DINAMICA DEL MOTO ROTATORIO

$$\frac{d\vec{L}_O}{dt} = \sum \vec{\tau}_{ext}$$

II EQ. CARDINALE DELLA MECC. (POLO FISSO)

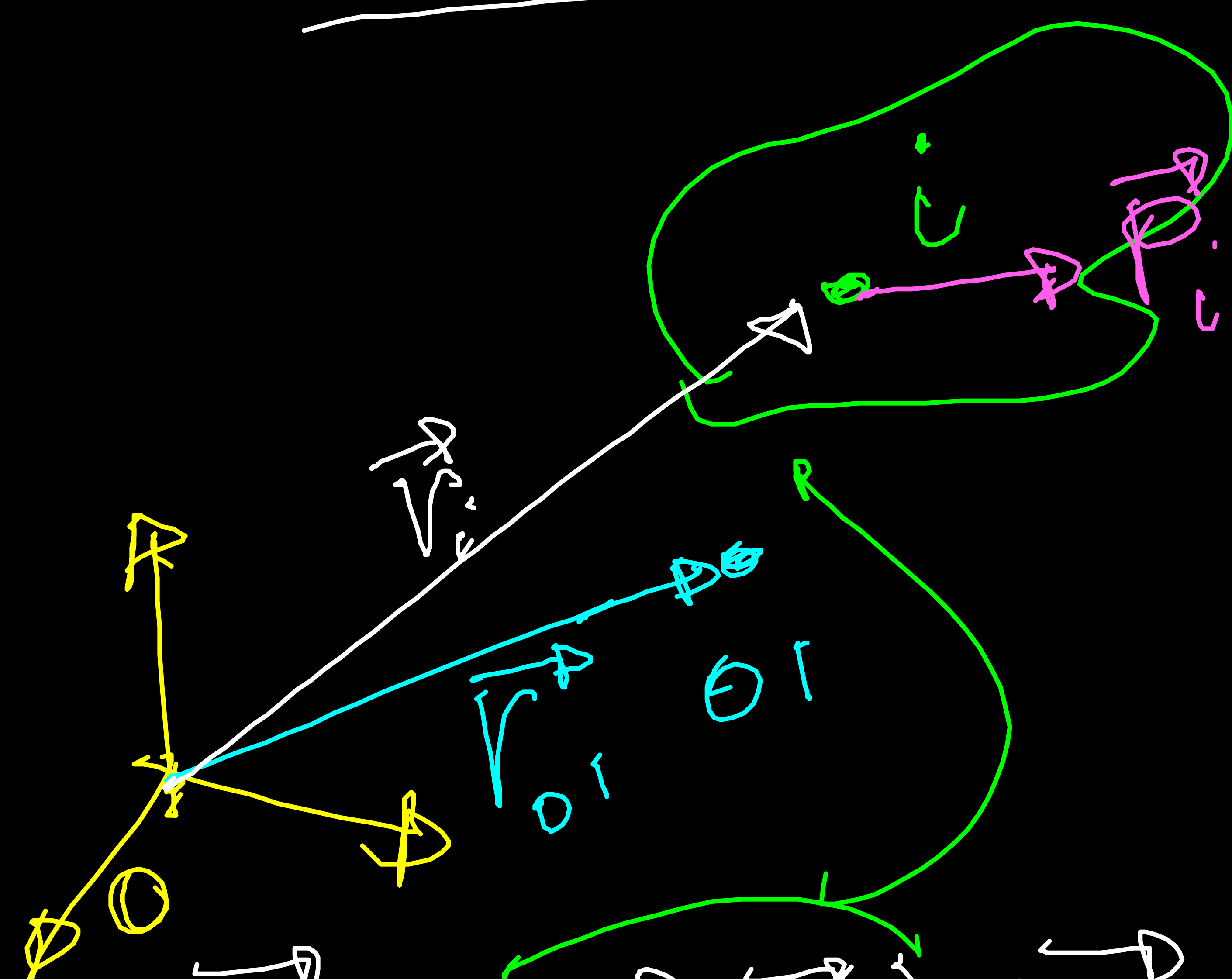
$$\vec{L}_O = \sum_{i=1}^{n} \vec{l}_{i,O}$$

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$$

I EQ. CARDINALE DELLA MECCANICA

- valgono per qualsiasi sistema meccanico
- condizioni necessarie
- se il corpo è rigido anche sufficienti (6 g.d.l.)

