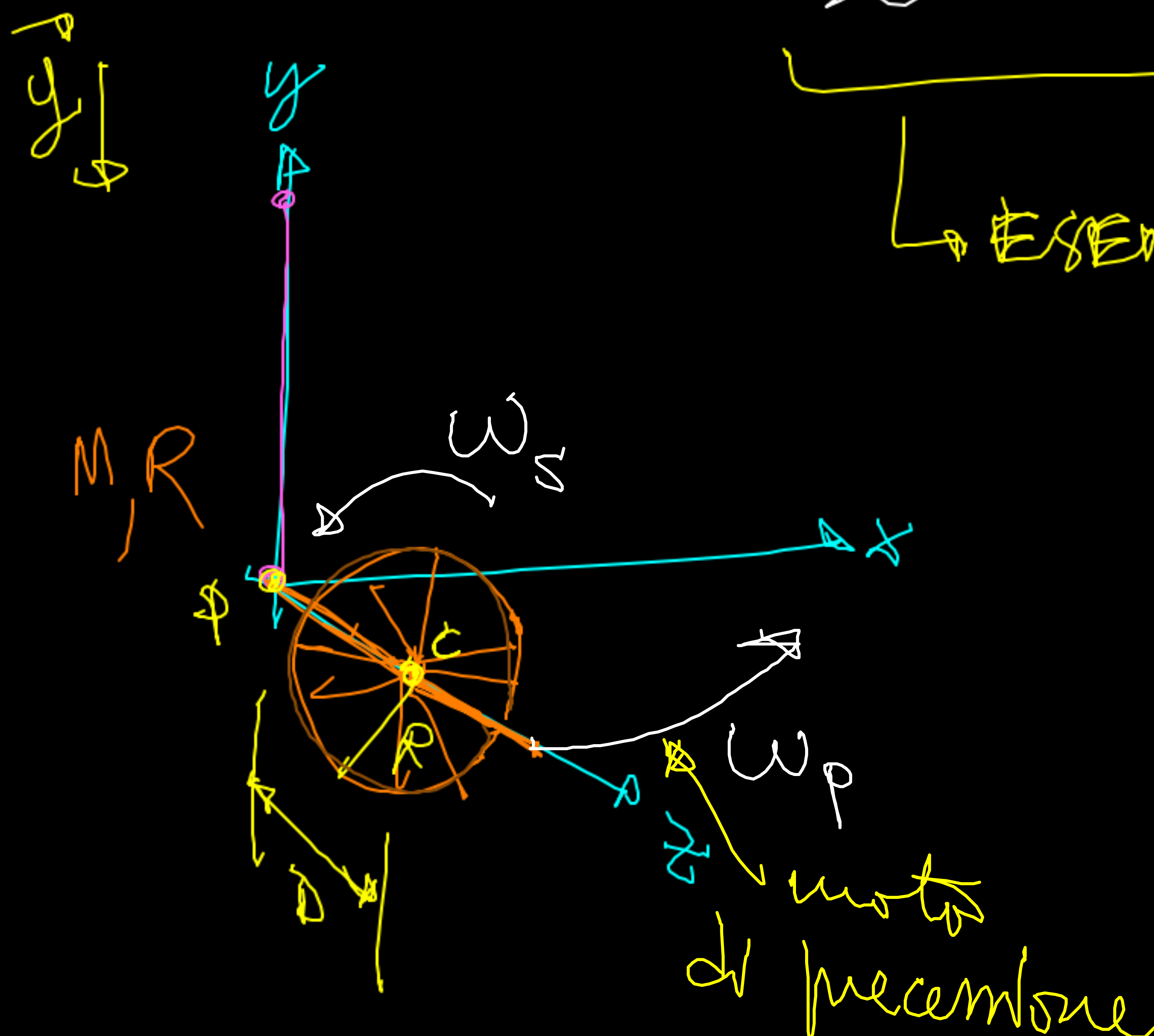
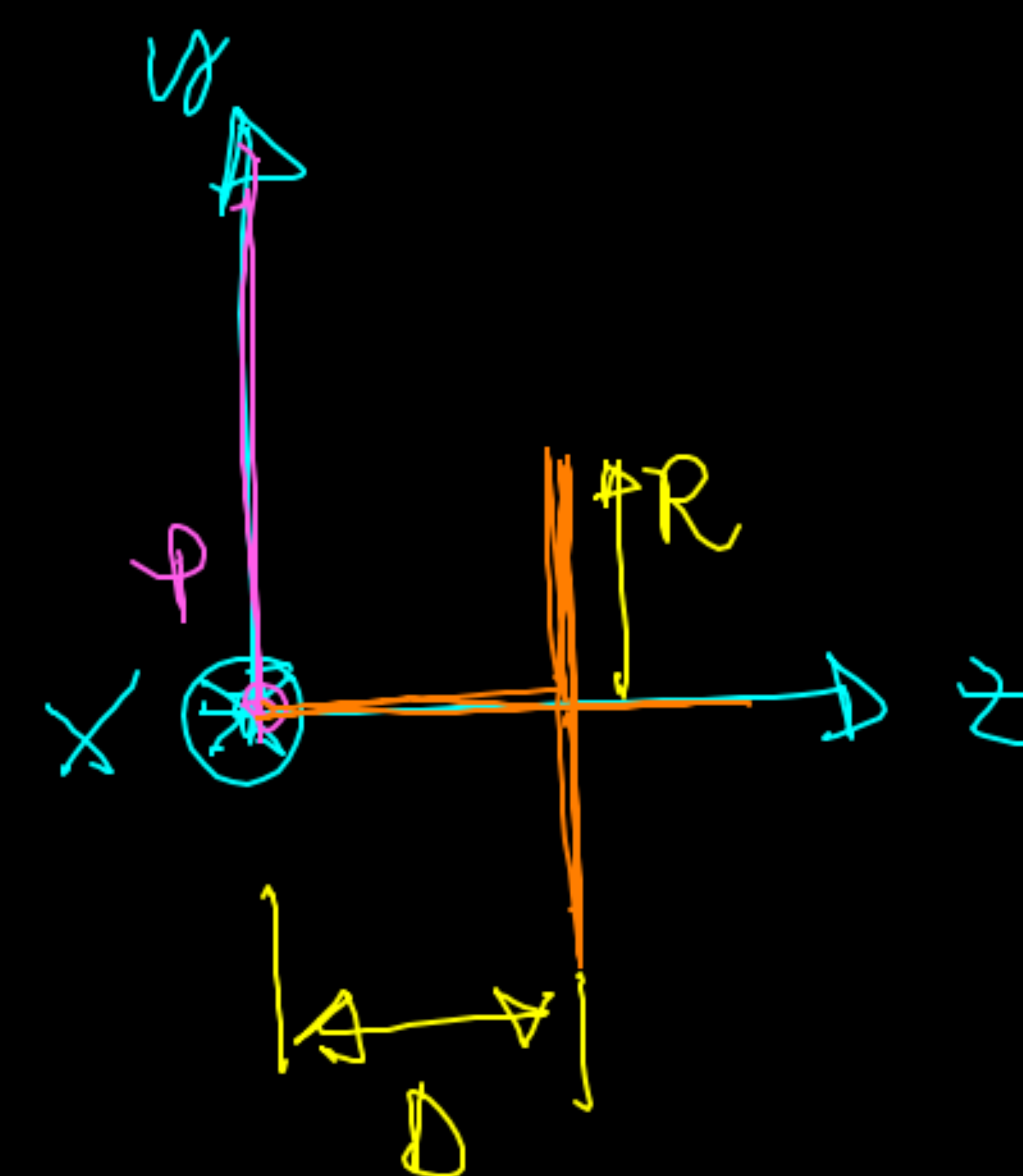
$$\sum \vec{\tau}_{O,eff} = \frac{d\vec{L}_O}{dt}$$

