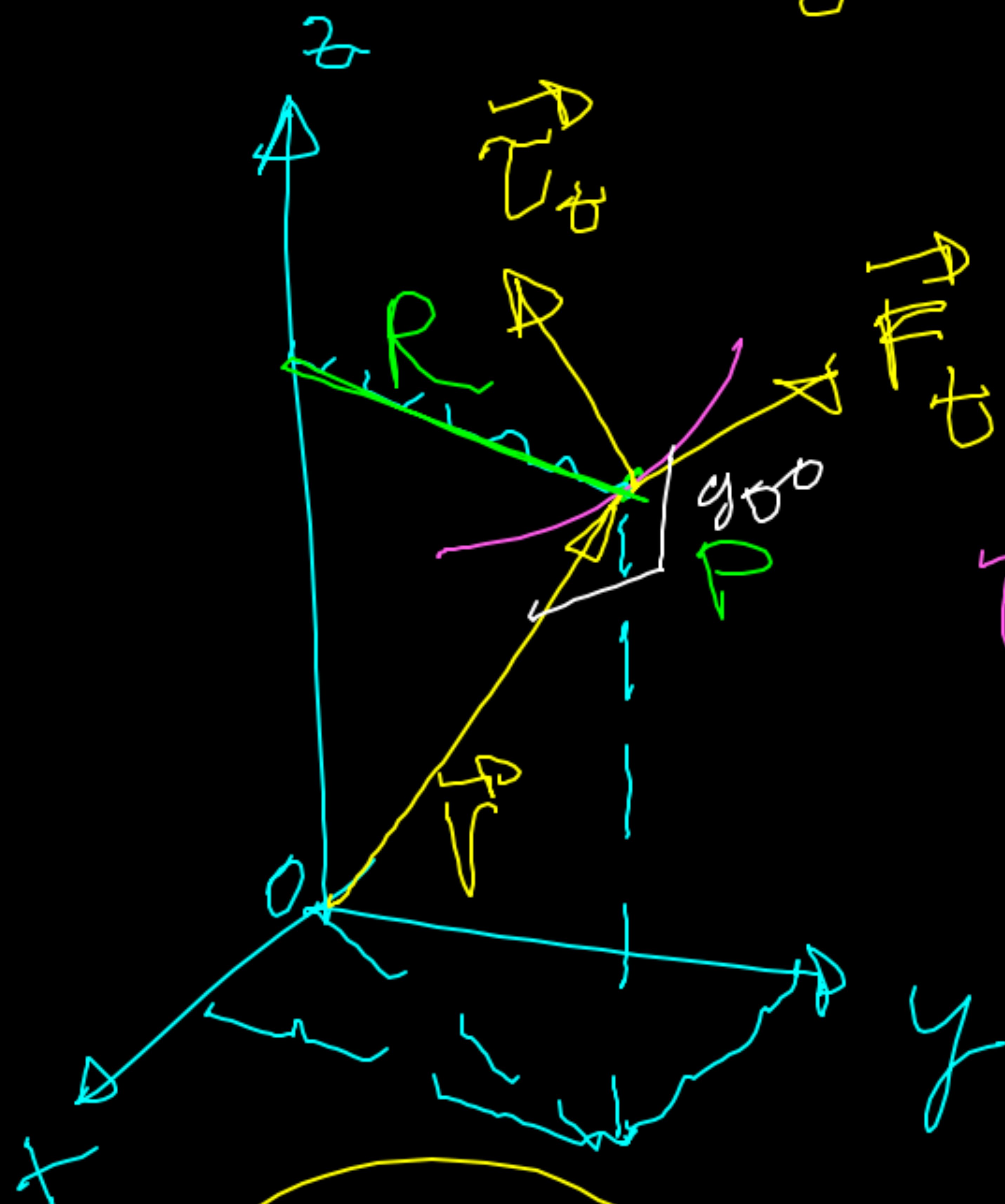
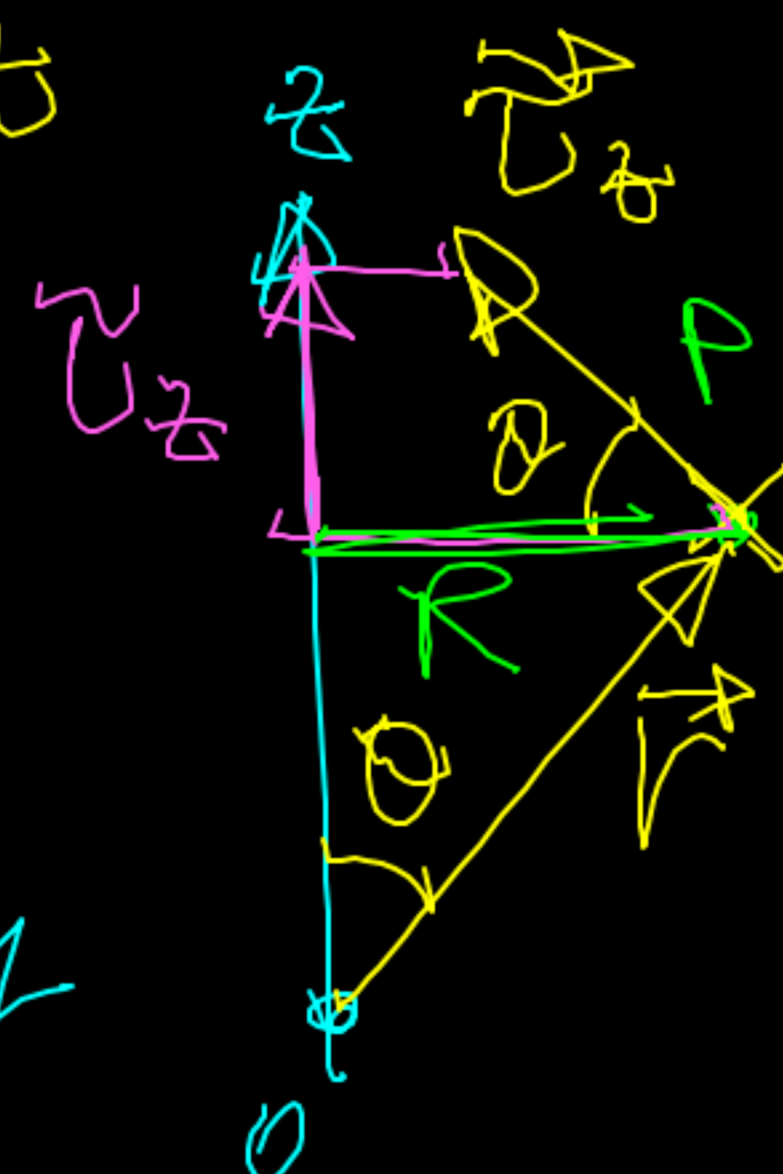
$$\tau_z = (\underbrace{\vec{r} \times \vec{F}_t}_{\vec{\tau}_t})_z \qquad |\vec{\tau}_t| = |\vec{r} \times \vec{F}_t| = r F_t (\sin 90°)$$