POLO MOBILE O'



$$\vec\ell_{O',i} = (\vec r_i - \vec r_{O'}) \times \vec p_i$$

$$\frac{d\vec\ell_{O',i}}{dt} = \vec r_i \times \left(\sum \vec F_i\right) - \frac{d\vec r_{O'}}{dt} \times \vec p_i$$

$$\vec L_{O'} = \sum \vec\ell_{O',i}$$

velocità di O'

q. d. moto totale

$$\boxed{\frac{d\vec L_{O'}}{dt} = \sum \vec\tau_{ext} - \vec V_{O'} \times \vec P}$$

II EQ. CARD. CON POLO O' MOBILE

SE $O' \equiv CM$ DEL SISTEMA

$$\frac{d\vec L_{CM}}{dt} = \sum \vec\tau_{ext} - \cancel{\vec V_{CM} \times \vec P} \qquad \vec P = M\vec V_{CM}$$

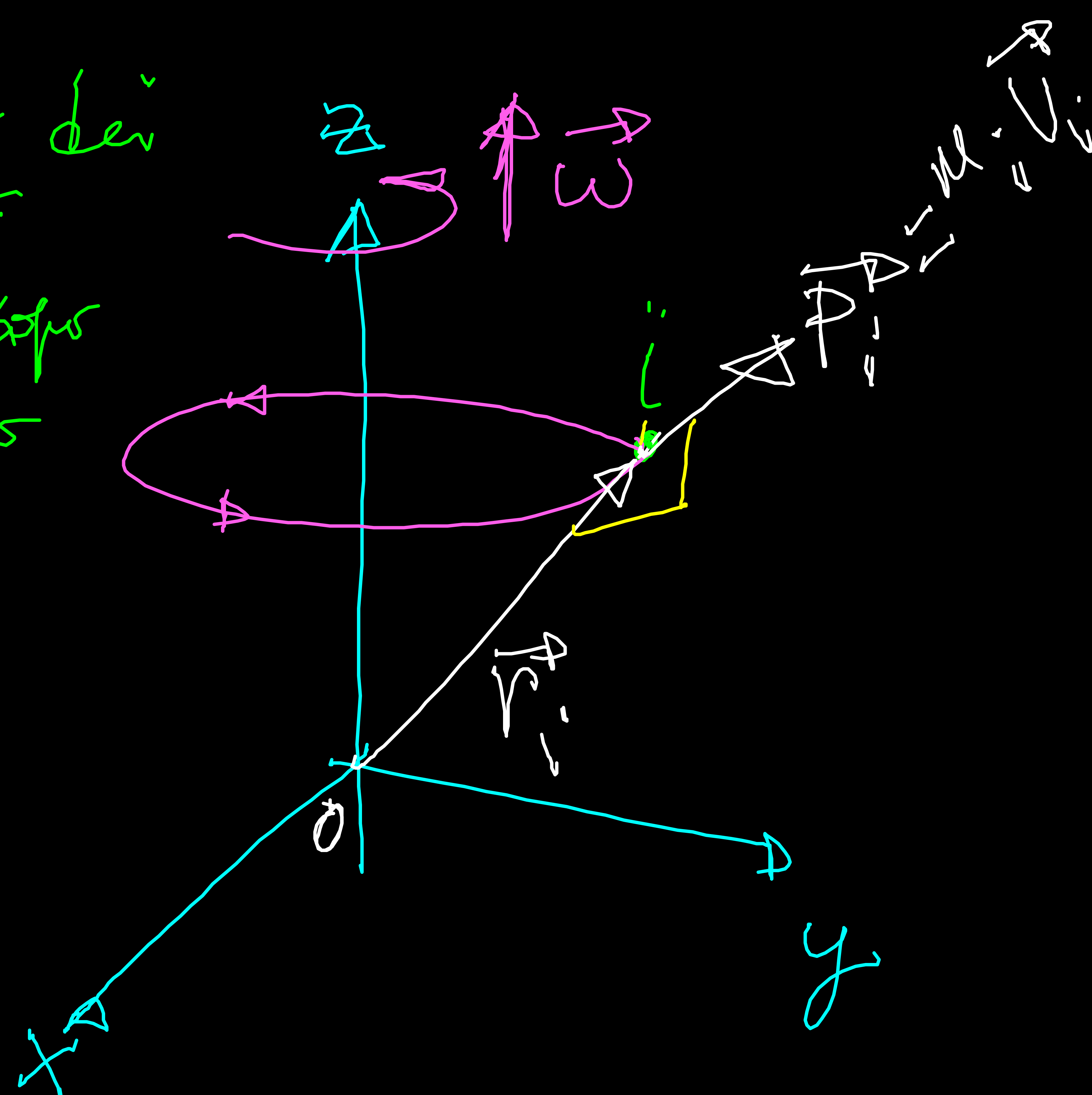
$$\boxed{\frac{d\vec L_{CM}}{dt} = \sum \vec\tau_{ext}}$$

