PRINCIPI DI CONSERVAZIONE

$$\mathcal{L}_{\text{NON CONS}} = \Delta(K+U) = \Delta E \Rightarrow \text{Se } \mathcal{L}_{N.C.} = 0 \text{ la } E \text{ si conserva}$$

CONS. DELL'EN. MECC.

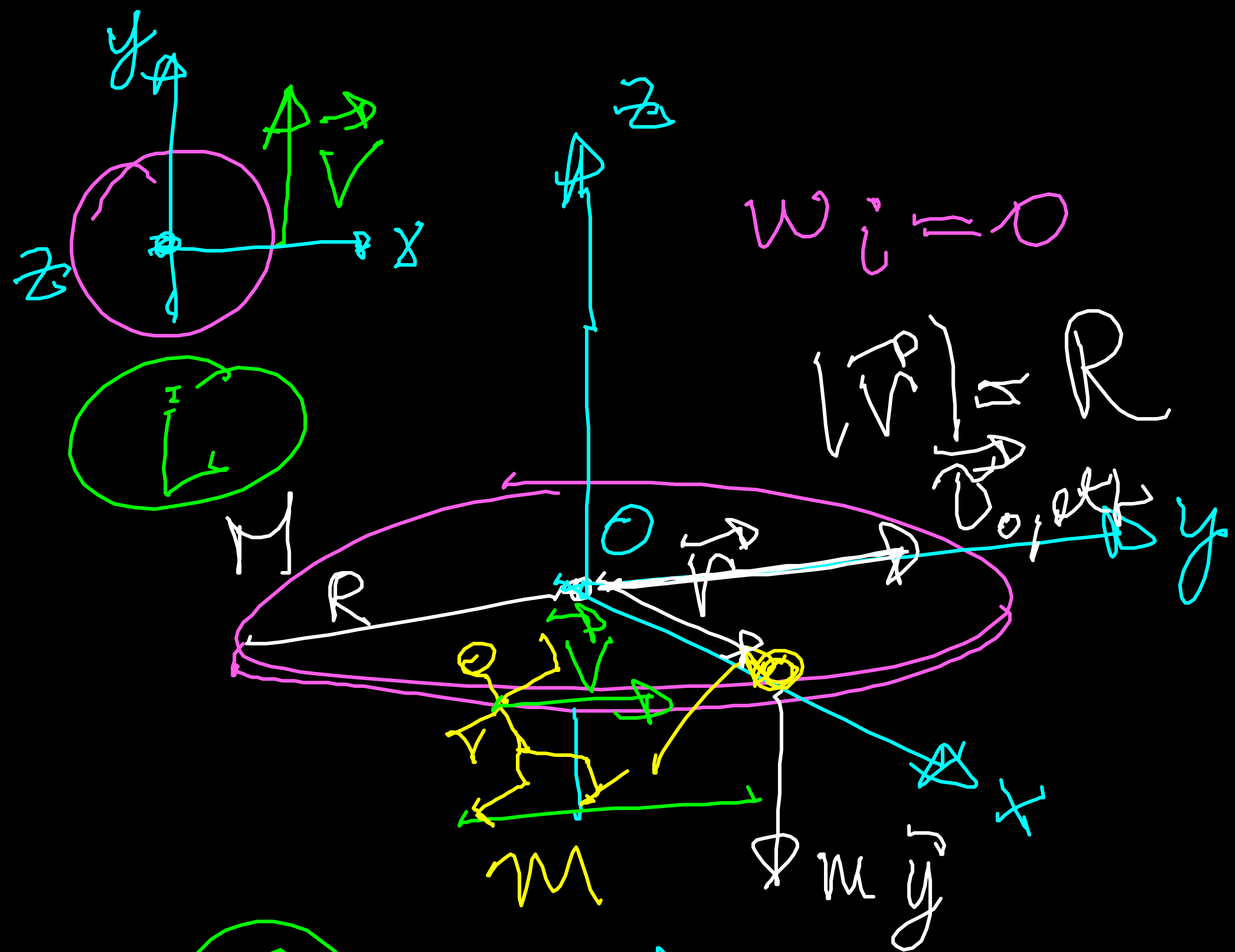
$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} \Rightarrow \text{Se } \sum \vec{F}_{ext} = 0 \text{ allora } \vec{P} \text{ si conserva}$$

CONS. DELLA Q. DI MOTO

$$\sum \vec{\tau}_{0\,ext} = \frac{d\vec{L}_0}{dt} \Rightarrow \text{Se } \sum \vec{\tau}_{ext} = 0 \text{ allora } \vec{L}_0 \text{ si conserva}$$