$$\tau_z = |\vec{\tau}_t| \sin\theta = r F_t \sin\theta$$

$$= (r \sin\theta) F_t$$





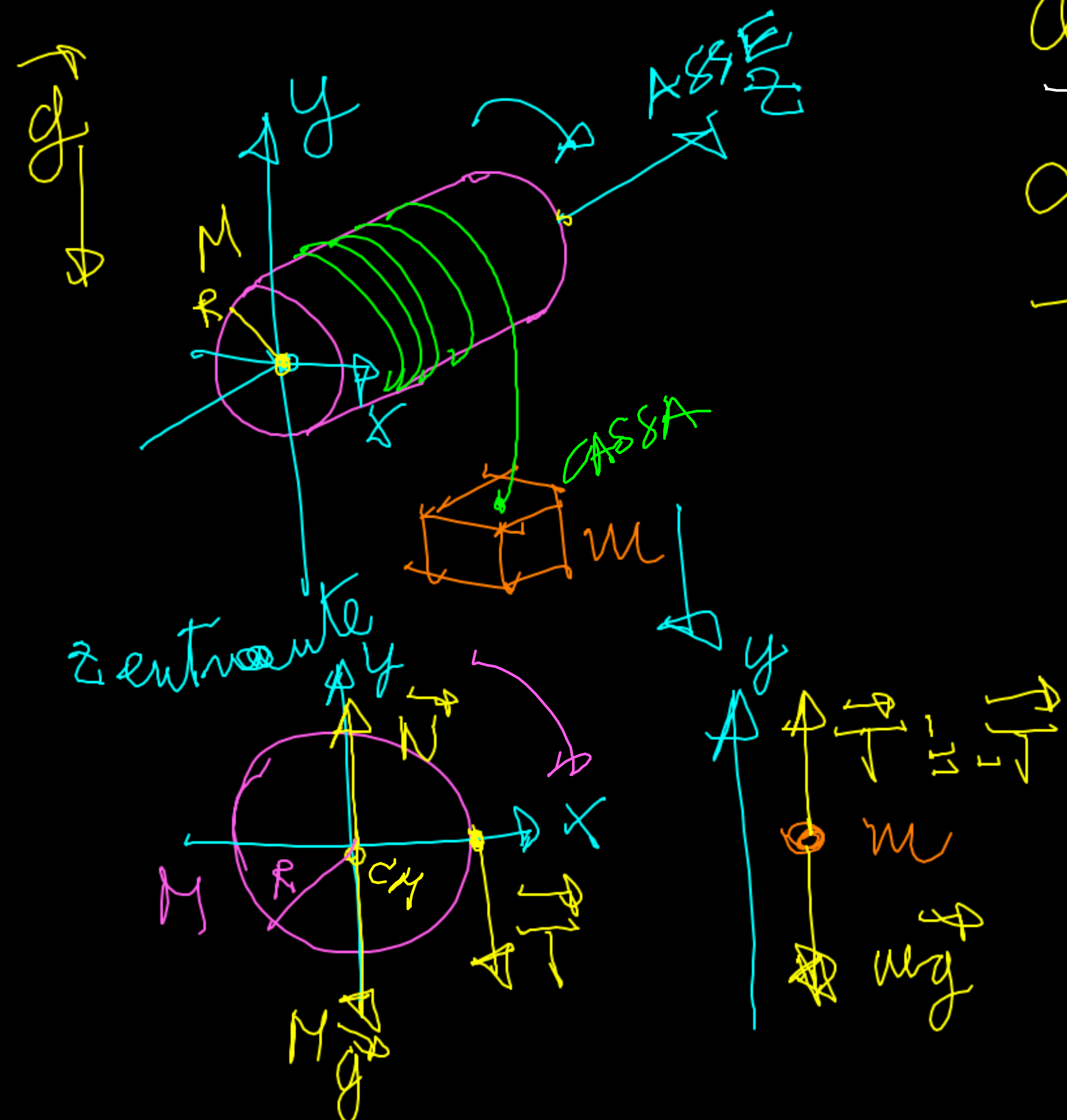
$\vec{F}_t$ è entrante $\qquad = R F_t$

$$\boxed{\tau_z = R F_t} = I_z \alpha_z$$

momento assiale

$\vec{r} \perp \vec{F}_t$

ESEMPIO



$a_y > 0$ cassa

α_z cilindro

T fune

CASSA $\quad T - mg = -m a_y$

CILINDRO $\quad N - Mg - T = 0$

I E.c. $\frac{d\vec{P}}{dt} = \sum \vec{F}_{est} = M\vec{a}_{cm}$

II E.c. $\sum \tau_z = I_z \alpha_z \qquad TR = \frac{1}{2} M R^2 \alpha_z$

$$\begin{cases} \text{i)} & T - mg = -m a_y \\ \text{ii)} & N - Mg - T = 0 \\ \text{iii)} & TR = \frac{1}{2} M R^2 \alpha_z \\ \text{iv)} & a_y = \alpha_z R \end{cases}$$

$$\Rightarrow$$

$$\text{(iii)} \oplus \text{(iv)} \quad T = \frac{1}{2} M R \frac{a_y}{R} = \frac{1}{2} M a_y$$