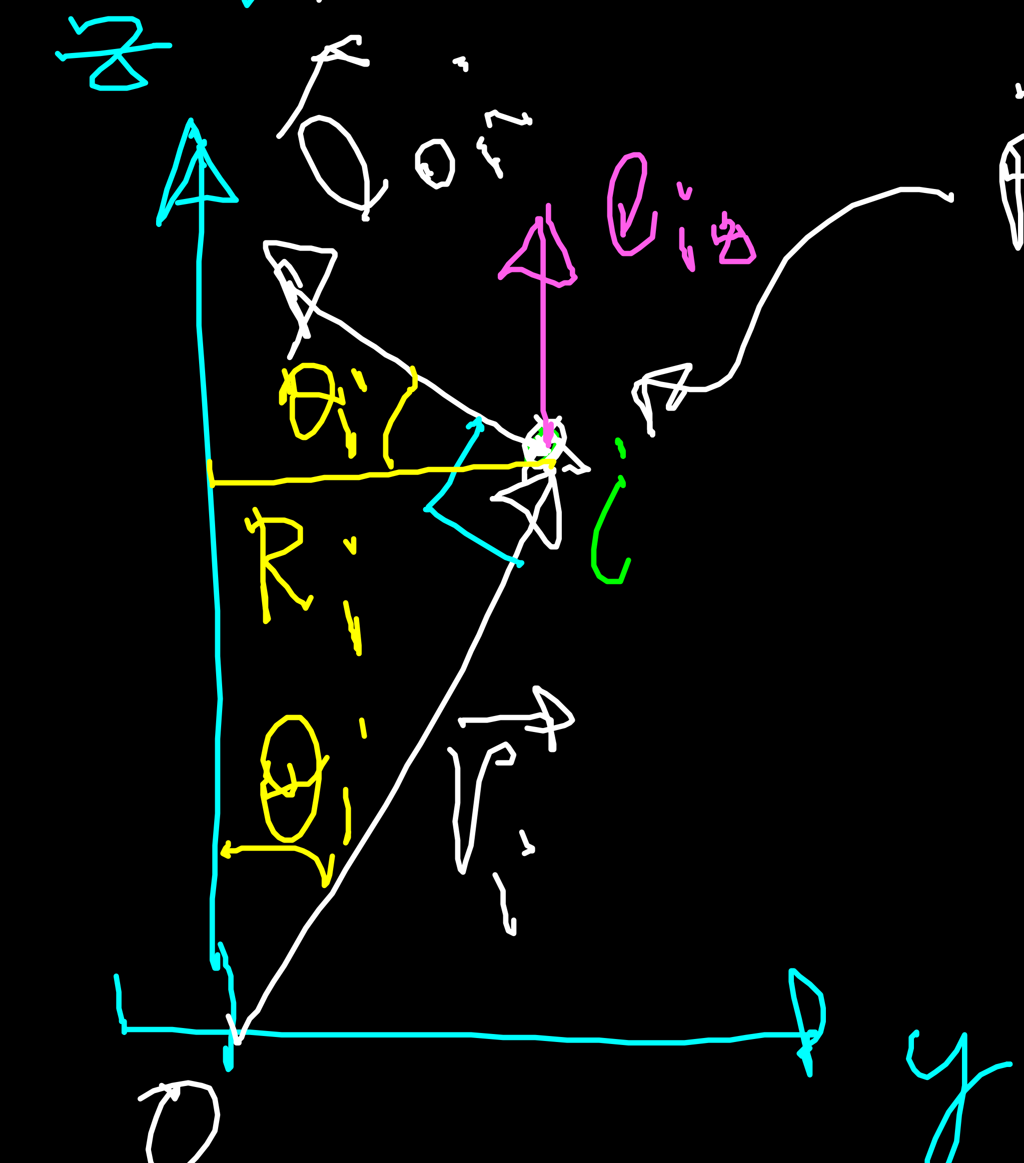
SE POLO NEL CM

fisso o mobile

ROTAZIONI DI UN CORPO RIGIDO INTORNO AD UN ASSE FISSO (z)

i = uno dei punti del corpo rigido





$\vec{\ell}_i = \vec{r}_i \times \vec{p}_i$; $|\vec{\ell}_i| = r_i p_i \cancel{\sin\theta_i}$ $\sin 90°$

$= \ell_i$

$\vec{p}_i$ è entrante

$\ell_{i,z} \to L_z = \Sigma(\ell_{i,z})$

$\ell_i = p_i r_i = m v_i r_i$

$\ell_i = m \omega_z R_i r_i$

$$\ell_{i,z} = \ell_i \sin\theta_i = m\omega_z R_i \underbrace{r_i \sin\theta_i}_{R_i} = m\omega_z R_i^2$$

$$\ell_{i,z} = m_i \omega_z R_i^2$$

$$\left(\vec{L}_O\right)_z = L_z = \sum_{i=1}^{N} \ell_{i,z} = \sum m_i \omega_z R_i^2 = \omega_z \underbrace{\left(\sum m_i R_i^2\right)}_{I_z}$$

$$\boxed{L_z = I_z \omega_z}$$

CORPO RIGIDO ROTANTE INTORNO A Z FISSO

"momento angolare assiale"

EQ. DEL MOTO

$$\left(\frac{d\vec{L}_0}{dt}\right)_z = \left(\sum \vec{\tau}_{ext}\right)_z \Rightarrow \boxed{\frac{dL_z}{dt} = \sum \tau_{ext,z}}$$