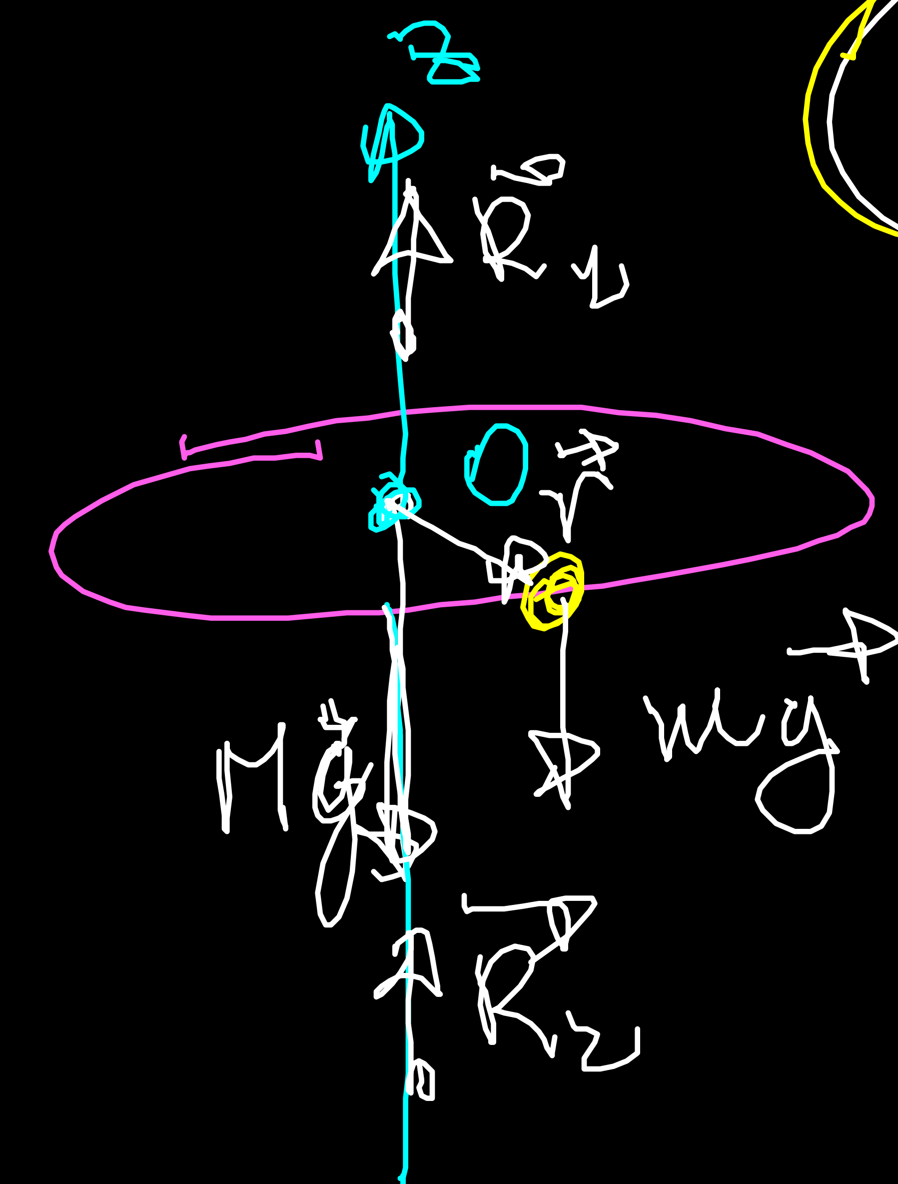
CONS. DEL MOMENTO ANGOLARE

ESEMPIO



$\omega_i = 0$

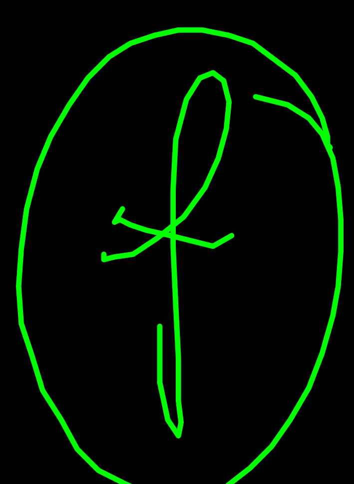
$|\vec r| = R$



i $\quad \sum F_{est} = m\vec g$

$M\vec g + \vec R_1 + \vec R_2 = 0$

(DISCO INIZ. FERMO)



ω_f?

$$\frac{d\vec L}{dt} = \sum \vec\tau_{O,est} = \vec r \times m\vec g = mgR\,\hat j$$