$$\text{i)} \quad \frac{1}{2} M a_y - mg = -m a_y$$

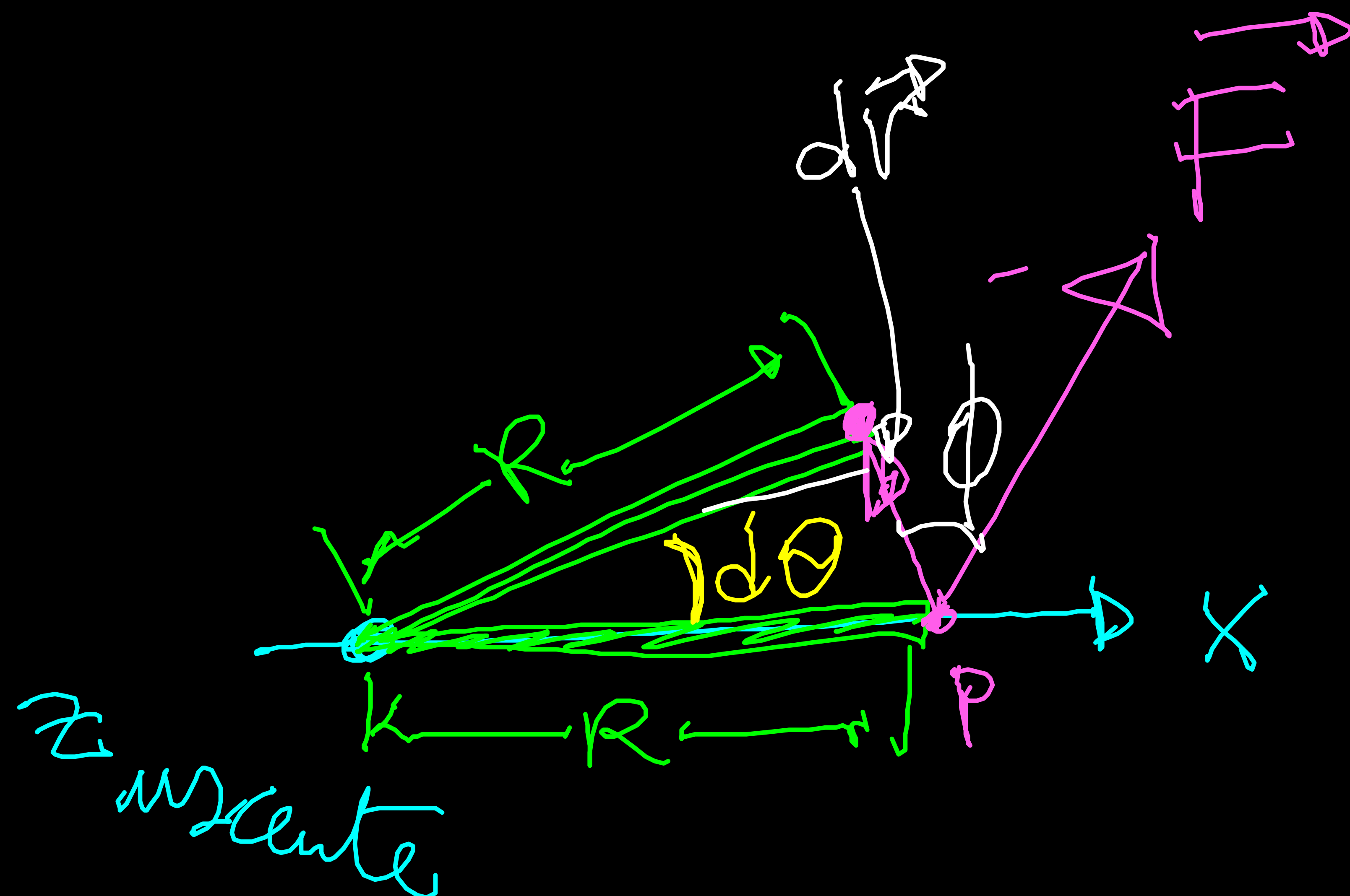
$$\left(\frac{1}{2} M + m\right) a_y = mg$$

$$a_y = \frac{mg}{\left(\frac{1}{2} M + m\right)} = \frac{2mg}{(M+2m)}$$

$$T = \frac{Mmg}{M+2m}$$

$$\alpha_z = \frac{2mg}{R(M+2m)}$$

LAVORO E ENERGIA (ROTAZIONI ASSIALI)