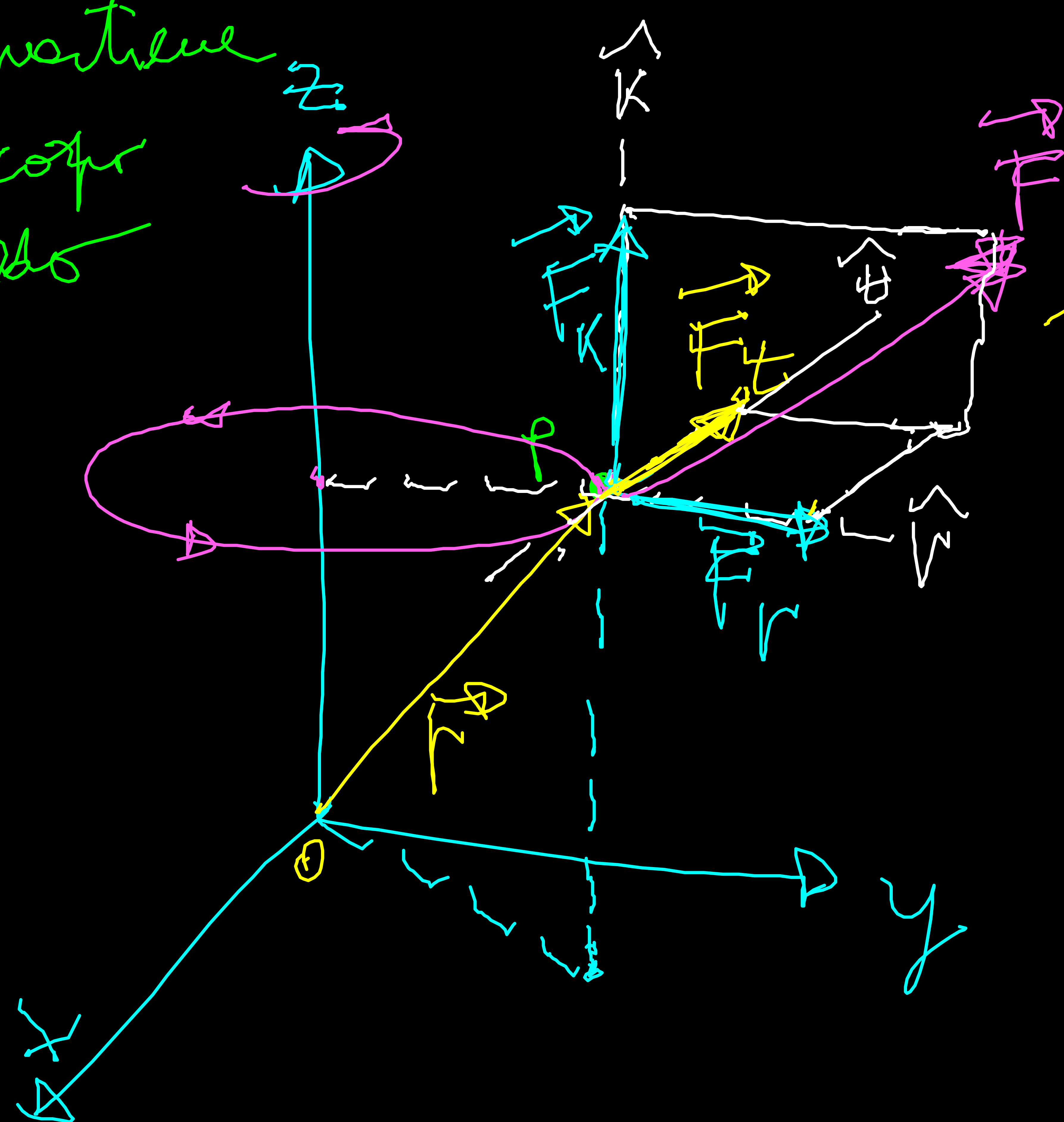
$$\frac{dL_z}{dt} = \frac{d}{dt}(I_z \omega_z) = I_z \frac{d\omega_z}{dt} = I_z \alpha_z$$

$$\boxed{\sum \tau_z = \frac{dL_z}{dt} = I_z \alpha_z}$$

EQ. DEL MOTO ROTATORIO INTORNO A z FISSO (CORPO RIGIDO)