$$0 = \sum(\tau_{est})_x = \frac{dL_x}{dt}$$

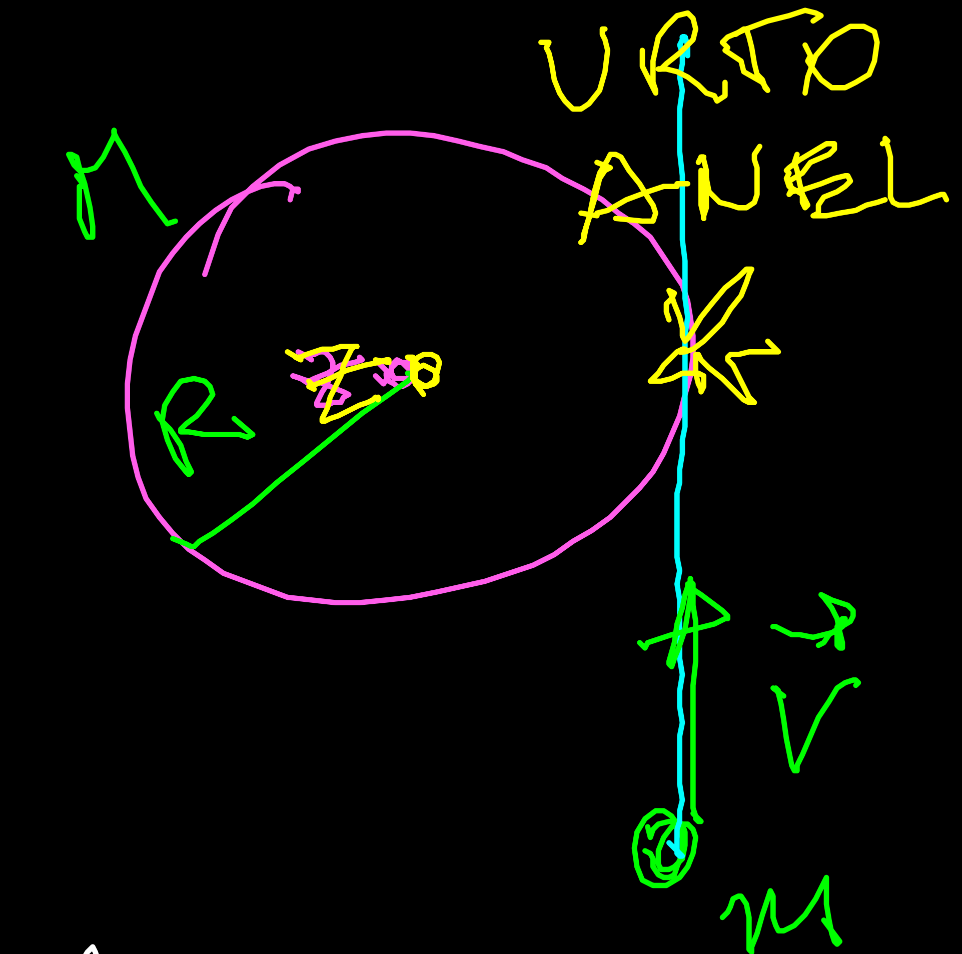
$$\sum(\tau_{est})_y = \frac{dL_y}{dt}$$

$$\sum(\tau_{O,est})_z = \frac{dL_z}{dt}$$

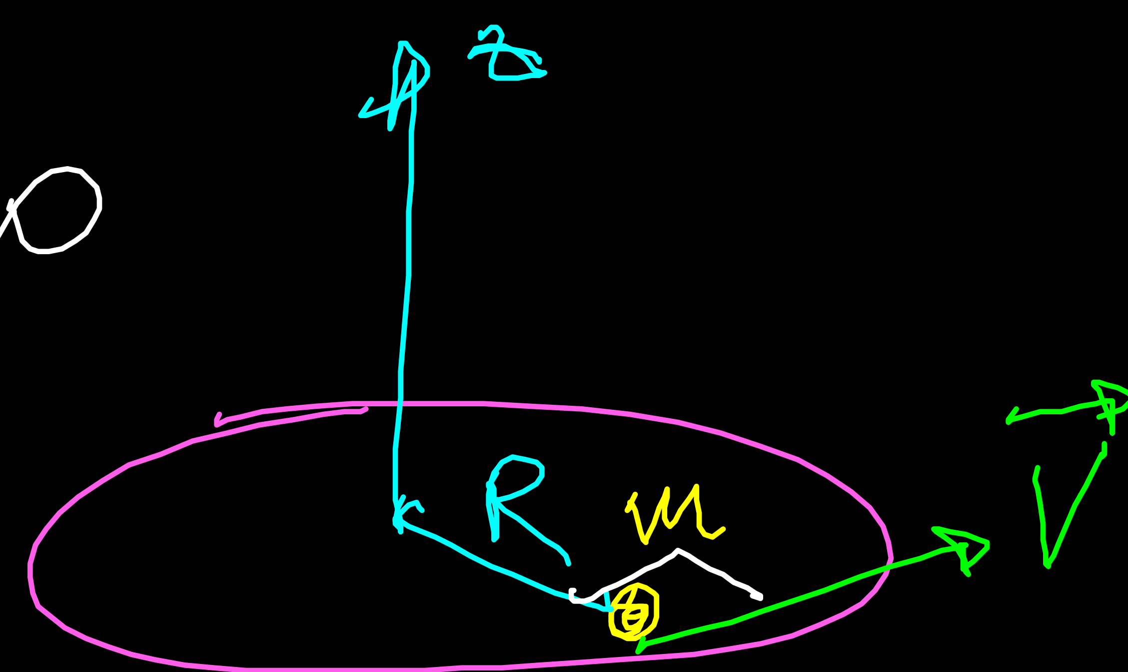
0''