$$dW = \tau_z \, d\theta$$

$$W_{\theta_i \to \theta_f} = \int_{\theta_i}^{\theta_f} \tau_z \, d\theta$$

$$|d\vec{r}| = R \, d\theta$$

$$dW = \vec{F} \cdot d\vec{r} = \overbrace{(F \cos\phi)}^{F_t} dr$$

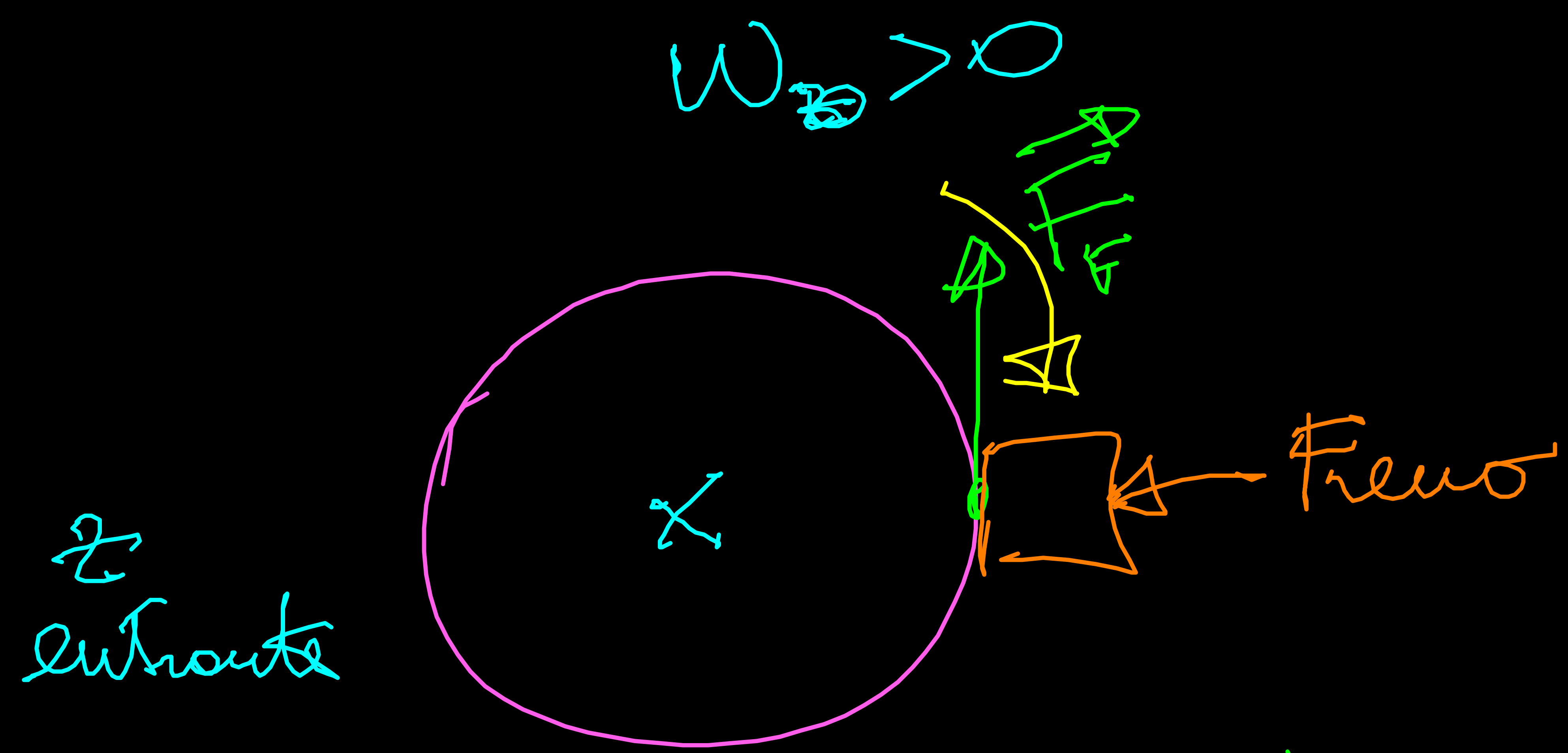
$$= F_t R \, d\theta = \tau_z \, d\theta$$

ESEMPIO PREC.

$$\tau_z = TR > 0$$

$$\Delta\theta = \pi/4 > 0$$

$$W_{0 \to \pi/4} = TR\Delta\theta > 0$$



$\vec{\omega}$ entra

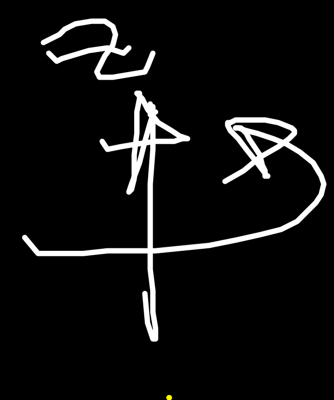
$$\tau_z(F_F) = -F_F \cdot R$$

$$\Delta\theta = \omega_0 \Delta t > 0$$

$$d\theta = \omega_0 \, dt$$

$$dW = \tau_z \, d\theta = -F_F R \omega_0 \, dt < 0$$

TEOREMA DELL'ENERGIA CIN. NEL CASO ROTATORIO INTORNO AD UN ASSE FISSO



$$dW_{TOT} = \left(\sum \tau_z\right) d\theta = I_z \alpha_z \, d\theta = I_z \frac{d\omega_z}{dt} \omega_z \, dt$$

$$\left(\frac{d\theta}{dt} = \omega_z\right)$$

$$dW_{TOT} = I_z \frac{d\omega_z}{dt} \omega_z \, dt$$

$$\boxed{\frac{d}{dt}\left(\frac{1}{2}\omega_z^2\right) = \frac{d\omega_z}{dt}\omega_z}$$

$$= I_z \frac{d}{dt}\left(\frac{1}{2}\omega_z^2\right) dt = \frac{1}{2} d\left(\omega_z^2\right) I_z$$