$\tau_z \longleftrightarrow$ FORZE ESTERNE

Porzione del corpo rigido



$\boxed{\text{Componente "z" del momento generato da } \vec F \text{ ?}}$

$$\tau_z = (\vec\tau)_z = (\vec r \times \vec F)_z$$

$$\vec F = \vec F_r + \vec F_K + \vec F_t$$

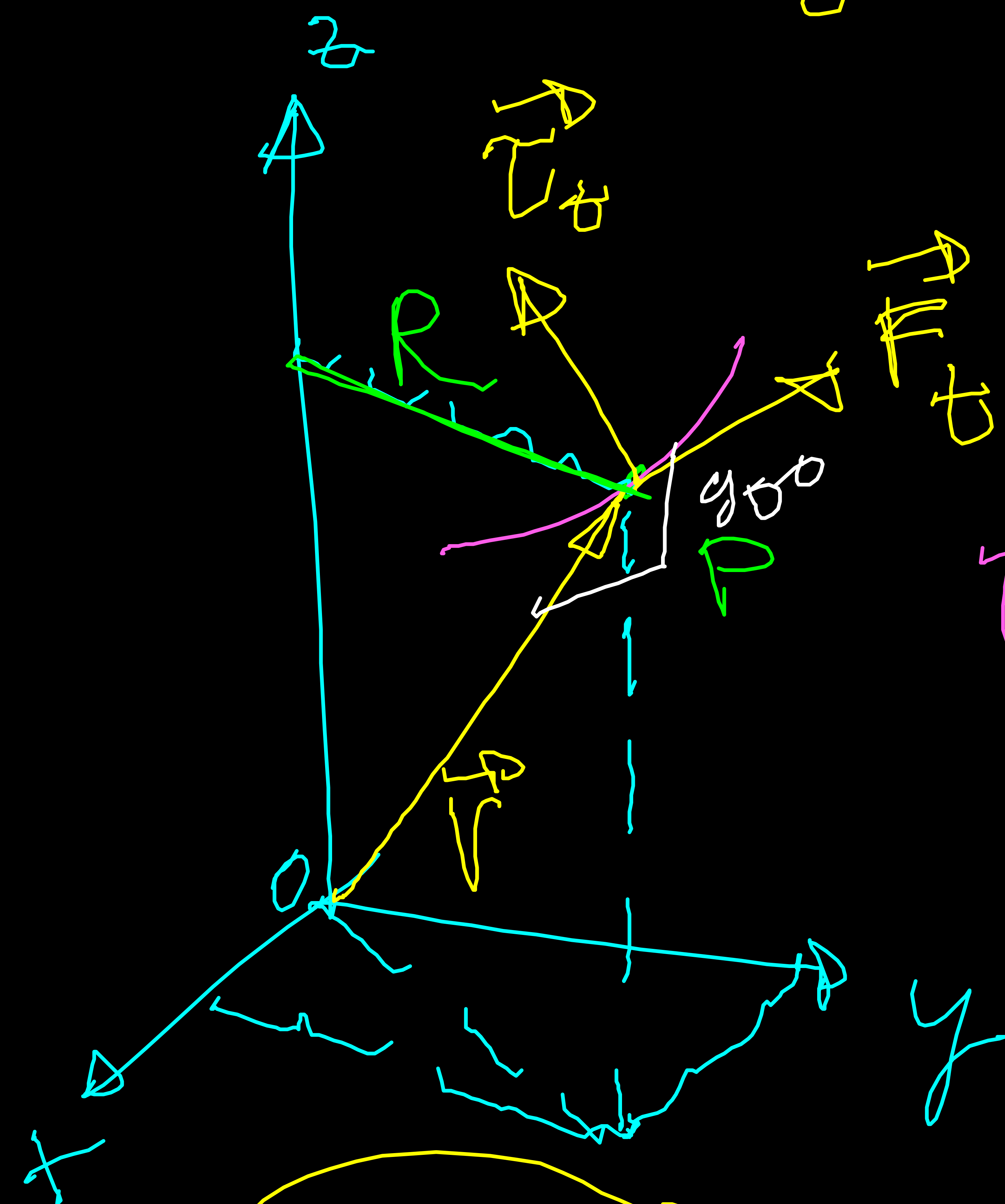
$$\tau_z = \left[\vec r \times (\vec F_r + \vec F_K + \vec F_t)\right]_z =$$