Se $d\vec{L}_O \parallel \vec{L}_O$ cambia solo $|\vec{L}_O|$

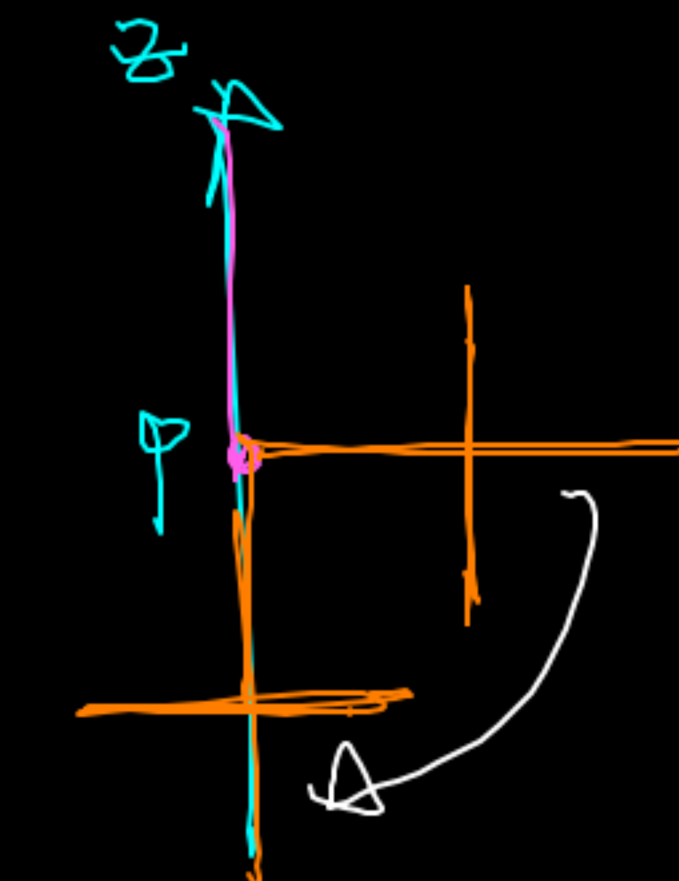
$\underbrace{\text{Se } d\vec{L}_O \nparallel \vec{L}_O}$ $\Rightarrow$ cambia la direzione di $\vec{L}_O$

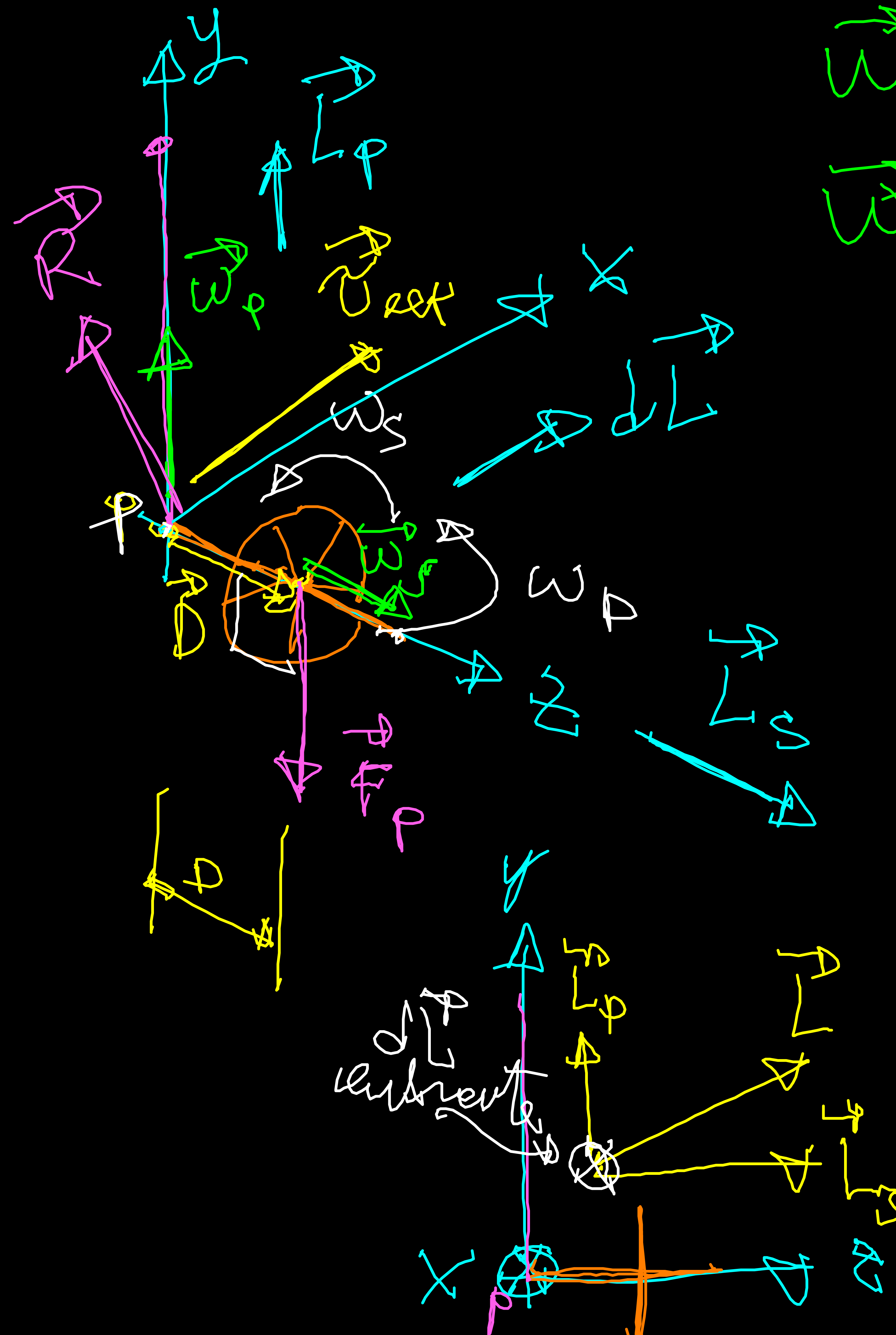
↳ ESEMPIO: IL GIROSCOPIO





Se no rot.