L_x
L_z si conservano

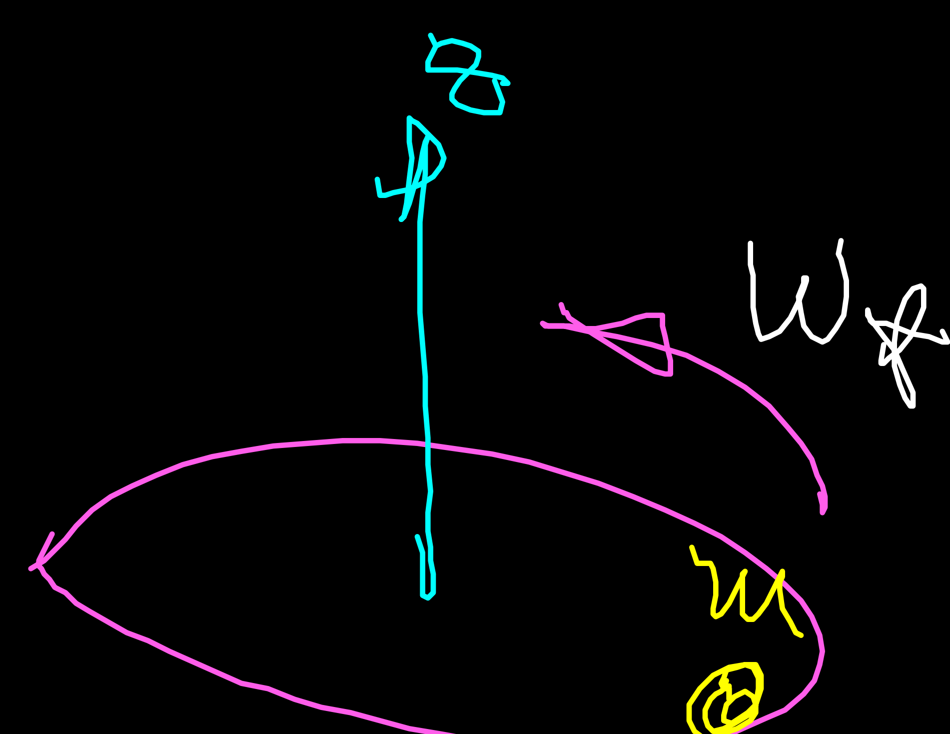
L_z (mom. ang. assiale) è costante



$\omega_i = 0$



$$L_{z,i} = mRV$$



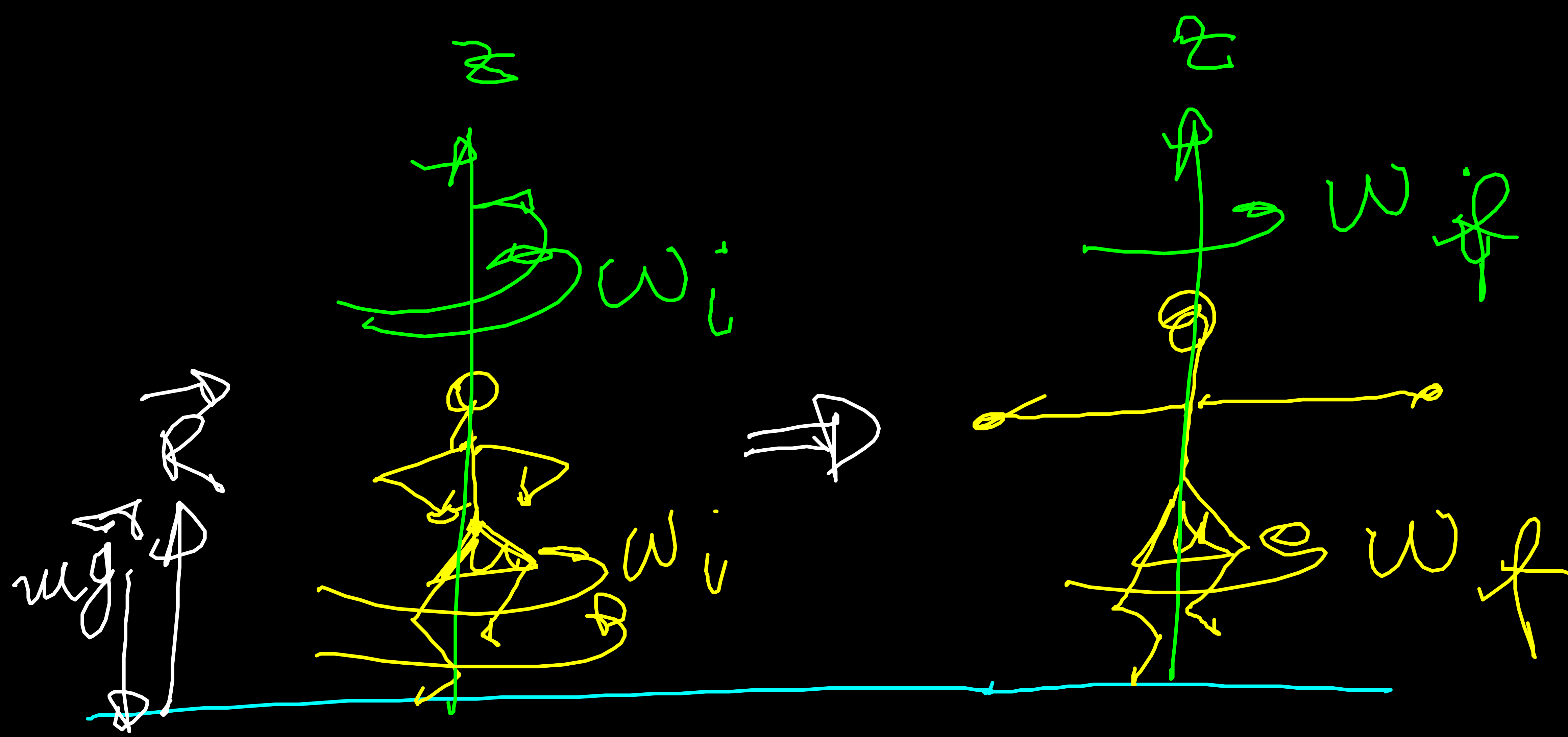
$$L_{z,f} = I_{tot,z}\,\omega_f$$

$$L_{z,i} = L_{z,f}$$

$$mRV = I_{tot,z}\,\omega_f$$

$$mRV = \left(\frac{1}{2}MR^2 + mR^2\right)\omega_f$$

$$\omega_f = \frac{mV}{\left(\frac{1}{2}M + m\right)R} = \left(\frac{2m}{M+2m}\right)\frac{V}{R}$$