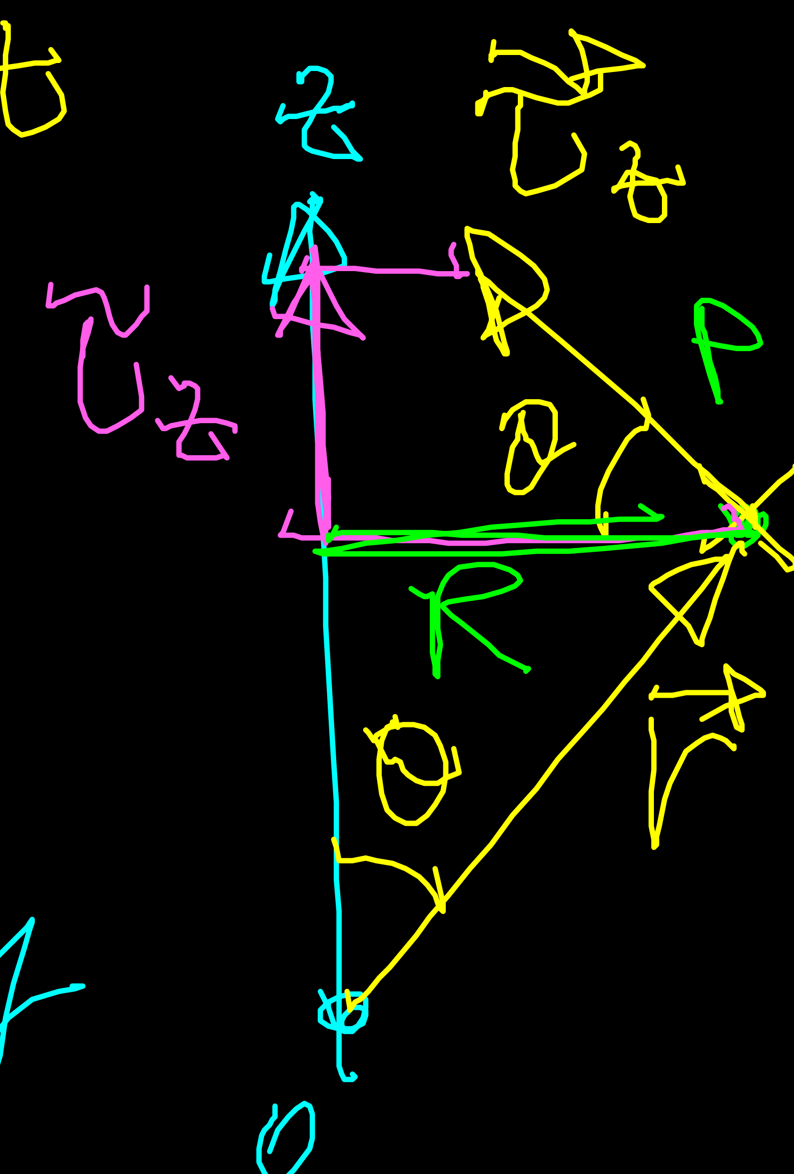
$$\tau_z = \cancel{(\vec r \times \vec F_r)_z} + \cancel{(\vec r \times \vec F_K)_z} + \underline{(\vec r \times \vec F_t)_z}$$

$$\tau_z = (\underbrace{\vec{r} \times \vec{F}_t}_{\vec{\tau}_t})_z \qquad |\vec{\tau}_t| = |\vec{r} \times \vec{F}_t| = r F_t \, (\sin 90°)$$