$\vec{\omega}_s$ velocità angolare di "spin"

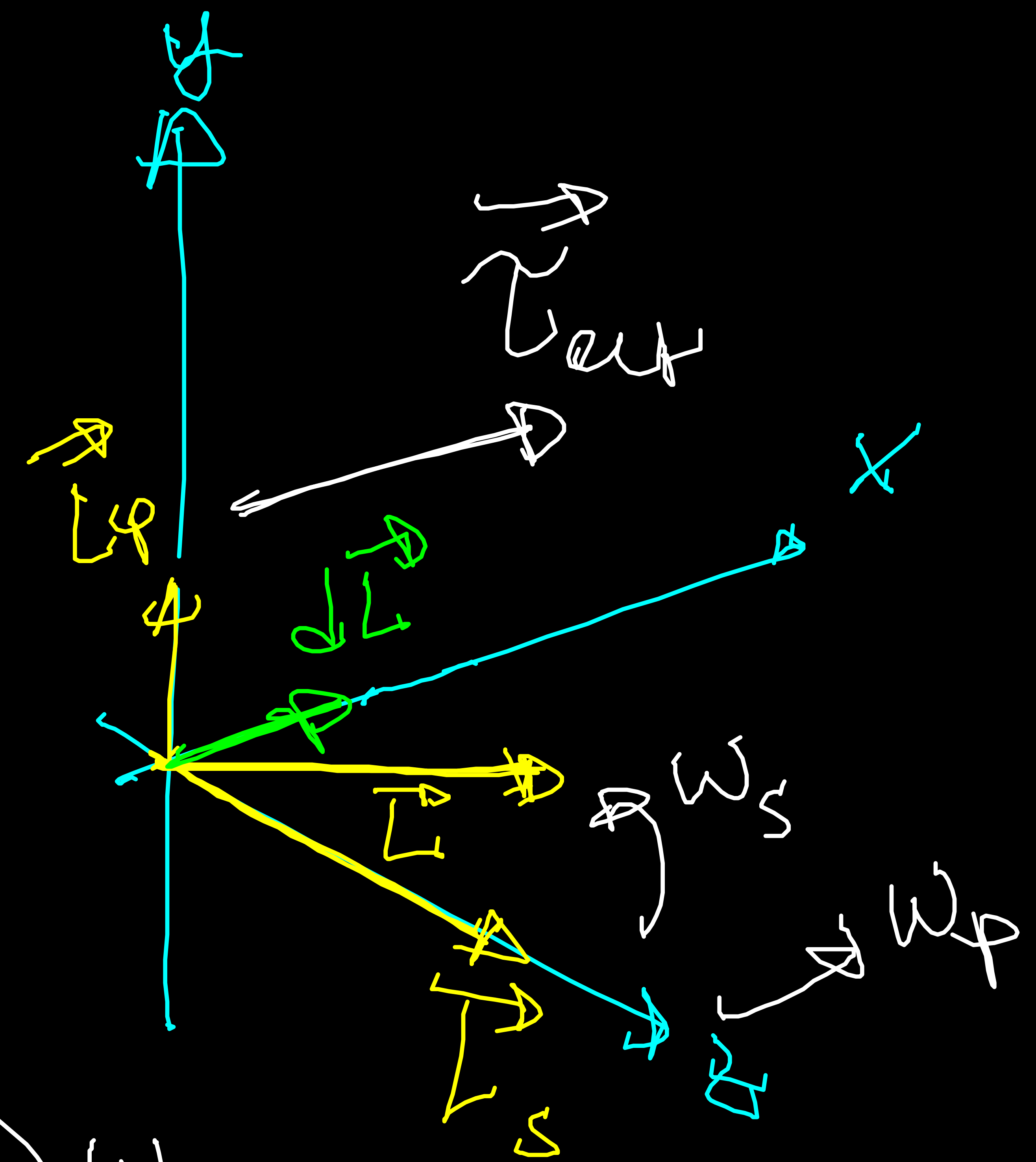
$\vec{\omega}_p$ velocità angolare di "precessione"

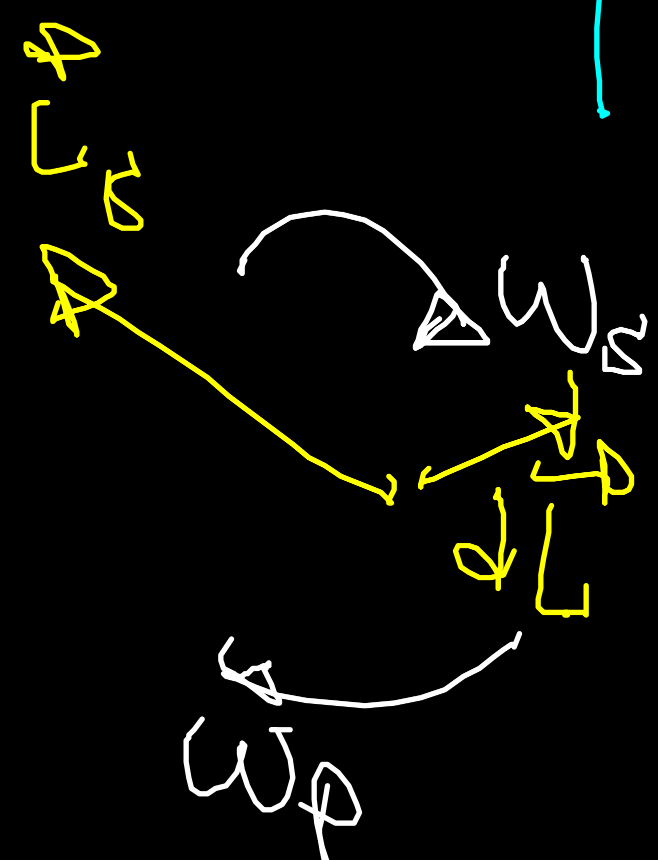
$$\vec{L}_s = I_z \omega_s \hat{K} \qquad \vec{L}_p = I_y \hat{j}$$