MOM. DI INERZIA VARIABILE

$$\tau_z = 0 \Rightarrow L_z \text{ è costante}$$

$$L_{z,i} = L_{z,f} \Rightarrow I_{z,i}\,\omega_i = I_{z,f}\,\omega_f$$

$$I_{z,f} > I_{z,i} \qquad \omega_f = \frac{I_{z,i}}{I_{z,f}}\,\omega_i < \omega_i$$

$$K_i = \frac{1}{2} I_{z,i}\,\omega_i^2 \qquad K_f = \frac{1}{2} I_{z,f}\,\omega_f^2$$

$$\Delta K = \frac{1}{2} I_{z,f}\,\omega_f^2 - I_{z,i}\,\omega_i^2$$

$$\frac{I_{f,z}}{I_{i,z}} = 2 \Rightarrow \omega_f = \frac{\omega_i}{2}$$

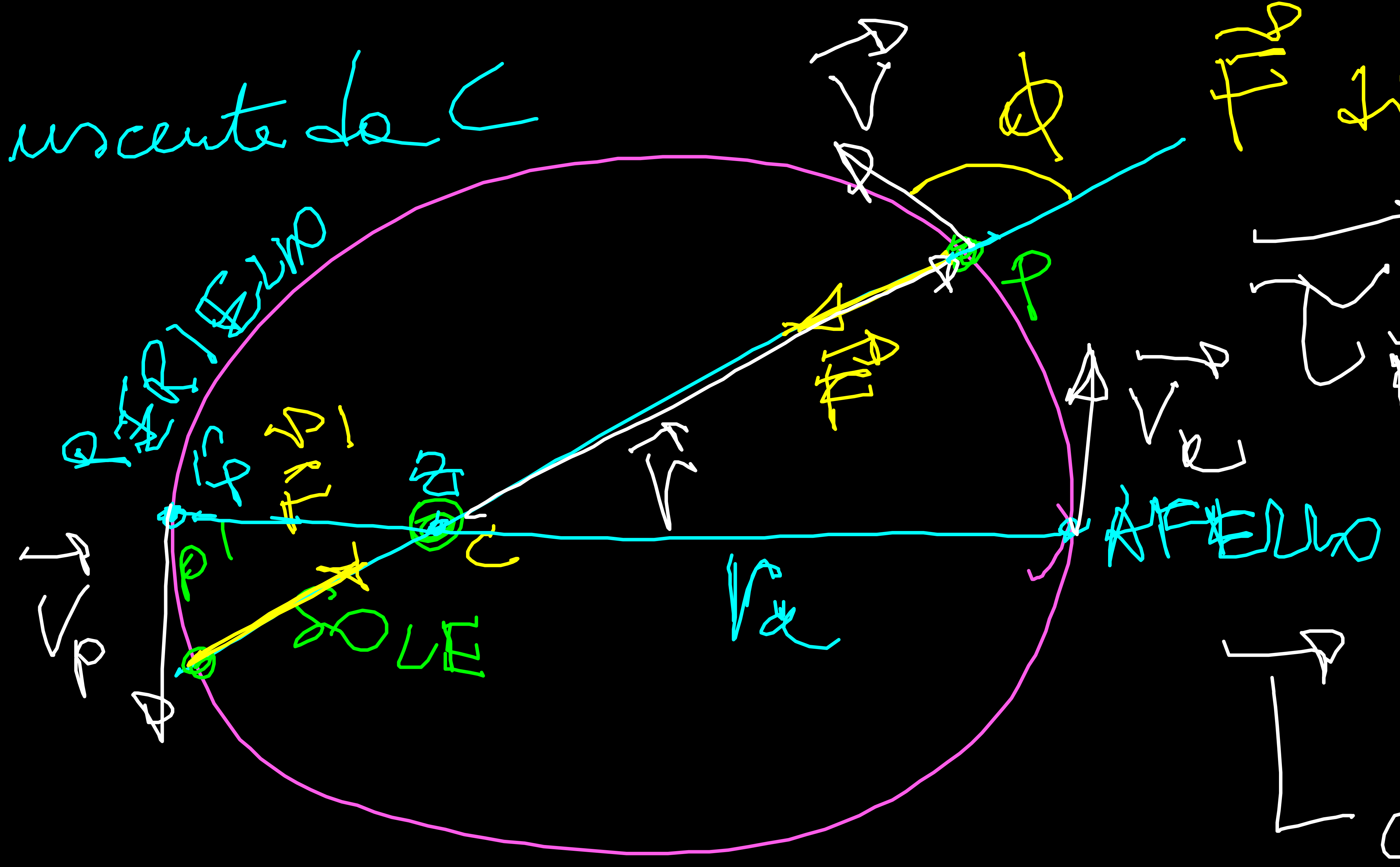
$$= \frac{1}{2}\left(2 I_{z,i}\frac{\omega_i^2}{4} - I_{z,i}\,\omega_i^2\right)$$

$$= \frac{1}{2}\left(-\frac{1}{2} I_{z,i}\,\omega_i^2\right) = -\frac{1}{4} I_{z,i}\,\omega_i^2$$