$$dW_T = \frac{1}{2} I_z \, d(\omega_z^2)$$

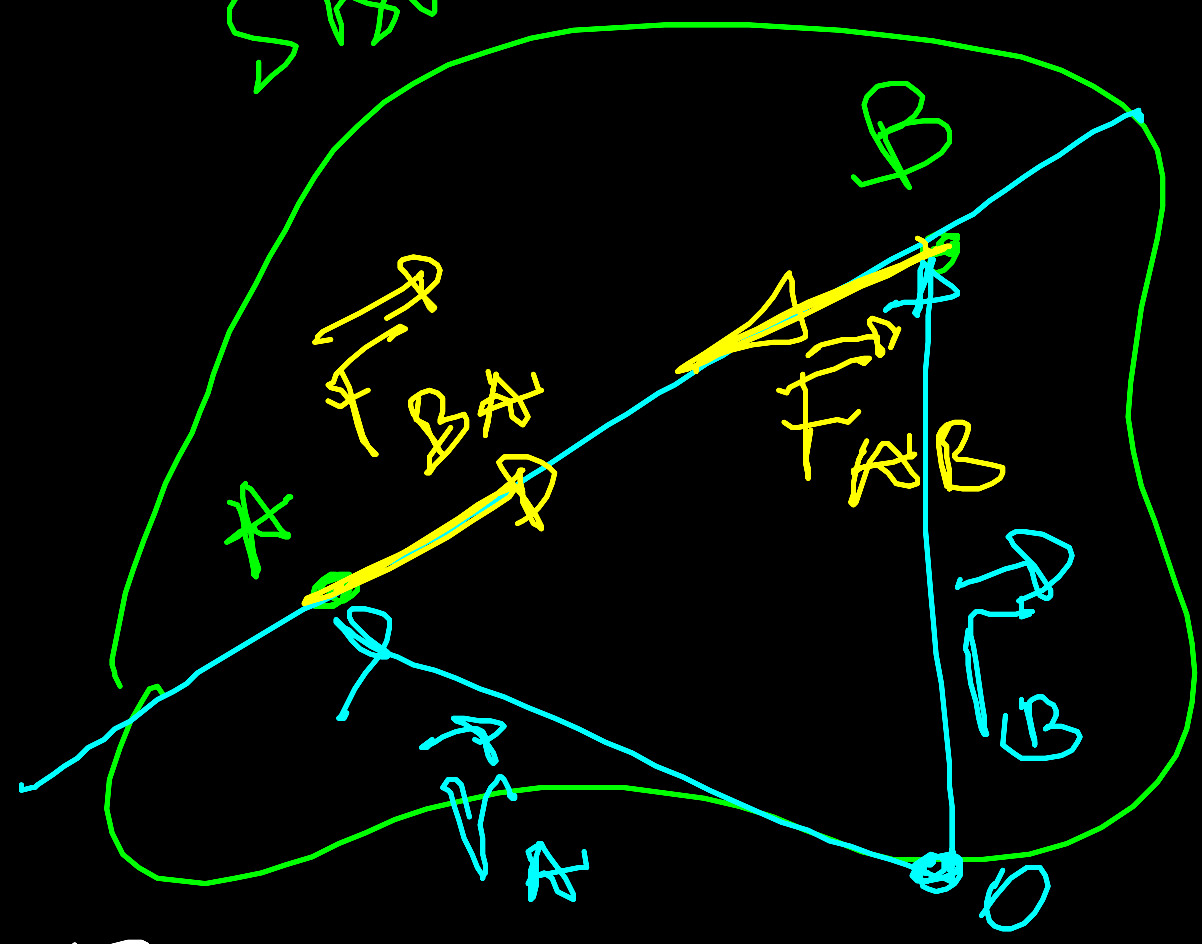
$$\omega_i \longrightarrow \omega_f$$

$$W_{tot} = \frac{1}{2} I_z \int_{\omega_i}^{\omega_f} d(\omega_z^2)$$

$$W_{tot} = \underbrace{\frac{1}{2} I_z \omega_f^2}_{\text{EN. CIN. "ROTAZIONALE"}} - \frac{1}{2} I_z \omega_i^2$$

POTENZA $P = \frac{dW}{dt} = \tau_z \frac{d\theta}{dt} = \tau_z \omega_z$

SISTEMA DI N PUNTI MAT.



$$W_T = \sum W_{j,T} = \sum (K_{j,f} - K_{j,i}) \qquad j = A, B \ldots$$

$W_{j,T}$: lavoro totale fatto da tutte le forze agenti sul "j-esimo" punto

Forze interne $\vec{F}_{AB} = -\vec{F}_{BA}$

$$dW_{int} = \vec{F}_{BA} \cdot d\vec{r}_A + \vec{F}_{AB} \cdot d\vec{r}_B$$

$$dW_{int} = \vec{F}_{BA} \cdot (d\vec{r}_A - d\vec{r}_B)$$

$$= \vec{F}_{BA} \cdot d(\underbrace{\vec{r}_A - \vec{r}_B}_{\text{distanza fra A e B}})$$

$$\boxed{W_T = K_{T,f} - K_{T,i}}$$

W_T: lavoro totale fatto nel sistema da tutte le forze (int + esterne)

$W_{int} \neq 0$, $W_{in} = 0$ per c. rigido

$$dW_{TOT} = \underbrace{dW_{TOT}^{CONS}}_{\to\, -dU_T^{EST} - dU_T^{INT}} + dW_{TOT}^{NON-CONS}$$

PER UN SISTEMA DI PUNTI

$$\underbrace{dW_{TOT} = dW_{TOT}^{NC} - d\left(U_T^{EST} + U_T^{INT}\right) = dK_{TOT}}$$

$$dW_{TOT}^{NC} = dK_{TOT} + d\left(U_T^{EST} + U_T^{INT}\right) = d\left(\underbrace{K_{TOT} + U_T^{EST} + U_T^{INT}}_{E_T}\right)$$

ENERGIA MECC. TOTALE

INT + EST $\to$

$$dW_{TOT}^{NC} = dE_T$$

SE $dW_T^{NC} = 0$

$\Rightarrow E_T$ SI CONSERVA

PER UN CORPO RIGIDO

$$dW_{TOT}^{NC} = d\left(K_T + U_T^{EST} + \cancel{U_T^{INT}}\right)$$

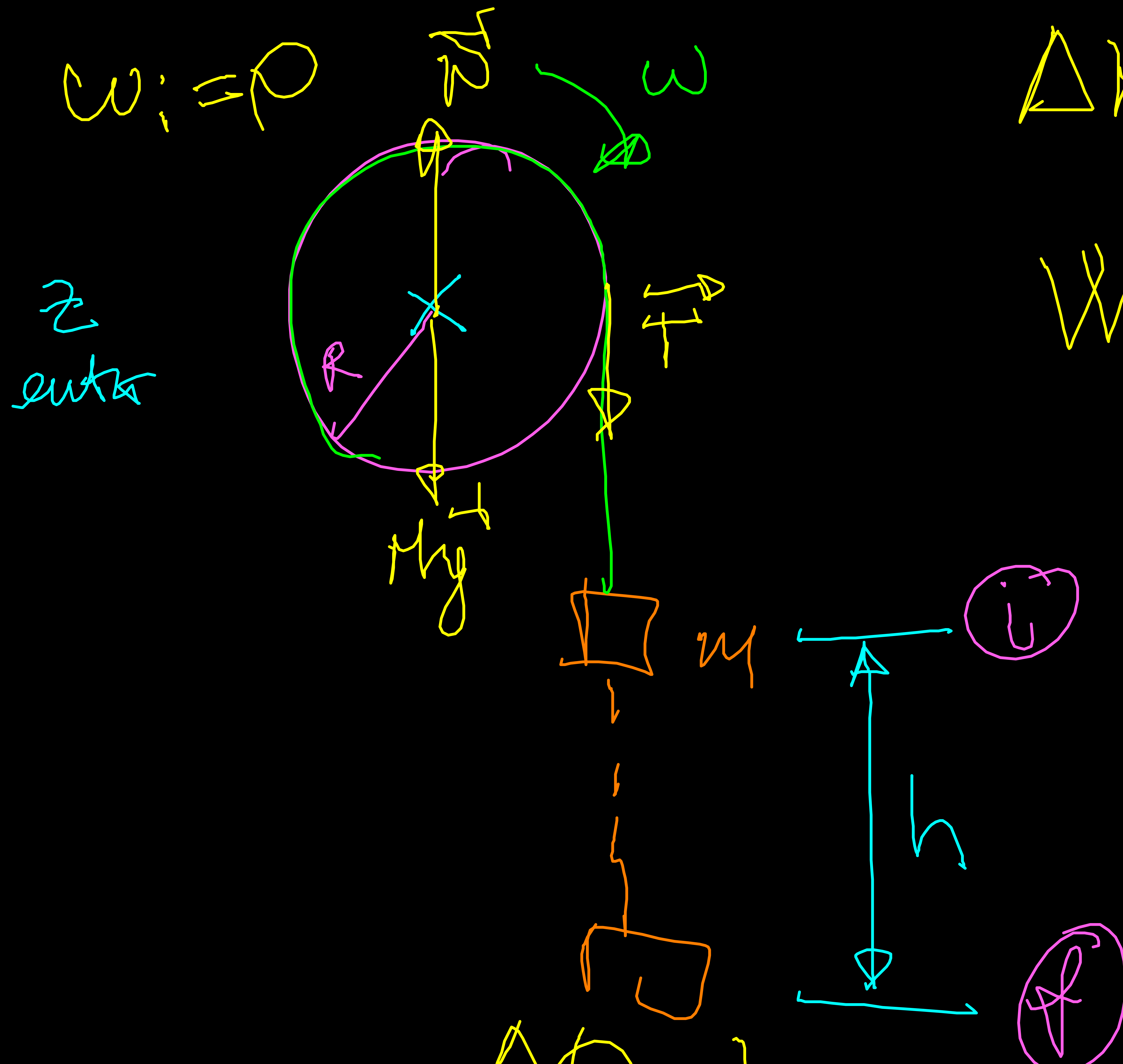
Supponiamo che $dW_{TOT}^{NC} = 0$

$\Rightarrow \boxed{K_T + U_T^{EST}}$ è costante

$$K_T = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM} \omega_{CM}^2$$

IN GENERALE

$$W_{TOT} = \Delta K_T = \int_{t_1}^{t_2} \left(\sum \vec{F}_{est}\right) \cdot \vec{v}_{CM}\, dt + \int_{t_1}^{t_2} \left(\sum \vec{\tau}_{est}\right) \cdot \vec{\omega}_{CM}\, dt$$



$$\Delta\Theta = \frac{h}{R}$$

$$\Delta K = \frac{1}{2} I_z \omega_f^2 - \frac{1}{2} I_z \omega_i^2 = \frac{1}{2} I_z \omega_f^2$$

$$W_T = \tau_z \cdot \Delta\Theta = T \cdot R \, \frac{h}{R}$$

$$\frac{1}{2} M R^2 \omega_f^2 = T h$$

$$\omega_f = \sqrt{\frac{T h}{\frac{1}{2} M R^2}}$$

$$P_f = \underbrace{T R}_{\tau_z} \cdot \omega_f =$$