$$\vec{L} = \vec{L}_s + \vec{L}_p$$

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}_{ext} = \vec{D} \times \vec{F}_p = MgD\,\hat{i}$$

$$d\vec{L} = (Mg\,D\,dt)\,\hat{i}$$





$$d\vec{L} = (Mg D\,dt)\hat{i}$$

$$\vec{L} = I_z \omega_s \hat{K} + I_y \omega_p \hat{j}$$

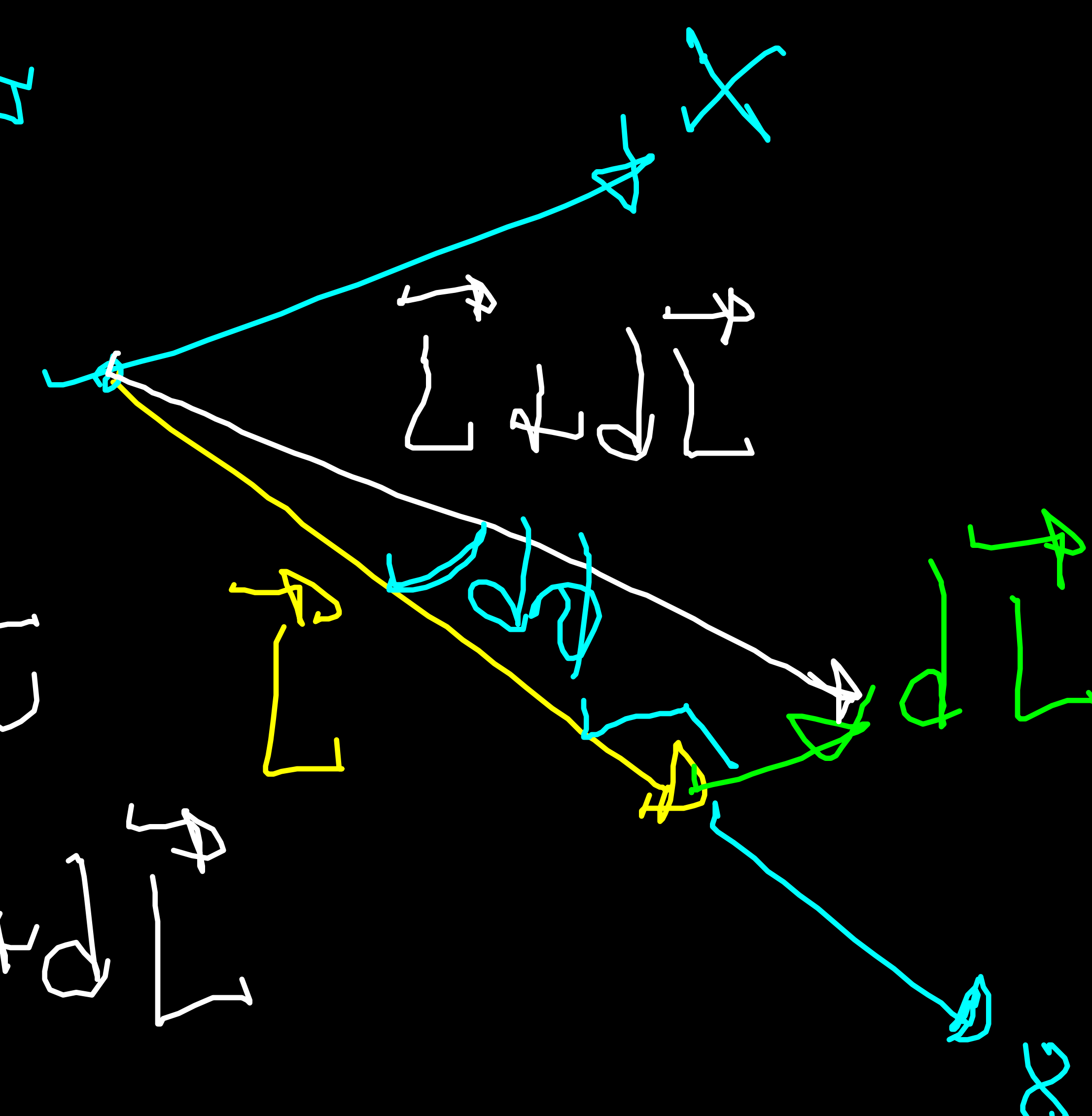
$$\omega_s >> \omega_p$$

$$\vec{L} \simeq \vec{L}_s = I_z \omega_s \vec{K}$$

$$d\phi = \frac{dL}{L} = \frac{Mg D\,dt}{I_z \omega_s}$$

in dt

$$\vec{L} \to \vec{L} + d\vec{L}$$



$$\boxed{\frac{d\phi}{dt} = \frac{MgD}{I_z \omega_s} = \omega_p}$$

MOTO ARMONICO

Legge oraria del moto armonico

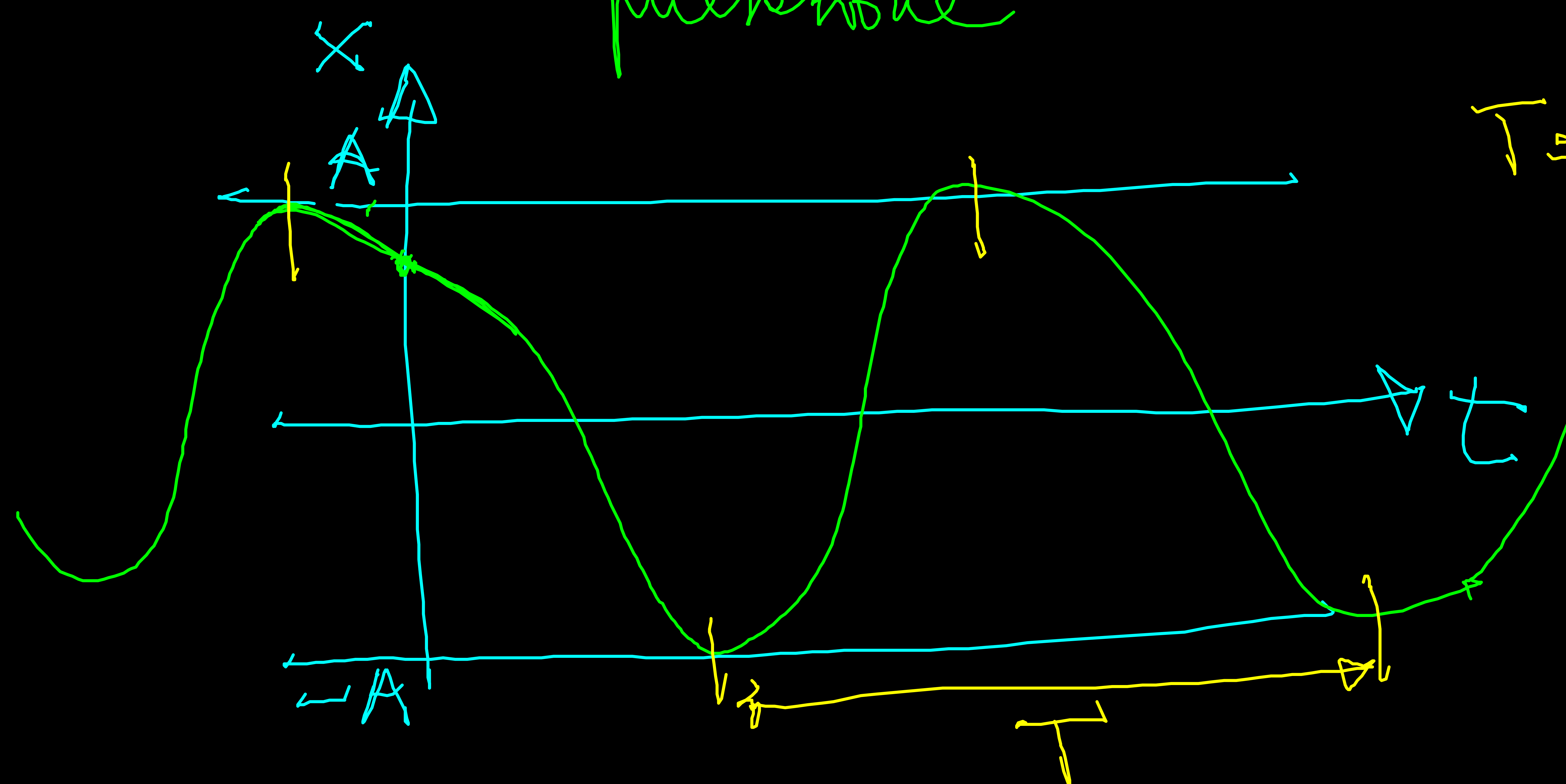
$$x(t) = A\cos(\omega t + \phi)$$

ampiezza

pulsazione

costante di fase

fase



T = periodo

$$A\cos(\omega(t+T)+\phi) = A\cos(\omega t+\phi)$$

$$x(t+T) = x(t)$$

$$\cos(\omega(t+T)+\phi) = \cos(\omega t+\phi)$$

$$\omega(t+T)+\phi = \omega t+\phi+2\pi$$

$\omega = [\text{rad}][s]^{-1}$ $\quad T = \frac{2\pi}{\omega}$ periodo $[s]$

Frequenza $\quad f = \nu = \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \boxed{2\pi\nu = \omega}$

[Hertz] $= [s]^{-1}$

$$x(t) = A\cos(\omega t + \phi)$$

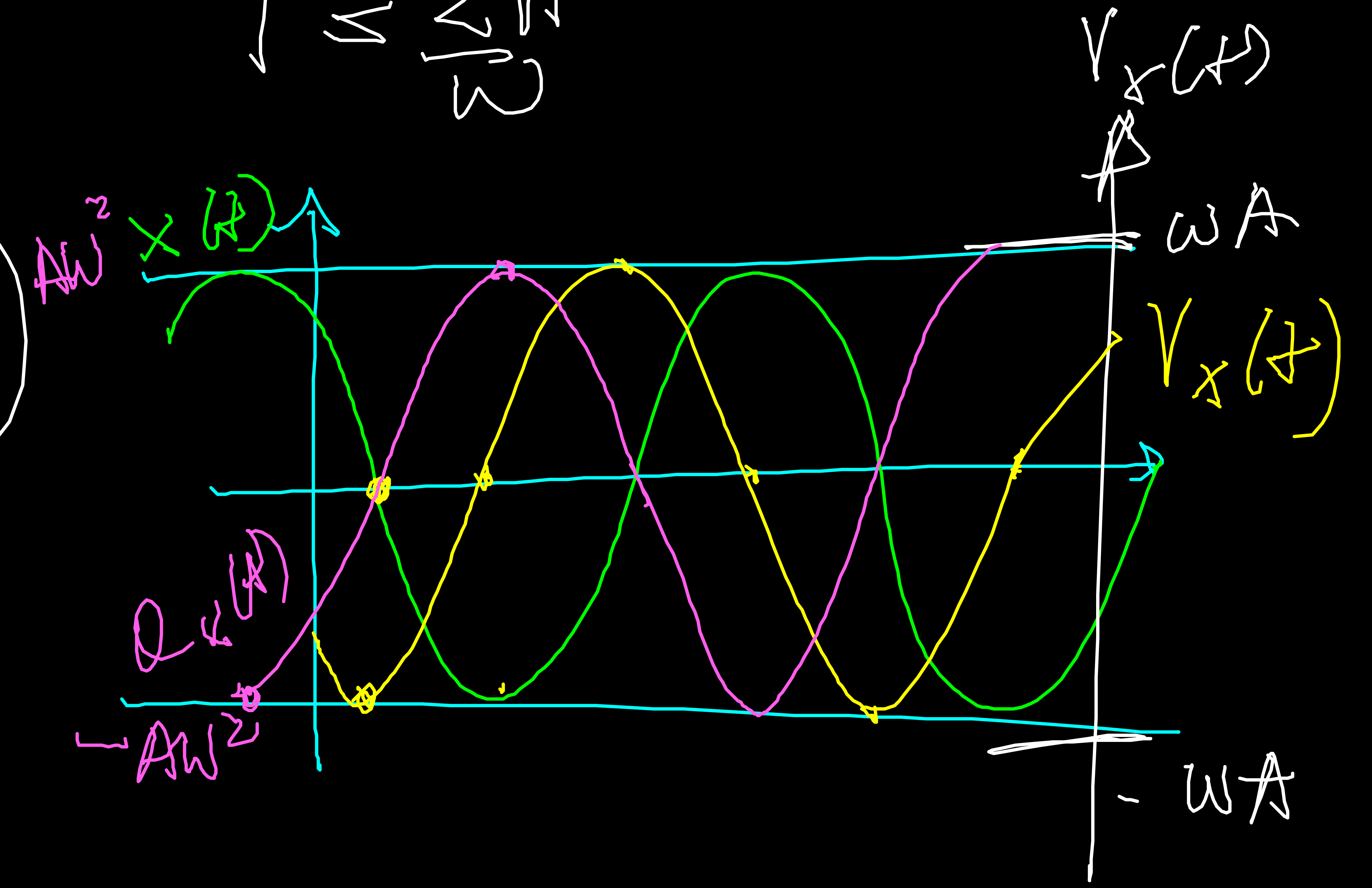
$$T = \frac{2\pi}{\omega}$$

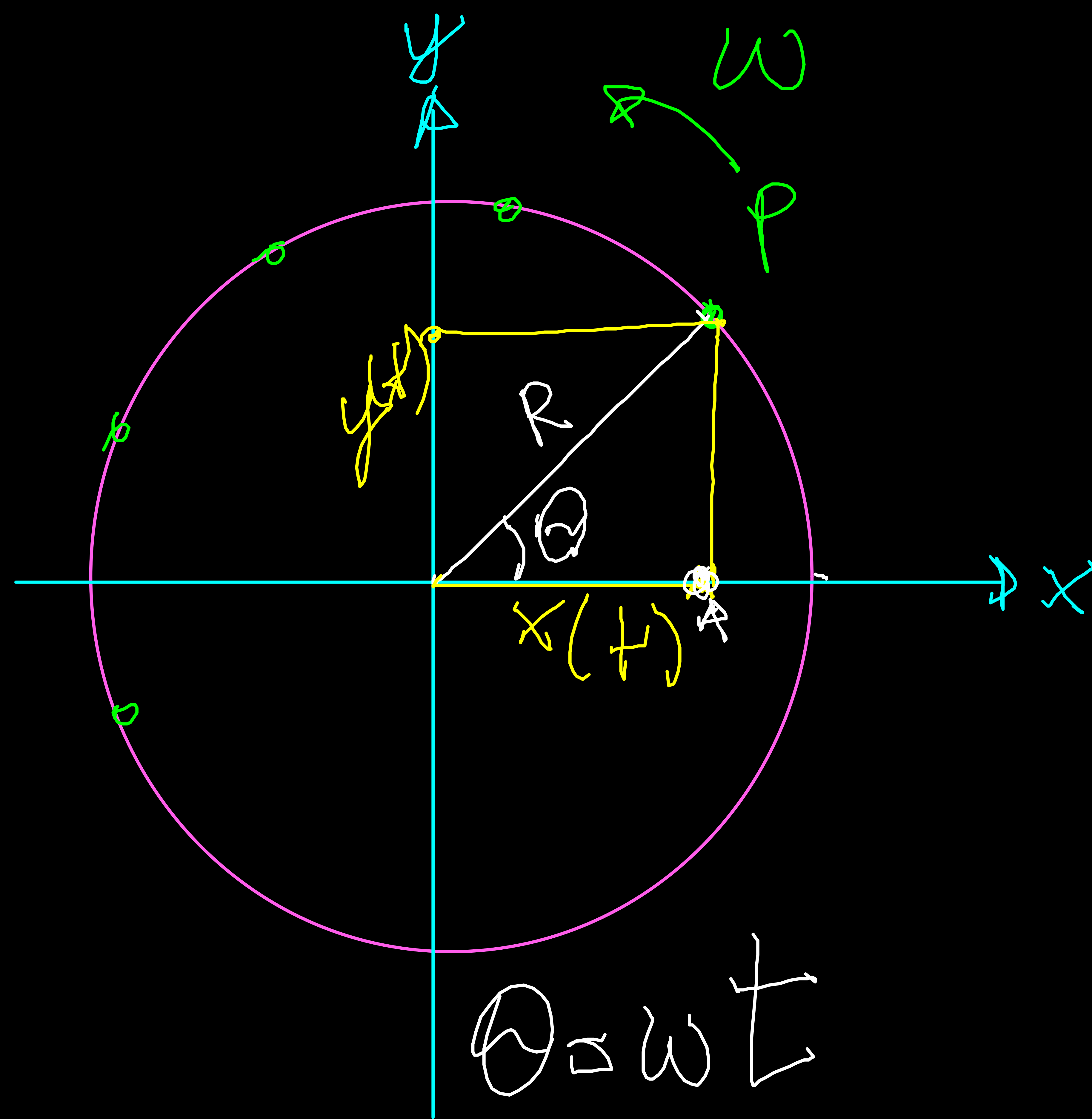
$$V_x = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$

$$a_x = \frac{dV_x}{dt} = \frac{d^2x}{dt^2} = \underbrace{-A\omega^2}\underbrace{\cos(\omega t + \phi)}$$

$$a_x = \boxed{\frac{d^2x}{dt^2} = -\omega^2 x(t)}$$

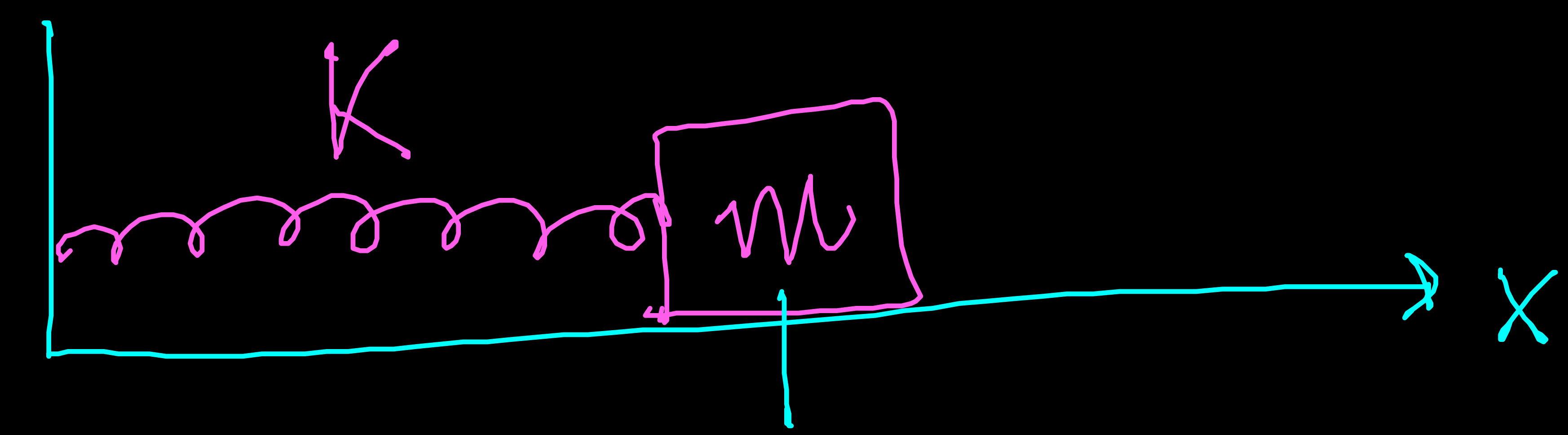
EQ. DIFF. DEL MOTO ARMONICO SEMPLICE



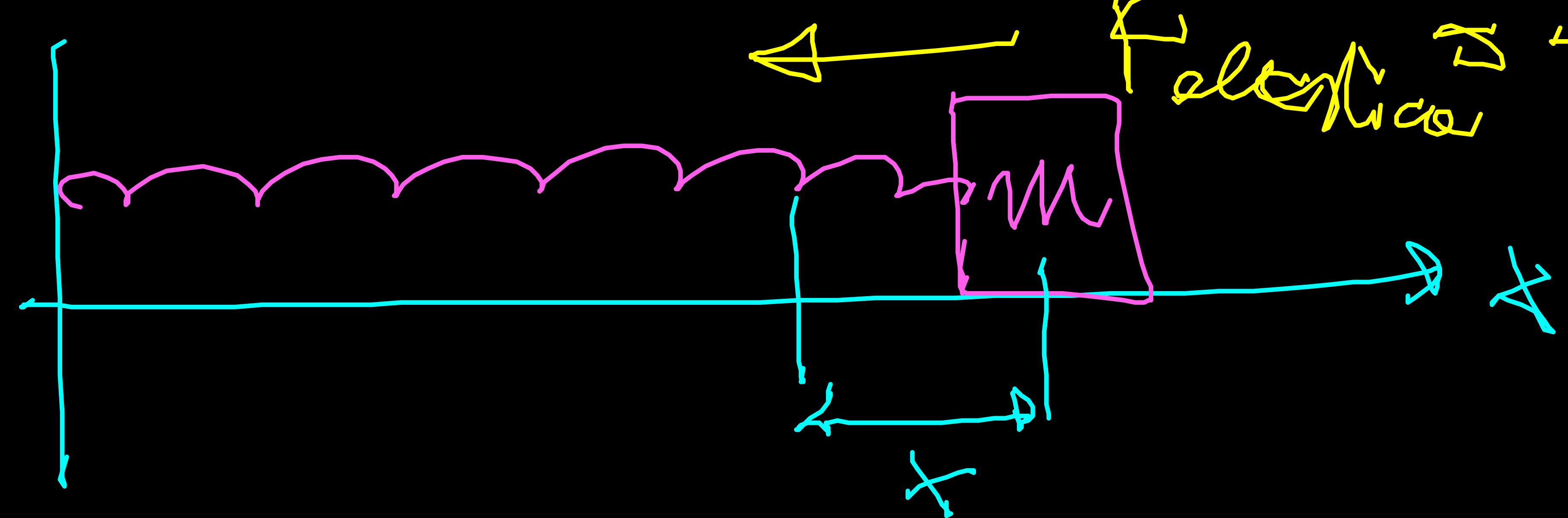


$$x(t) = R\cos\theta$$
$$y(t) = R\sin\theta$$

$$x(t) = R\cos(\omega t + \theta_0)$$
$$y(t) = R\sin(\omega t + \theta_0)$$



$x = 0$ molla a riposo



$\vec F_{elastica} = -Kx\hat i$

No attrito

$$\sum \vec F = \vec F_{elastica} = m\vec a$$

$$-Kx\hat i = m a_x \hat i$$

$$m a_x = -Kx$$

$$\frac{d^2x(t)}{dt^2} = -\frac{K}{m}x(t)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega^2 = \frac{K}{m} \Rightarrow \omega = \sqrt{\frac{K}{m}}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$x(t) = A\cos(\omega t + \phi)$$

COND. INIZIALI

$$V_x(t) = -\omega A \sin(\omega t + \phi)$$

$$x(0) = x_0$$

$$V_x(0) = V_{x0}$$

$$x_0 = A\cos\phi$$

$$V_{x0} = -\omega A \sin\phi$$

$$\Rightarrow \boxed{\tan\phi = -\frac{V_{x0}}{x_0 \omega}}$$

$$x_0^2 = A^2\cos^2\phi$$

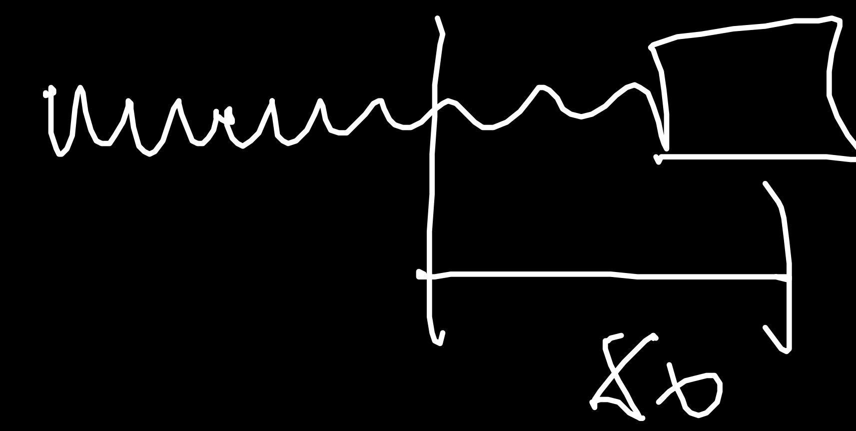
$$\left(\frac{V_{x0}}{\omega}\right)^2 = +A^2\sin^2\phi$$

$$A^2(\sin^2\phi + \cos^2\phi) = x_0^2 + \frac{V_{x0}^2}{\omega^2}$$

$$\boxed{A = \sqrt{x_0^2 + \frac{V_{x0}^2}{\omega^2}}}$$

$$\tan\phi = -\frac{V_{x0}}{x_0\omega}$$

$$A = \sqrt{x_0^2 + \frac{V_{x0}^2}{\omega^2}}$$



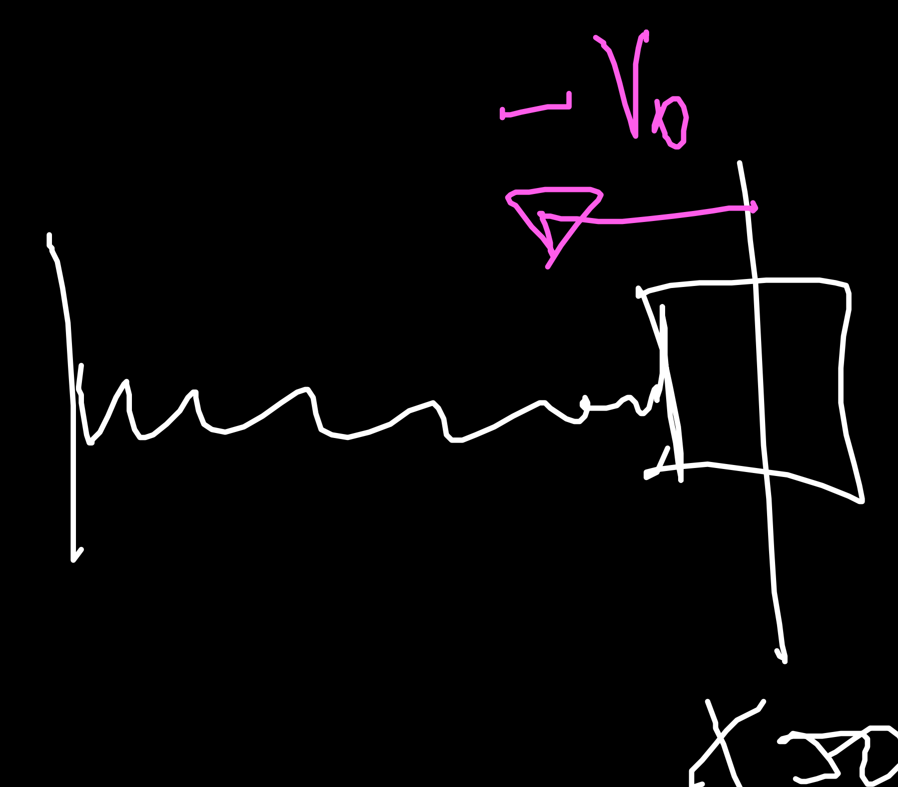
$$x(0) = x_0$$

$$V_{x0} = 0$$

$$\tan\phi = 0 \Rightarrow \phi = 0$$

$$A = \sqrt{x_0^2} = x_0$$

$$x(t) = x_0 \cos\omega t$$



$$x(0) = 0$$

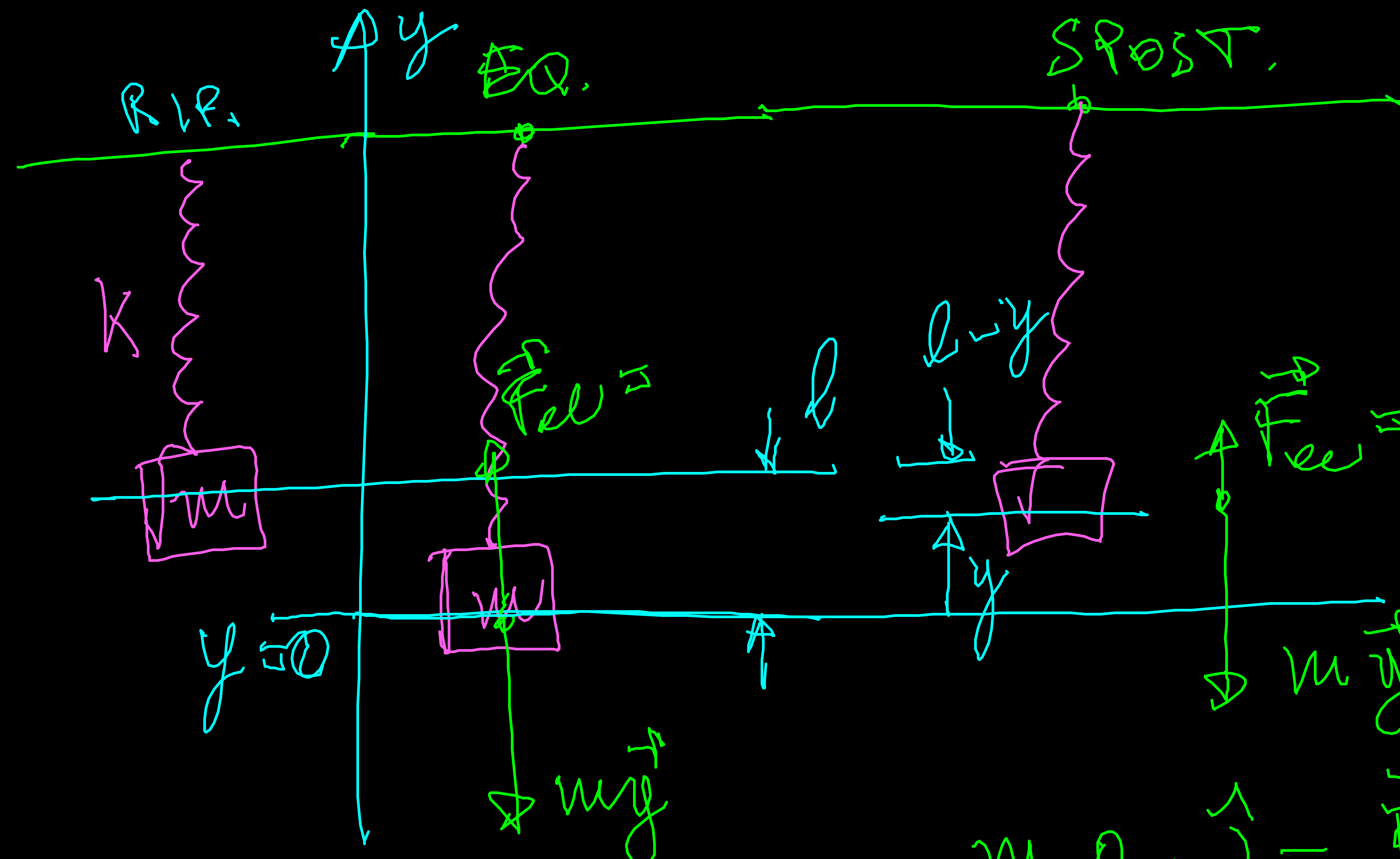
$$V_{x0} = -V_0$$

$$\phi \to \frac{\pi}{2}$$

$$A = \sqrt{\frac{V_0^2}{\omega^2}} = \frac{V_0}{\omega}$$

$$x(t) = \frac{V_0}{\omega}\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{V_0}{\omega}\sin\omega t$$



$$\vec{F}_{el} = K(l-y)\,\hat{j}$$

$$m\vec{g} = -mg\,\hat{j}$$

$$m\,a_y\,\hat{j} = \vec{F}_{el} + m\vec{g} \neq 0$$

$$\vec{F}_{el} + m\vec{g}$$

$$K l\,\hat{j} - mg\,\hat{j} = 0$$