FORZE CENTRALI

z uscente da C

$\vec{F}$ diretta sempre verso C



$$\vec{\tau}_{\vec{F},C} = 0 \Rightarrow \frac{d\vec{L}_C}{dt} = 0$$

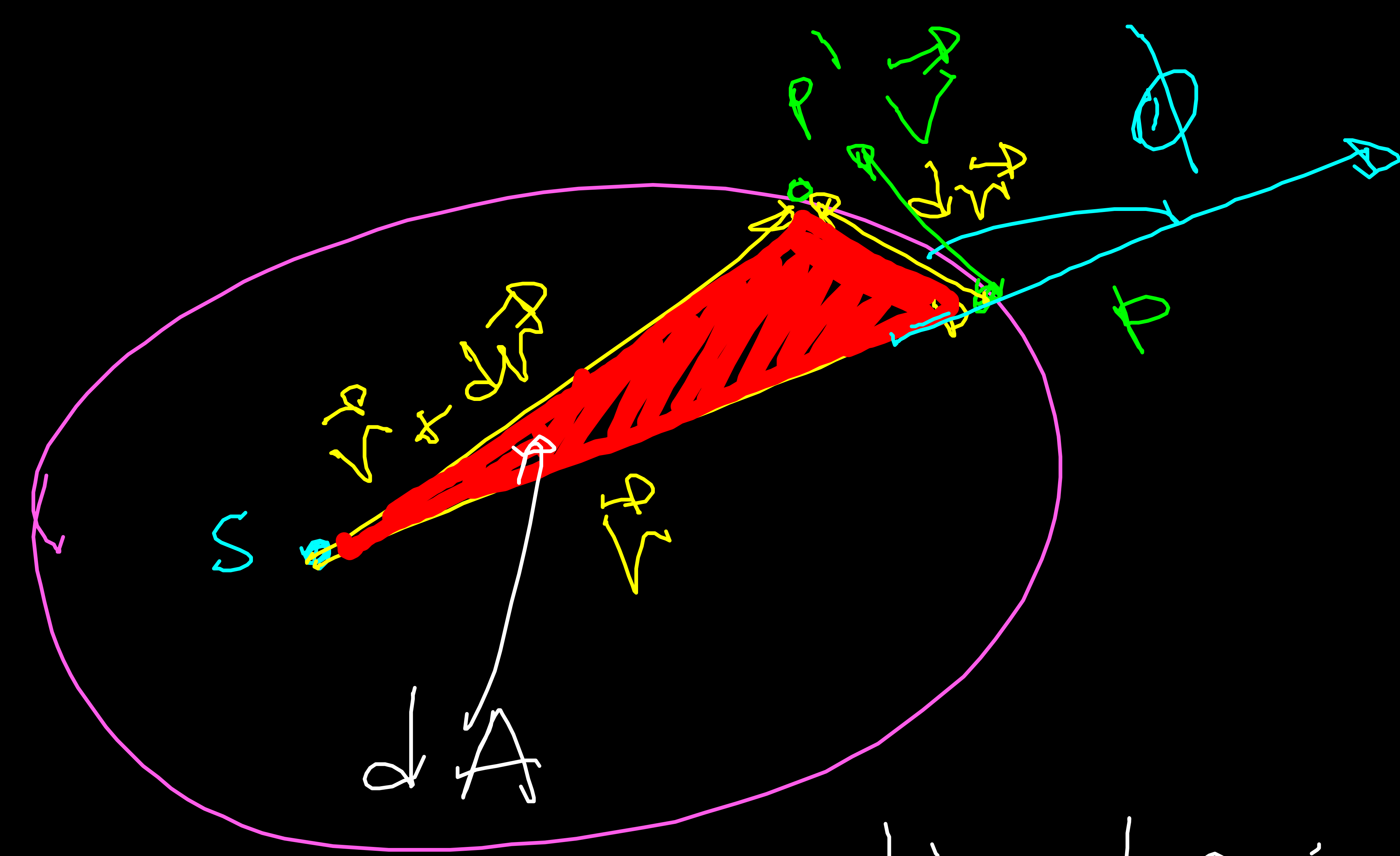
$\vec{L}_C$ è costante

$$\vec{L}_C = (r m v \sin\phi)\hat{k}$$

$L_z = r m v \sin\phi$ è costante

$$m r_a v_a = m r_p v_p \Rightarrow v_p = \frac{r_a}{r_p} v_a \Rightarrow v_p > v_a$$