$$\tau_z = |\vec{\tau}_t| \sin\theta = r F_t \sin\theta$$

$$= (r \sin\theta) F_t$$





$\vec{F}_t$ è entrante

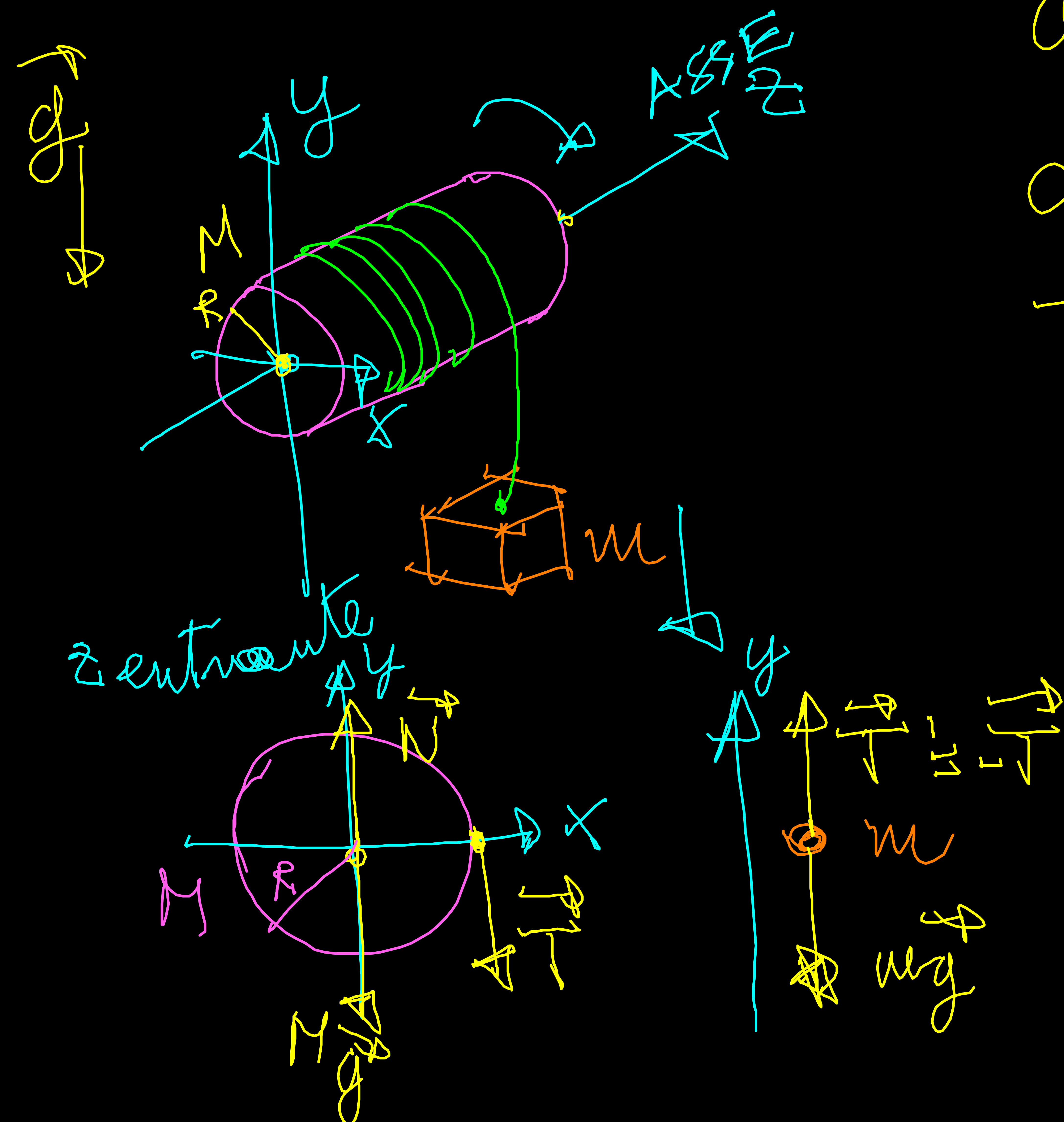
$$= R F_t$$

$\vec{r} \perp \vec{F}_t$

$$\boxed{\tau_z = R F_t} = I_z \alpha_z$$

momento assiale

ESEMPIO



a_y corpo

α_z cilindro

T fune