$P \to P'$ in dt



$$dA = \frac{1}{2} r \sin\phi \, dr$$

$$v = \frac{dr}{dt} \Rightarrow dr = v \, dt$$

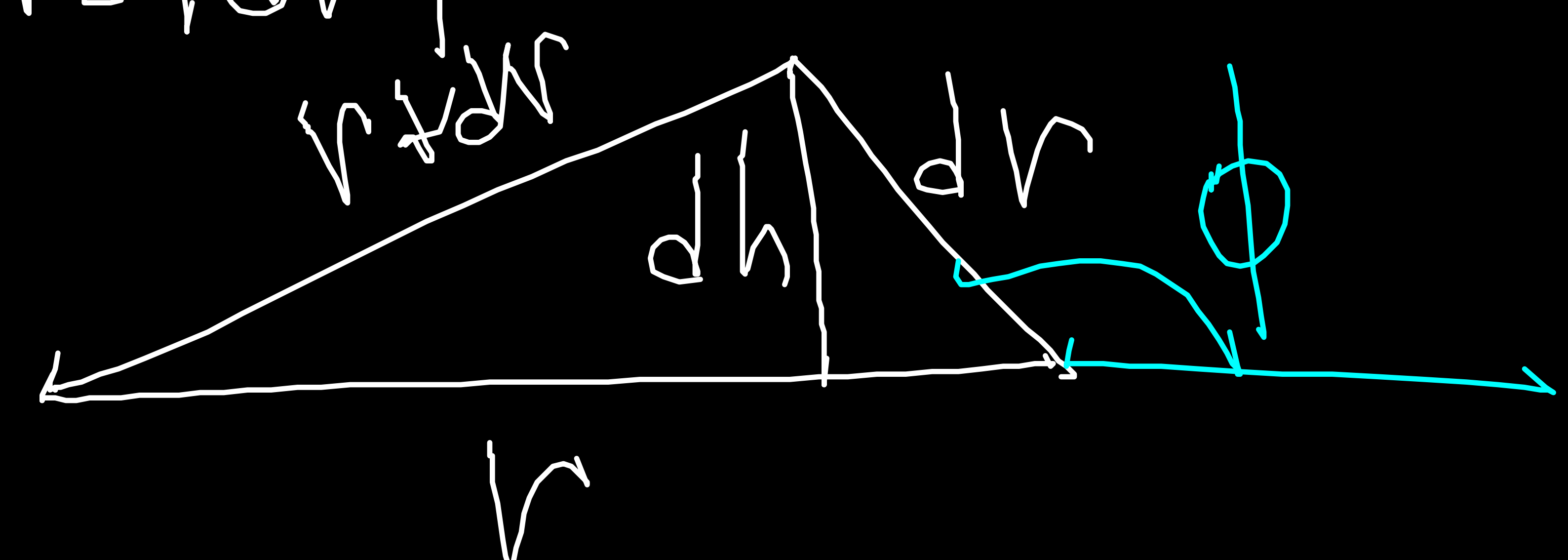
$$dA = \frac{1}{2} r \sin\phi \, v \, dt$$

$$\frac{dA}{dt} = \frac{m r v \sin\phi}{2} = \text{cost}$$

L_z = costante

$dr = |d\vec{r}|$

$dh = dr \sin\phi$



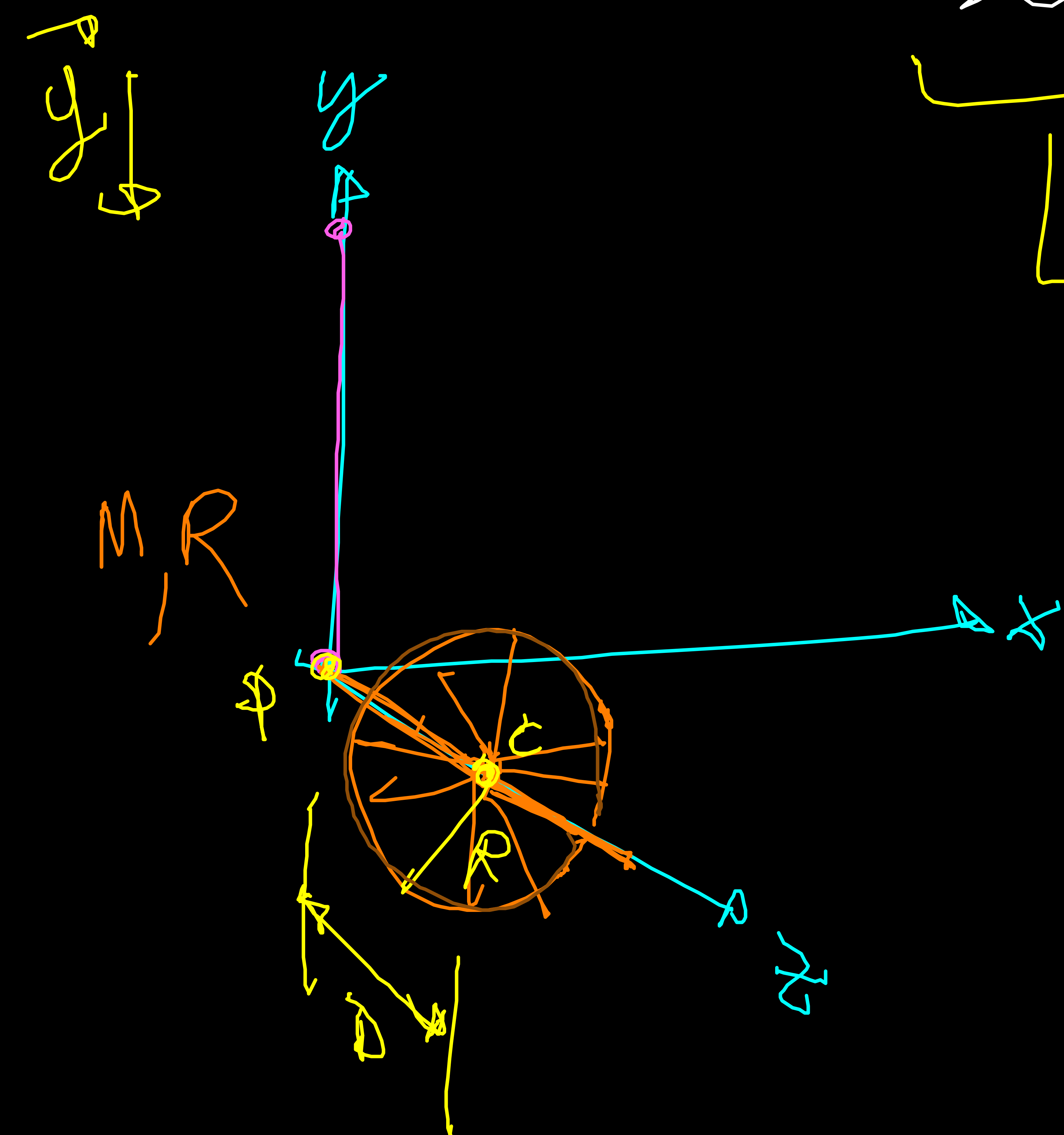
$$dA = \frac{\text{"base} \times \text{altezza"}}{2} = \frac{1}{2} r \, dr \sin\phi$$

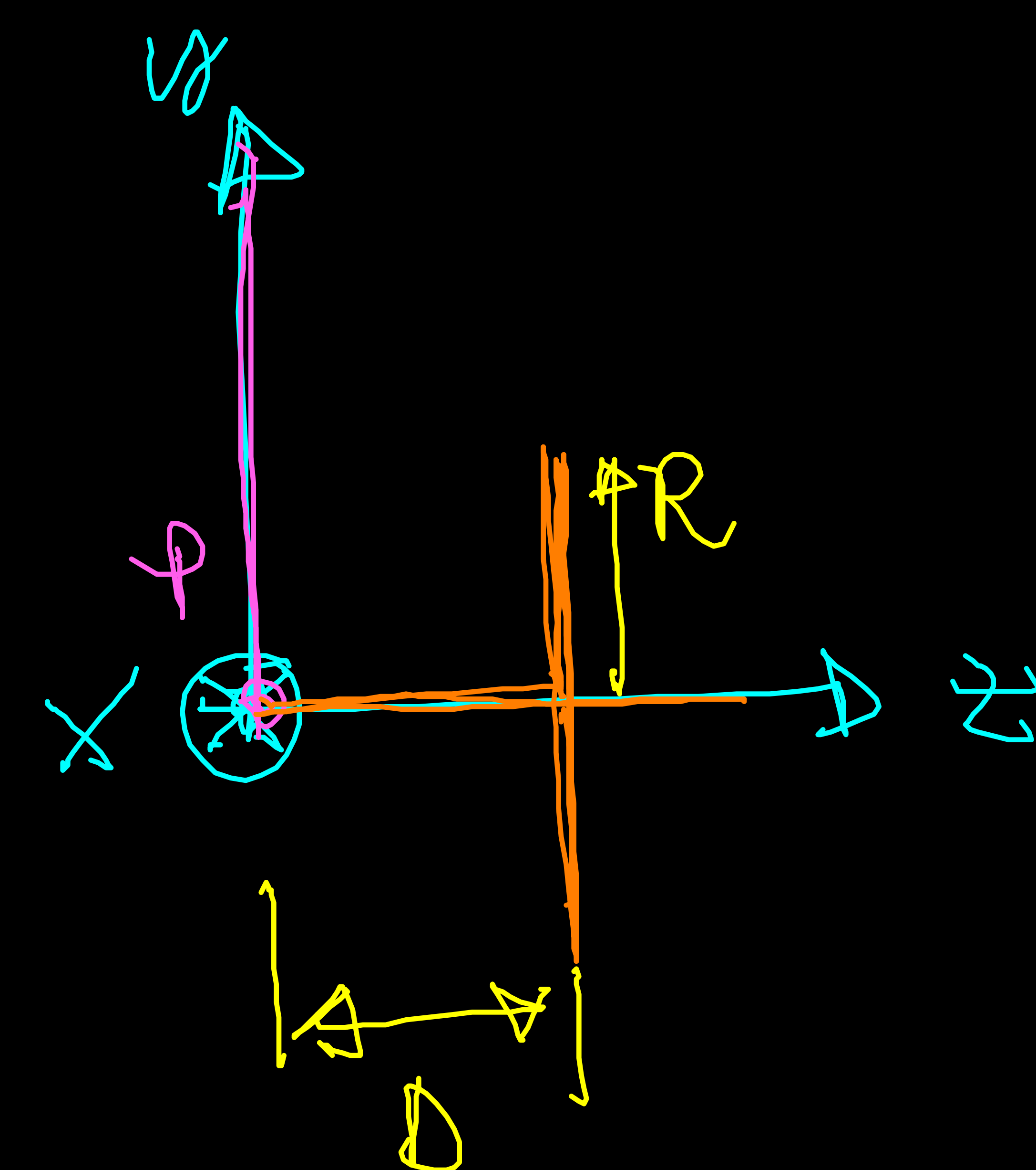
$$\sum \vec{\tau}_{O,ext} = \frac{d\vec{L}_O}{dt}$$

Se $d\vec{L}_O \parallel \vec{L}_O$ cambia solo $|\vec{L}_O|$

Se $d\vec{L}_O \nparallel \vec{L}_O \Rightarrow$ cambia la direzione di $\vec{L}_O$

↳ ESEMPIO: IL GIROSCOPIO





Se no rot.: