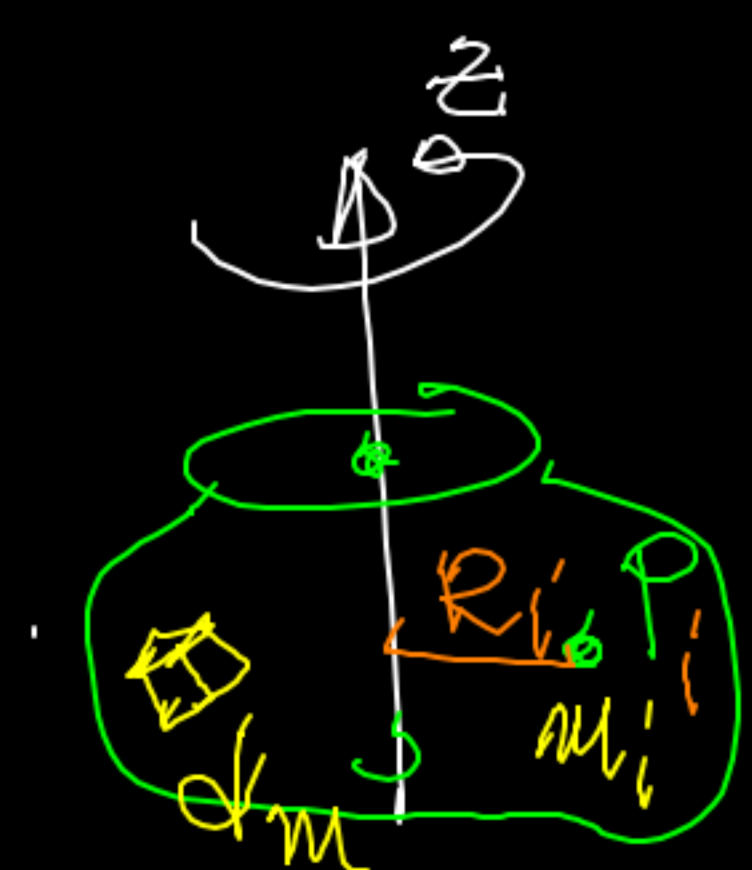
MOMENTO DI INERZIA

$K = \frac{1}{2} I_z \omega_z^2$

ROT. DI CORPO RIGIDO INTORNO A Z FISSO



$I_z = \sum_{i=1}^{N} m_i R_i^2$ CASO DISCRETO

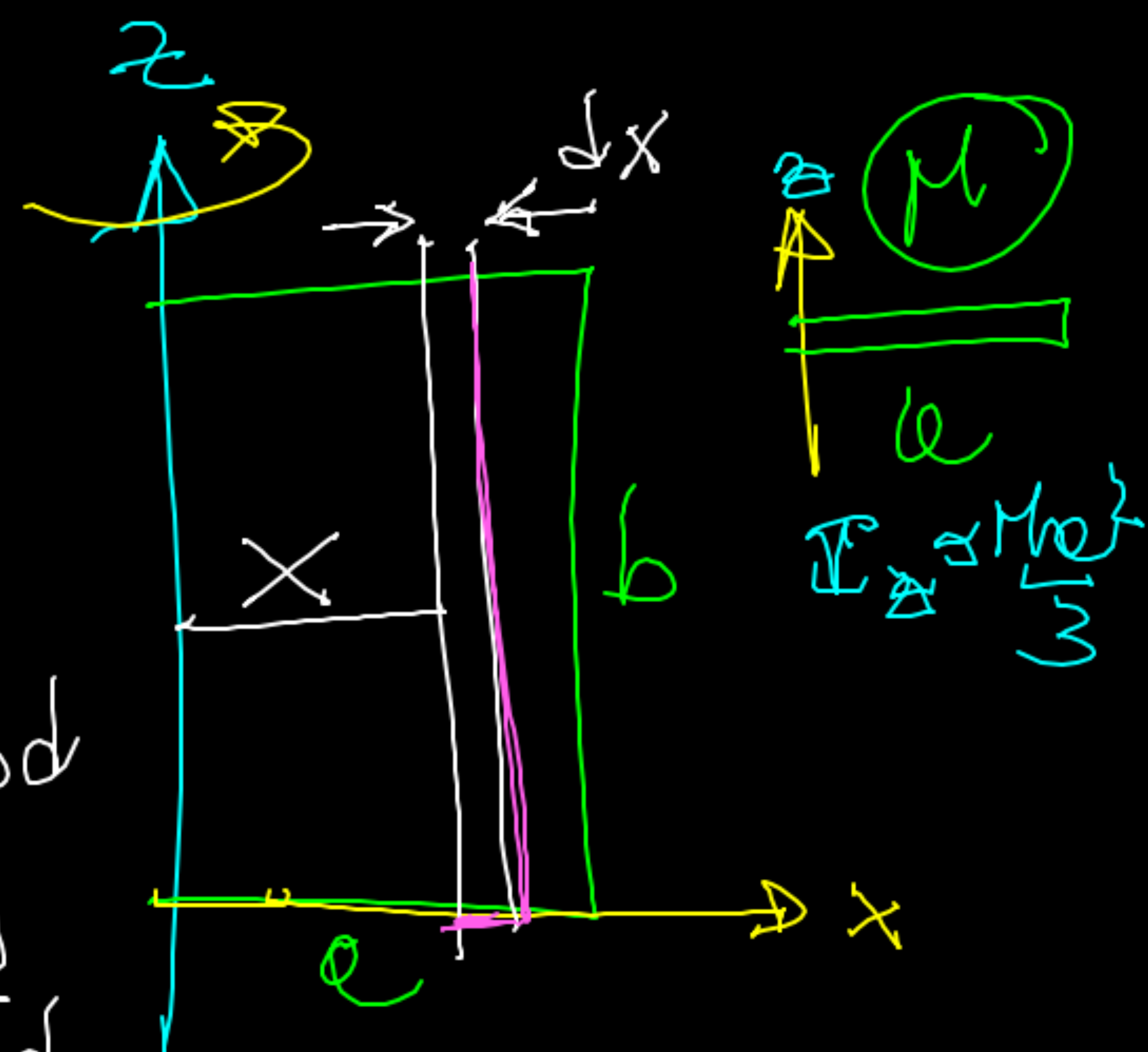
$I_z = \int_{CORPO} R^2 dm = \int_{\text{Vol. del corpo}} \rho R^2 dV$

ESEMPIO DI CALCOLO

I_z della parte (omogenea) sottile

M

ρ



$V = ebd$

$\rho = \frac{M}{ebd}$

$$I_z = \int \rho R^2 dV$$

Vol del corpo

$$dV = b\,d\,dx$$

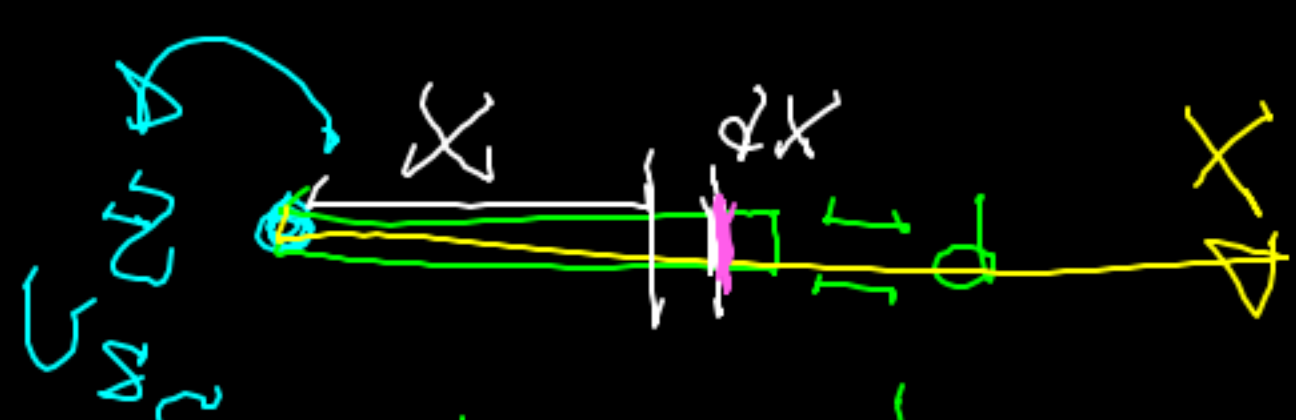
$$R = x$$

$$\rightarrow I_z = \int_0^e \rho\, b d\, x^2 dx$$

$$I_z = \rho b d \int_0^e x^2 dx =$$

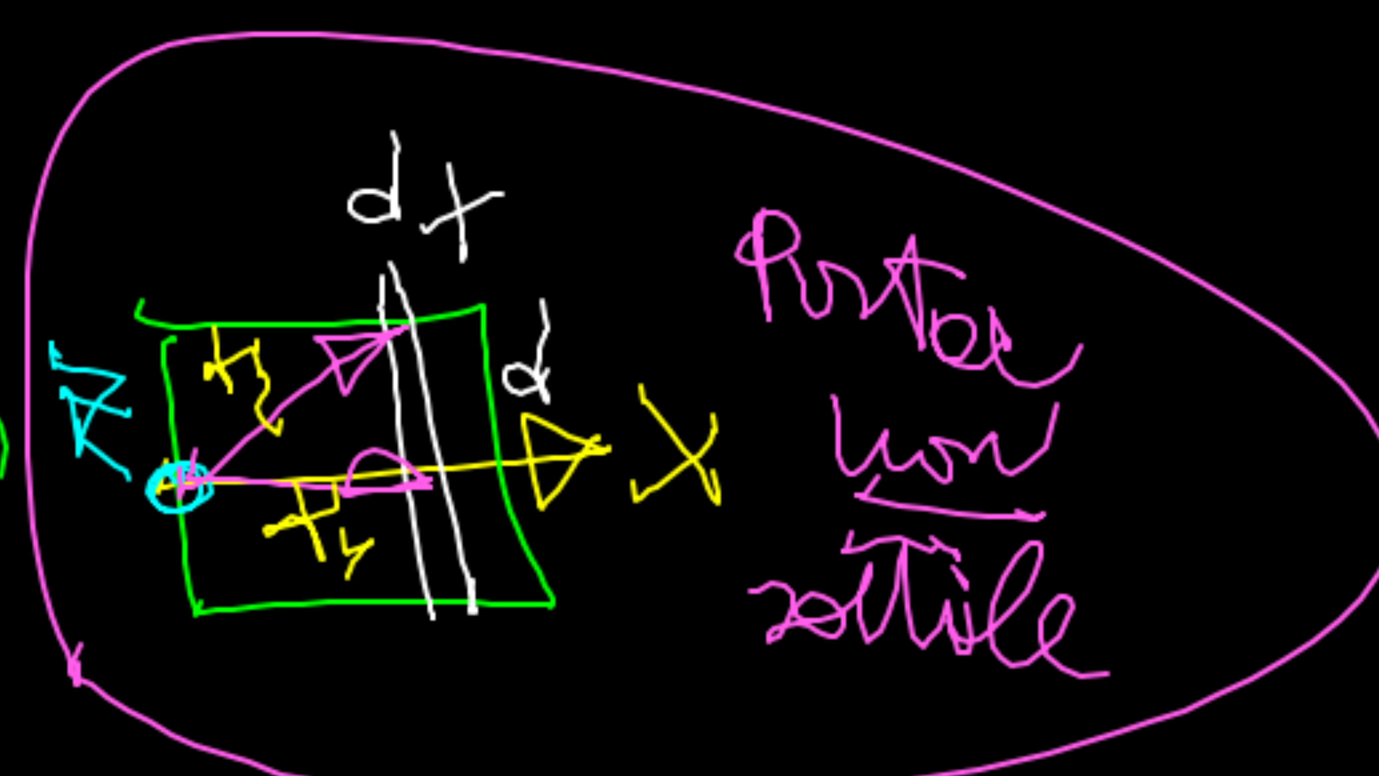
$$= \rho b d \frac{e^3}{3} =$$

$$I_z = \frac{M}{ebd} bd \frac{e^3}{3} = \frac{Me^2}{3}$$



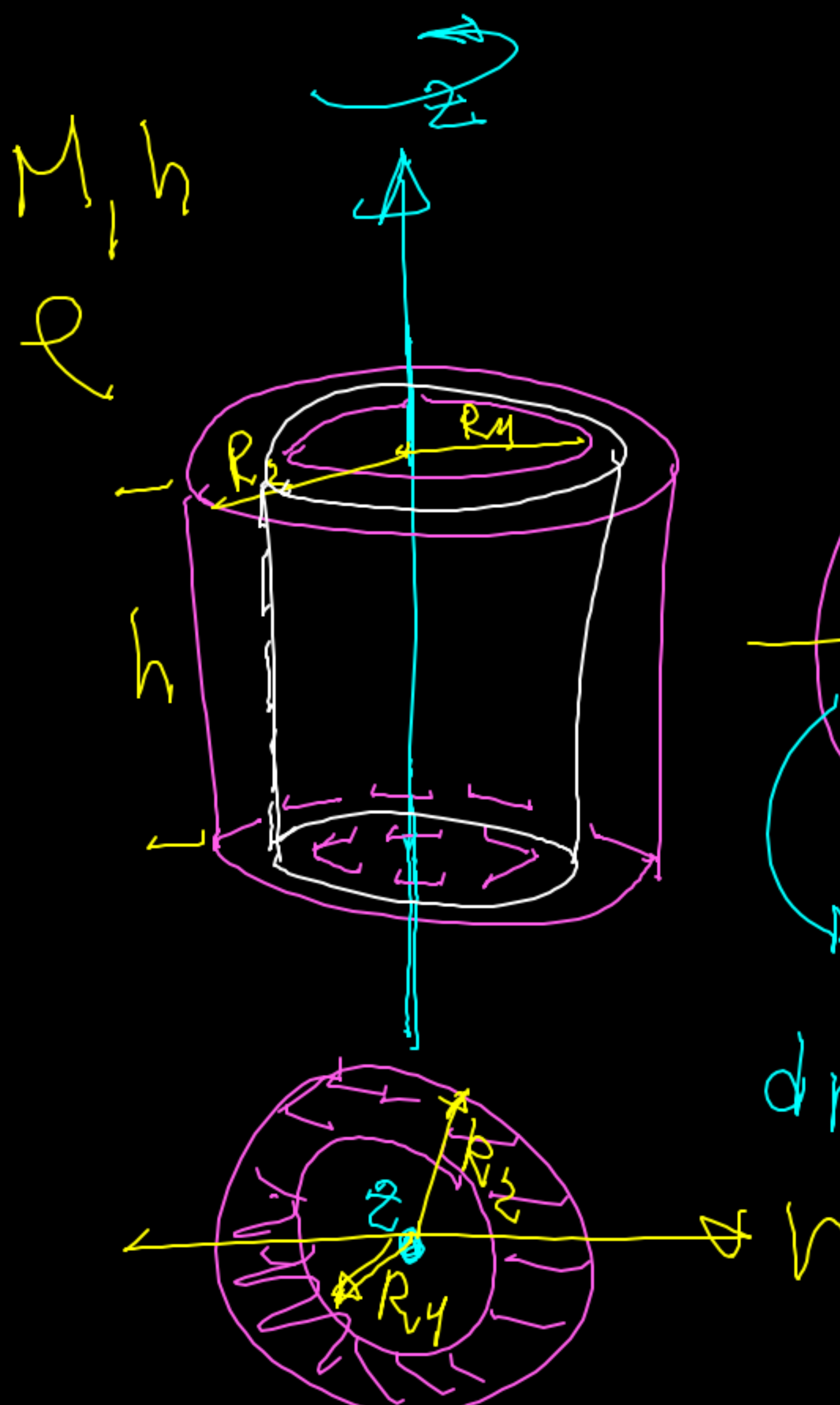
$d \ll e, b$

parte sottile



ESEMPIO

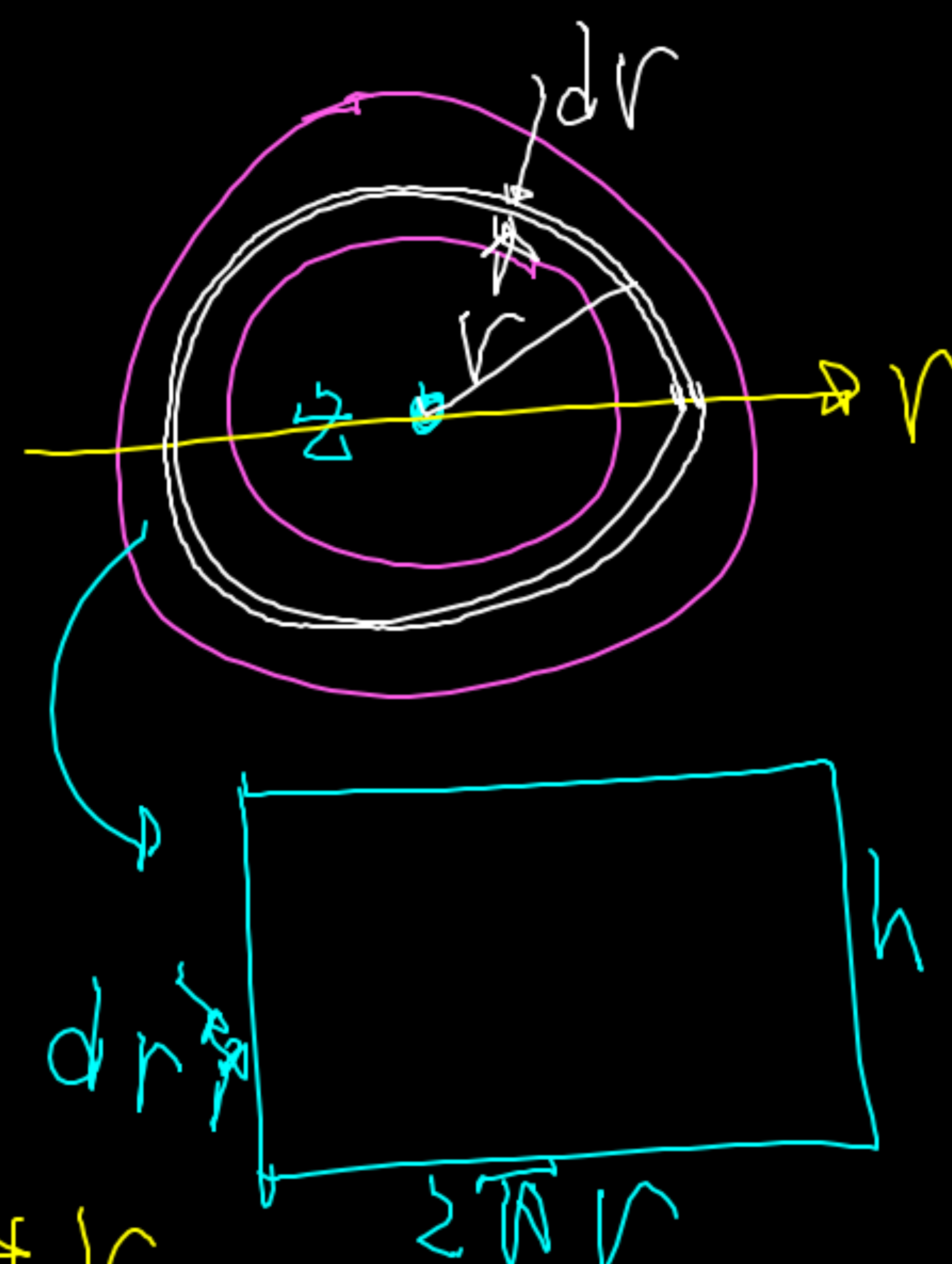
MOM. DI INERZIA DEL CILINDRO (omogeneo)



$$I_z = \int_{Vol} \rho R^2 dV$$

$$dV = 2\pi h r dr$$

$$R^2 = r^2$$



$$I_z = \int_{Vol} \rho 2\pi h r^3 dr =$$

$$= 2\pi h \rho \int_{R_1}^{R_2} r^3 dr = \frac{2\pi h \rho}{4}\left(R_2^4 - R_1^4\right)$$

$$\rho = \frac{M}{Vol} = \frac{M}{\pi R_2^2 h - \pi R_1^2 h}$$

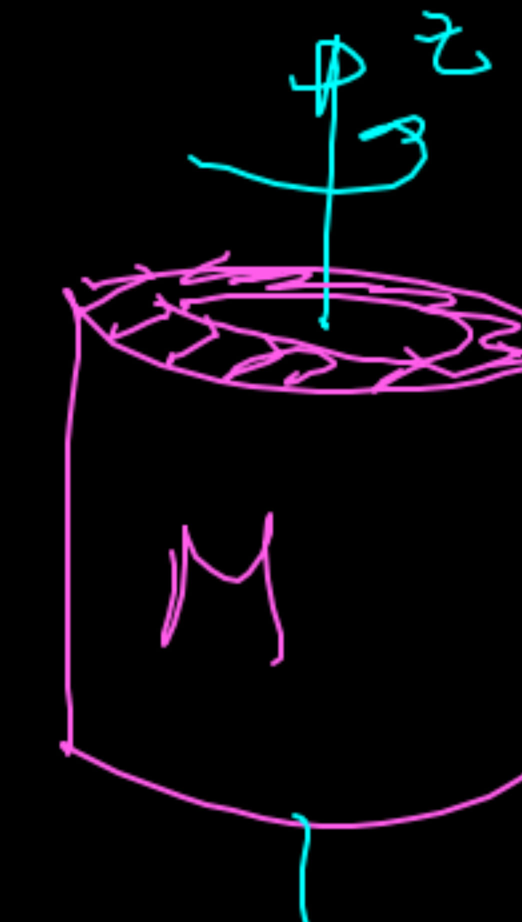
$$I_z = \frac{\pi h \rho}{2}\left(R_2^4 - R_1^4\right) = \frac{\pi h M}{2}\,\frac{\left(R_2^4 - R_1^4\right)}{\left(R_2^2 - R_1^2\right)\pi h} =$$

$$\rho = \frac{M}{\pi h \left(R_2^2 - R_1^2\right)}$$

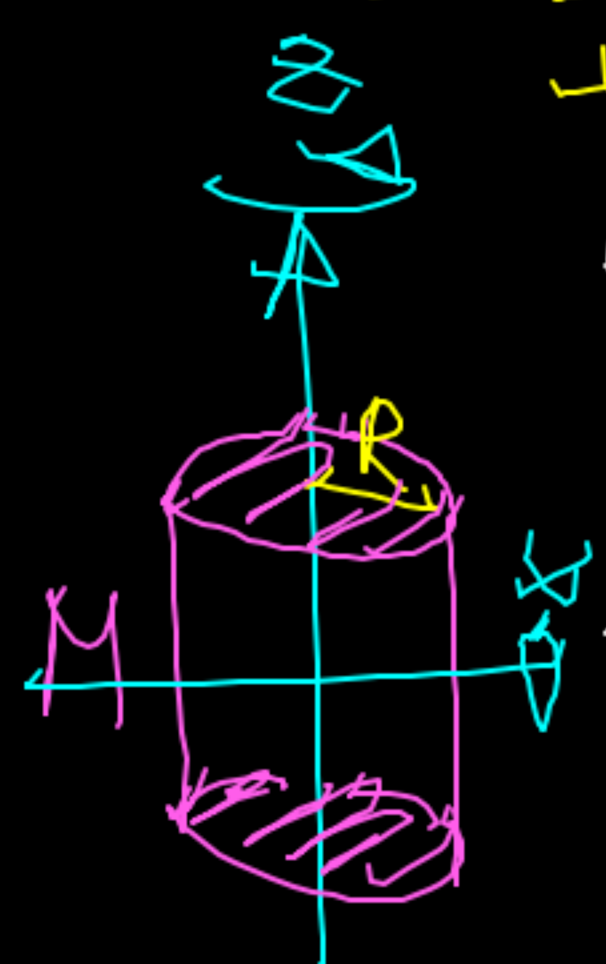
$$= \frac{M}{2}\,\frac{\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)}{\left(R_2^2 - R_1^2\right)} =$$

CIL. CAVO

$$I_z = \frac{M}{2}\left(R_2^2 + R_1^2\right)$$



NB $I_z \neq I_x$

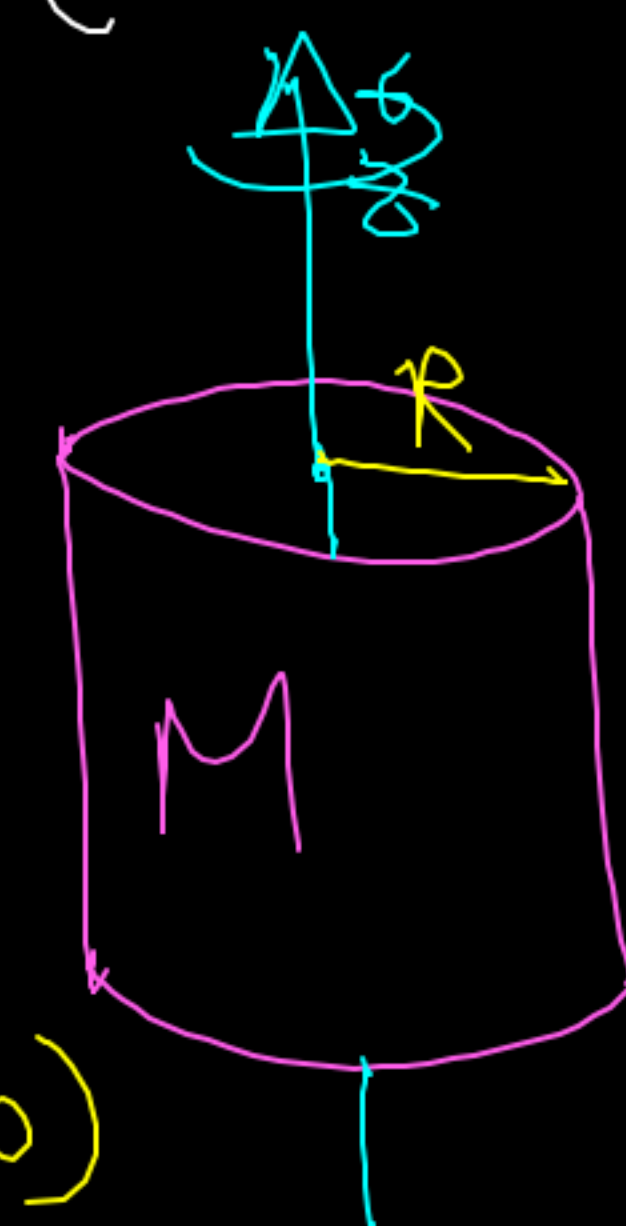


$R_1 \to 0$

$R_2 \to R$

$$I_z = \frac{1}{2} M R^2$$

CIL. PIENO (DISCO)



$R_1 \to R$

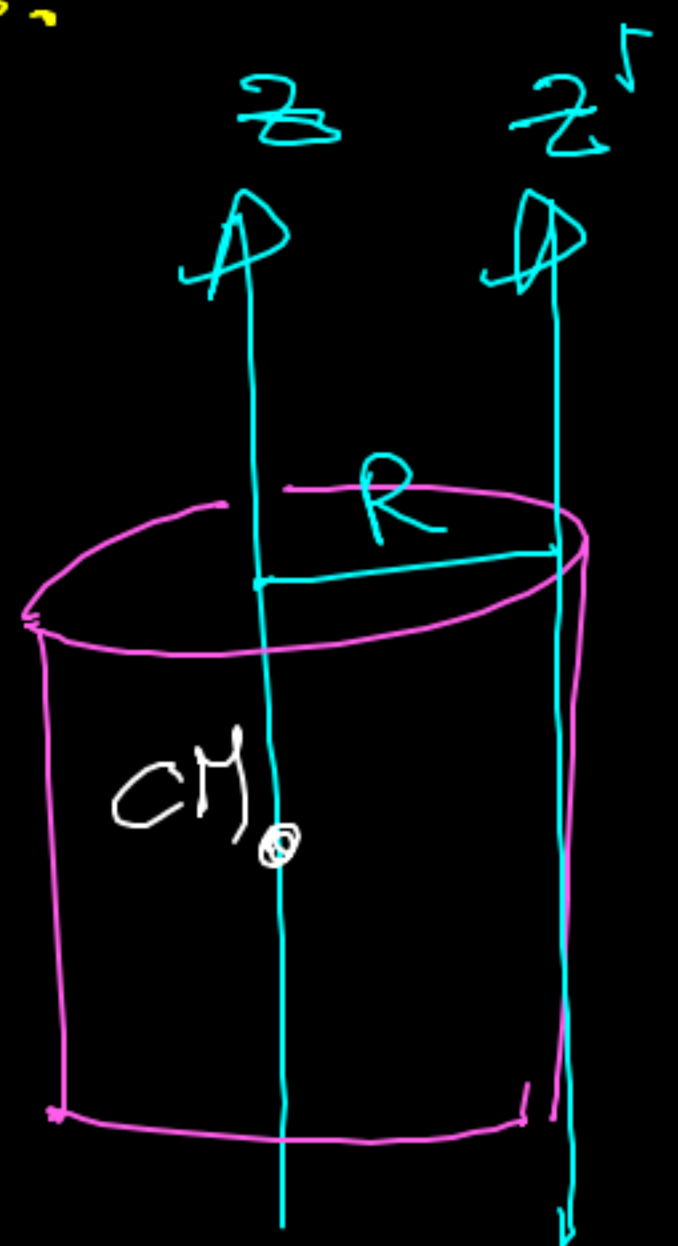
$R_2 \to R$

$$I_z = M R^2$$

STRATO CILIND. SOTTILE

TEOREMA DI HUYGENS-STEINER O DEGLI ASSI PARALLELI

Es.



$$I_z \neq I_{z'}$$

$$I_z = \frac{MR^2}{2}$$

$$I_{z'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

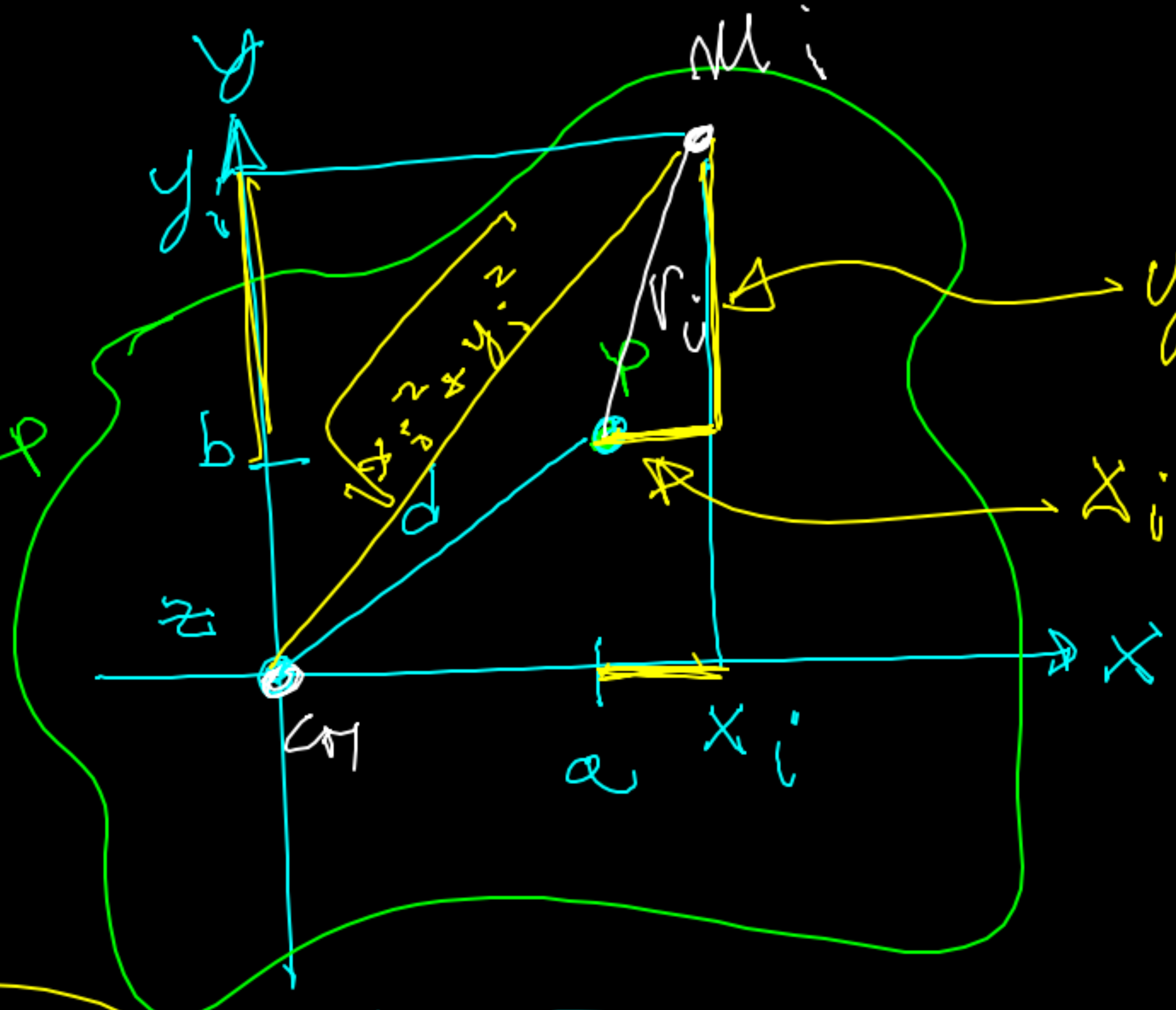
$z' \parallel z$, distanza d

$$\boxed{I_p = I_{cm} + Md^2}$$

ASSE Z uscente

DI ROT

$I_{CM} + Md^2 = I_P$



$$I_P = \sum_{i=1}^{N} m_i r_i^2$$

$$\Rightarrow r_i^2 = (x_i - a)^2 + (y_i - b)^2 =$$

$$r_i^2 = x_i^2 - 2a x_i + a^2 + y_i^2 - 2 y_i b + b^2$$

$$= (x_i^2 + y_i^2) - 2a x_i - 2b y_i + a^2 + b^2$$

$$d = \sqrt{a^2 + b^2}$$

$$I_P = \sum m_i (x_i^2 + y_i^2) + m_i (a^2 + b^2) - 2a m_i x_i - 2b m_i y_i$$

$$= \underbrace{\sum m_i (x_i^2 + y_i^2)}_{I_{CM}} + \underbrace{\sum m_i (a^2 + b^2)}_{Md^2}$$

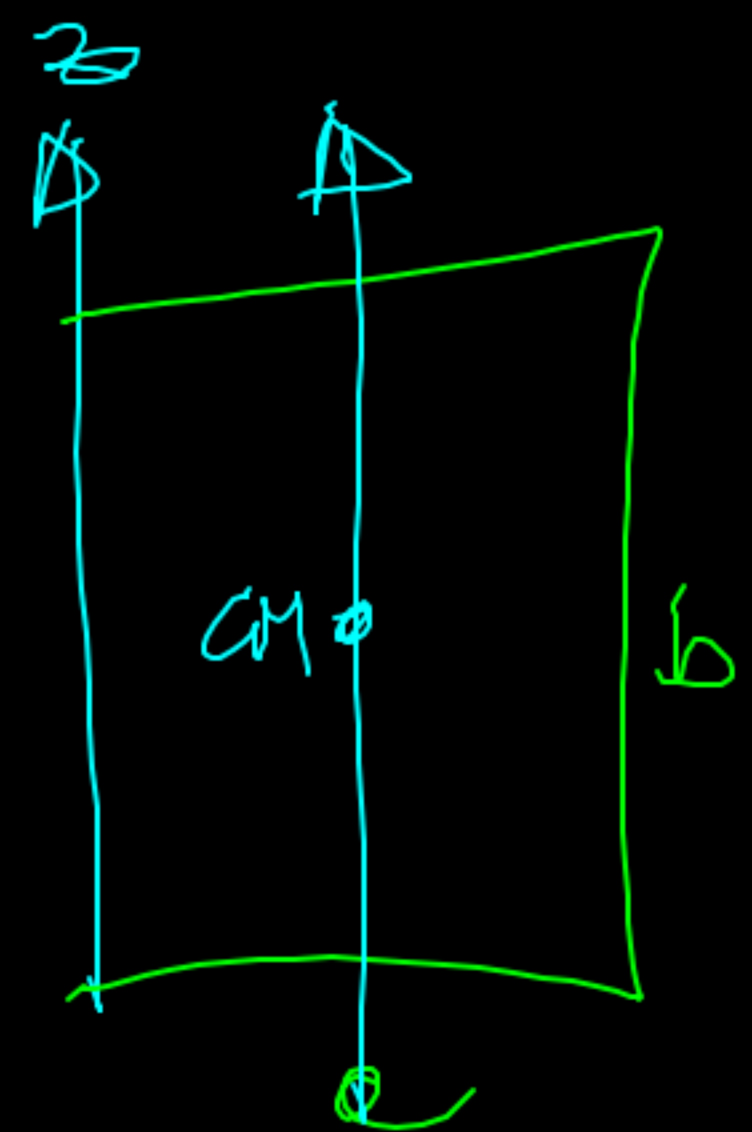
$$- 2a \underbrace{\cancel{\sum m_i x_i}}_{=0} - 2b \underbrace{\cancel{\sum m_i y_i}}_{=0}$$

$x_{CM} = 0$

$y_{CM} = 0$

$$x_{CM} = \frac{\sum m_i x_i}{M}$$

$$\underline{M x_{CM} = \sum m_i x_i = 0}$$



$$I_z = \frac{Ma^2}{3}$$

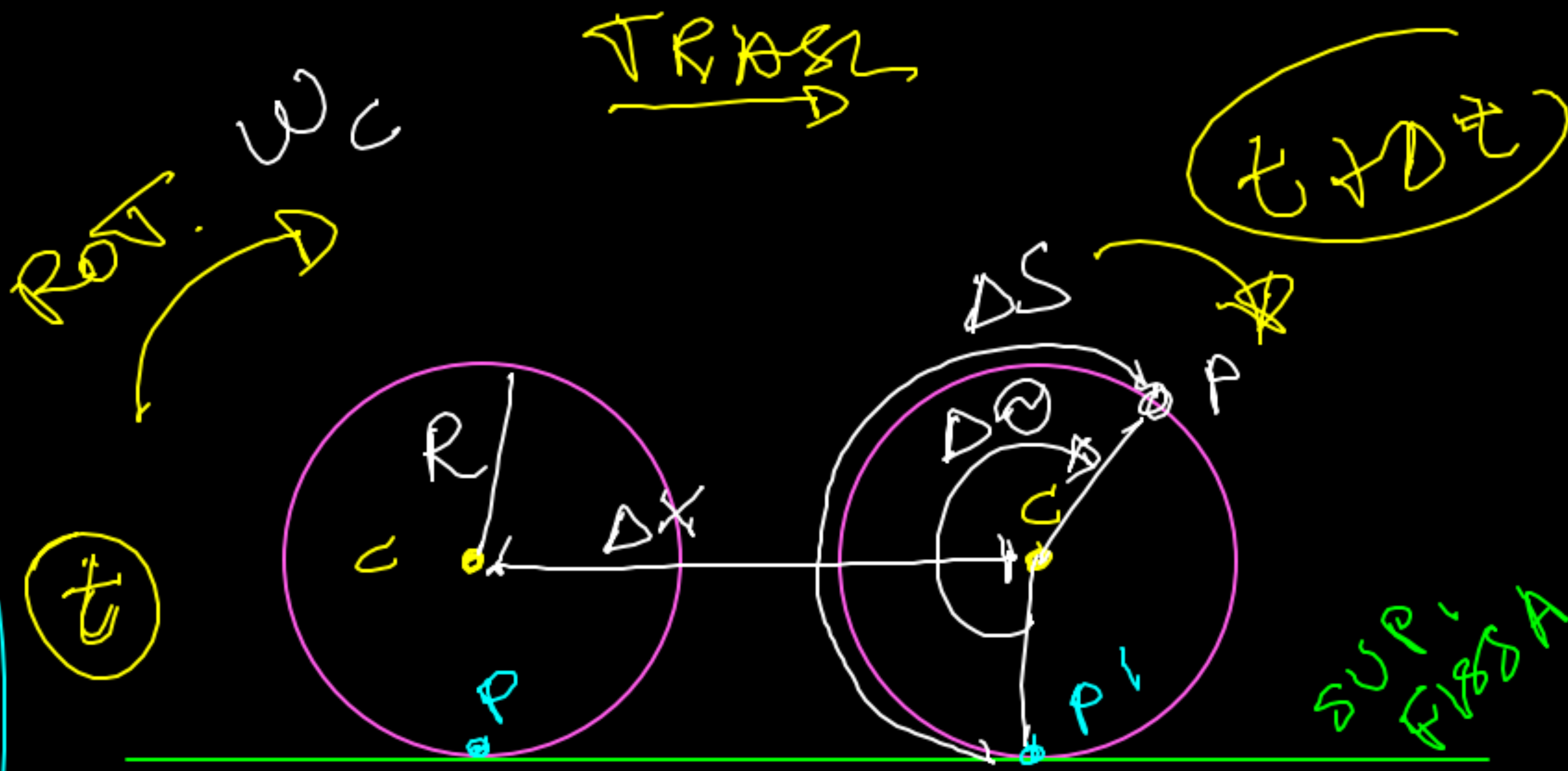
$$I_{cm} = ?$$

$$I_z = I_{cm} + M\frac{a^2}{4}$$

$$I_{cm} = I_z - M\frac{a^2}{4} =$$

$$= \frac{Ma^2}{3} - \frac{Ma^2}{4} = Ma^2\frac{1}{12}$$

PURO ROTOLAMENTO DI UN CORPO RIGIDO



$\vec{v}_P(t) = 0$

"ROTOLARE SENZA STRISCIARE"

$$\frac{\Delta S}{\Delta t} = \frac{\Delta X}{\Delta t} = v_C$$

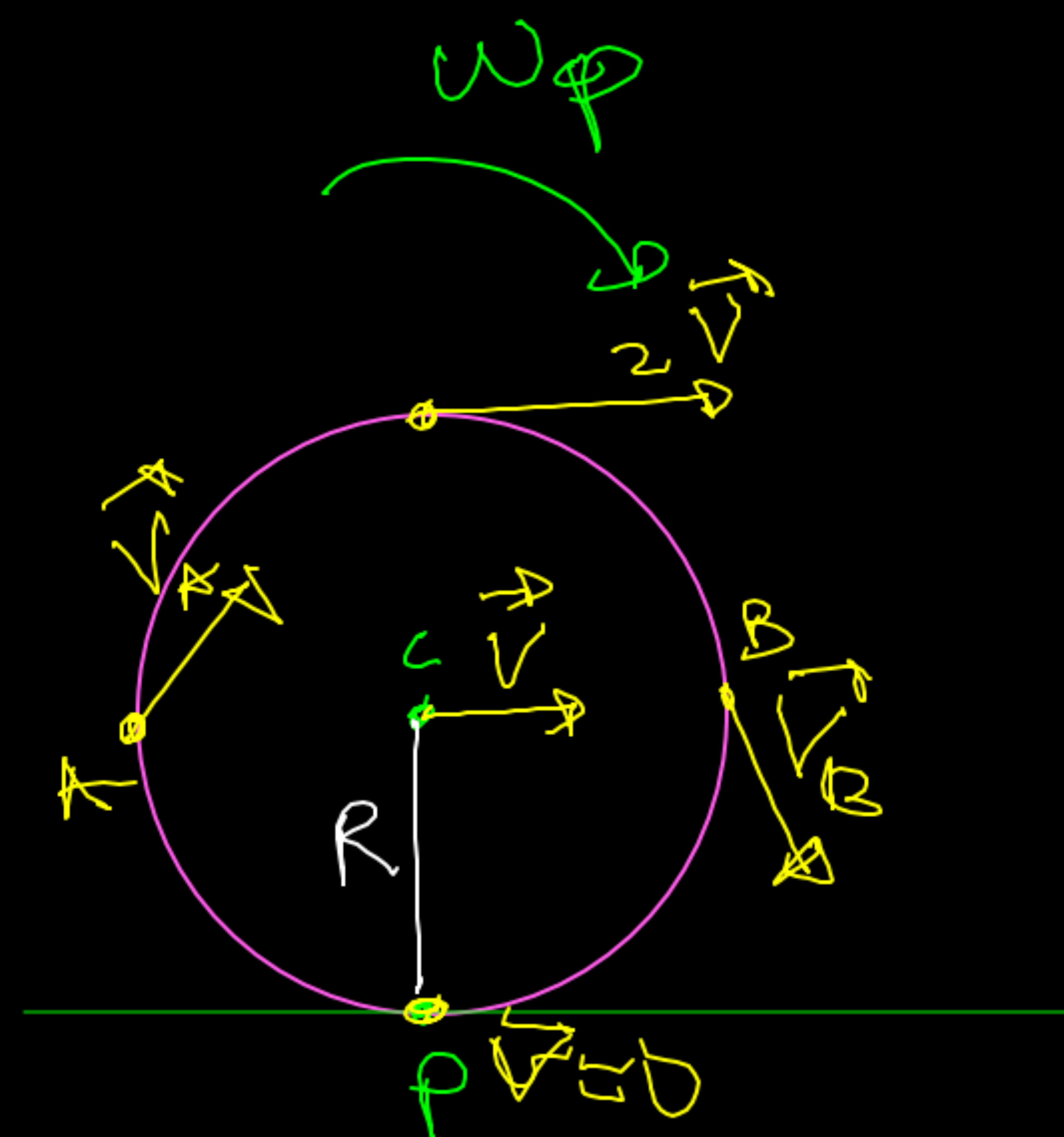
$$\frac{\Delta S}{R} = \Delta\theta \Rightarrow \frac{R\,\Delta\theta}{\Delta t} = R\,\omega_C$$

$$\boxed{v_C = \omega_C R}$$

Condizione di puro rotolamento

(l'asse passante per C rimane parallelo a se stesso durante il moto)

VISTA DA "P"



ROTAZIONE ISTANTANEA INTORNO A P

$V = \omega_P R$ "C" visto da "P"

$V = \omega_C R$ "P" visto da "C"

$\Rightarrow \omega_C = \omega_P = \omega$

Se puro rotolamento

ENERGIA CINETICA (puro rotolamento)

$$K = \frac{1}{2} I_P \omega^2 = \frac{1}{2}\left(I_{CM} + MR^2\right)\omega^2 =$$

$$\text{CECM} = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M \underbrace{R^2\omega^2} = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M v_{CM}^2$$

$$v_{CM} = \omega R \qquad K = \underbrace{\frac{1}{2} I_{CM}\omega^2}_{\text{E.c. rot.}} + \underbrace{\frac{1}{2} M v_{CM}^2}_{\text{E.c. trasl.}}$$

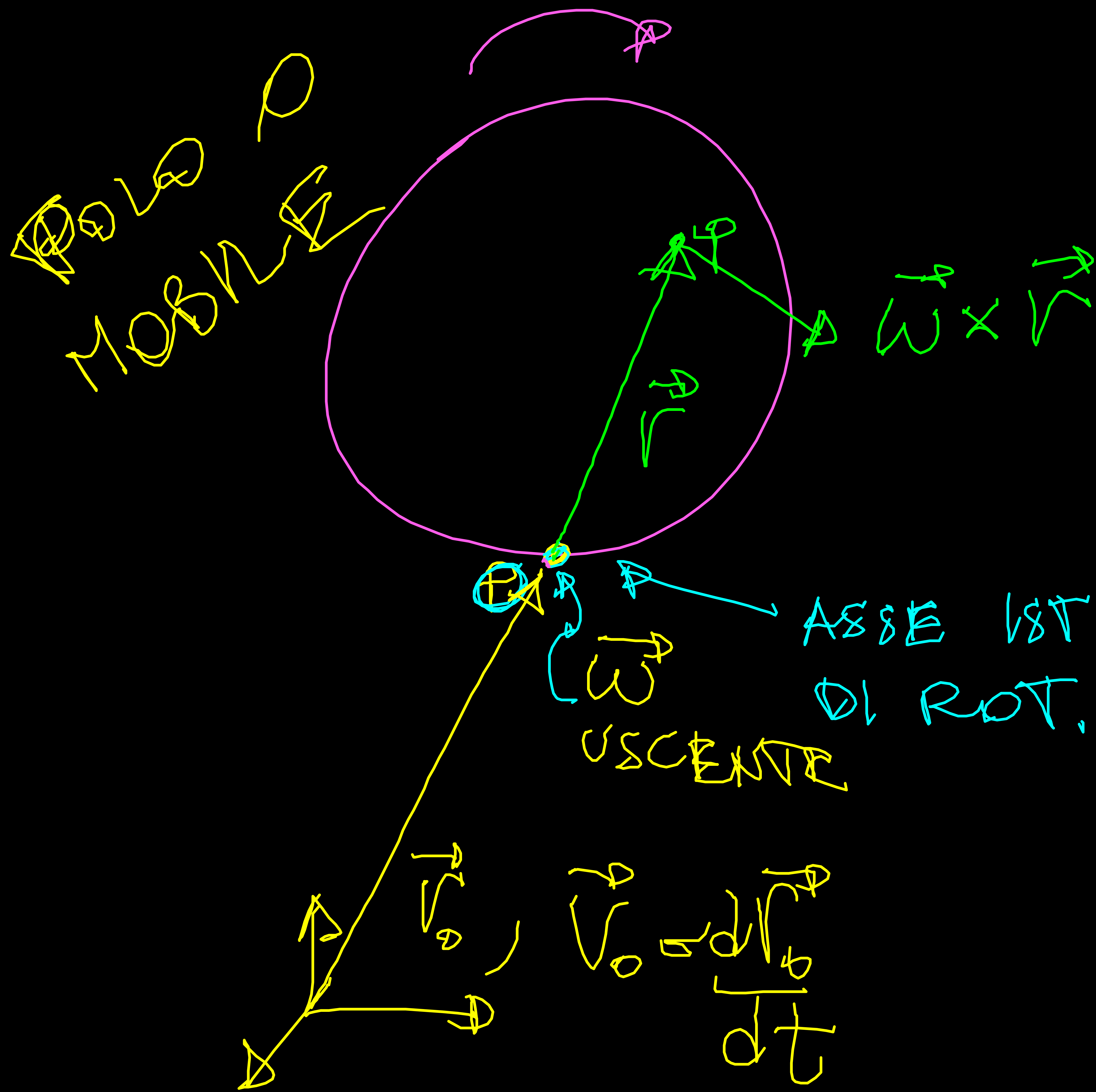
Se no puro rot.

$$v \neq \omega R \Rightarrow K = \frac{1}{2} I_{CM}\omega^2 + \frac{M}{2} v^2$$

$$\neq v_{CM}$$

TEOREMA DI KÖNIG

(ASSE DI ROT. SI MANT. ∥)

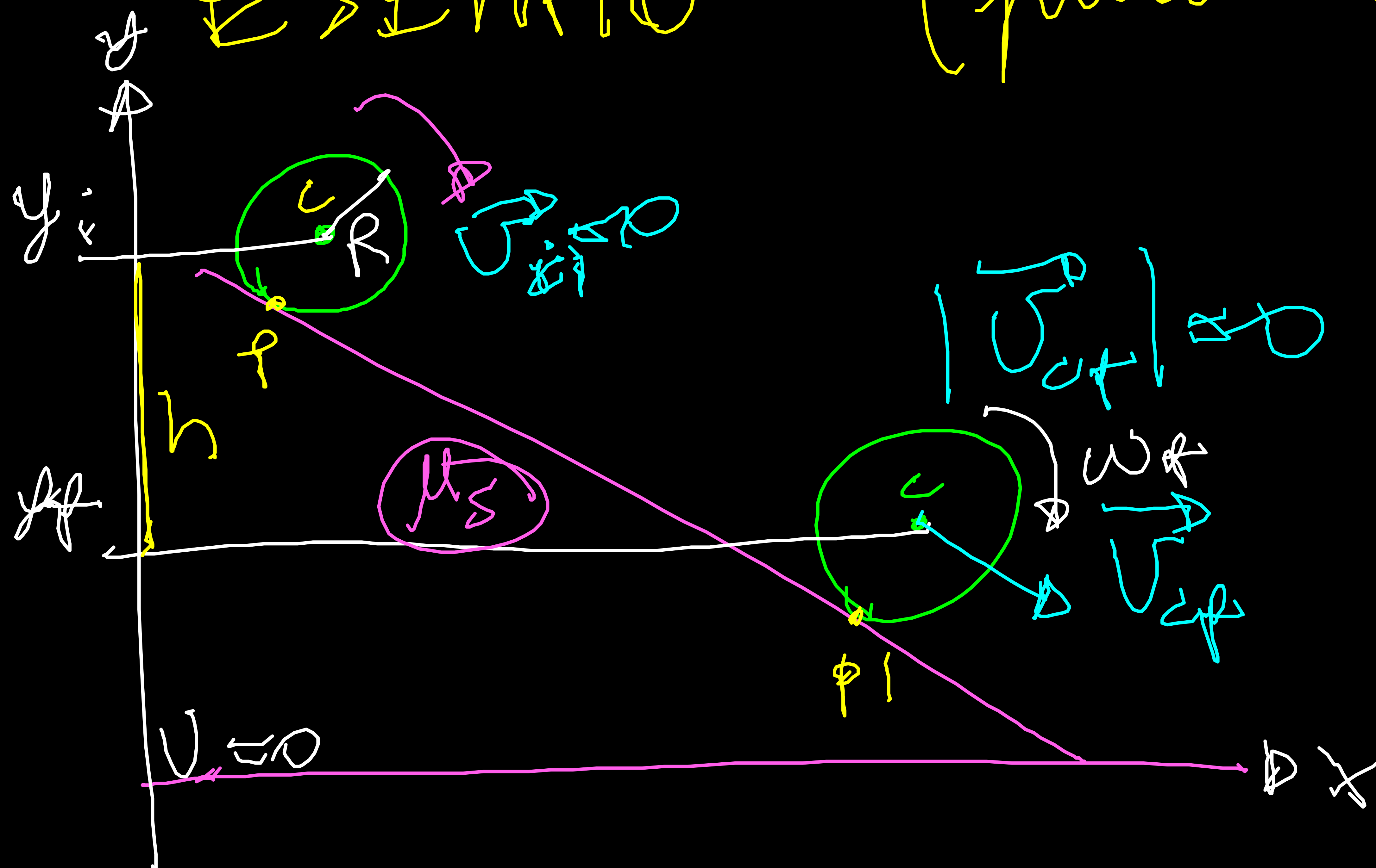


$$\boxed{\vec{V}_P = \vec{V}_0 + \vec{\omega} \times \vec{r}}$$

Vel. di P vista da un sistema fisso

$\vec{V}_0$ Vel. del polo mobile vista dal sistema fisso

ESEMPIO (puro rotolamento)



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}Mv_{cf}^2 + \frac{1}{2}I_{cm}\omega_f^2 + Mgy_f = Mgy_i$$

$$\omega_f = \frac{v_{cf}}{R}$$

$$\frac{1}{2}Mv_{cf}^2 = Mg(y_f - y_i) - \frac{1}{2}I_{cm}\omega_f^2$$

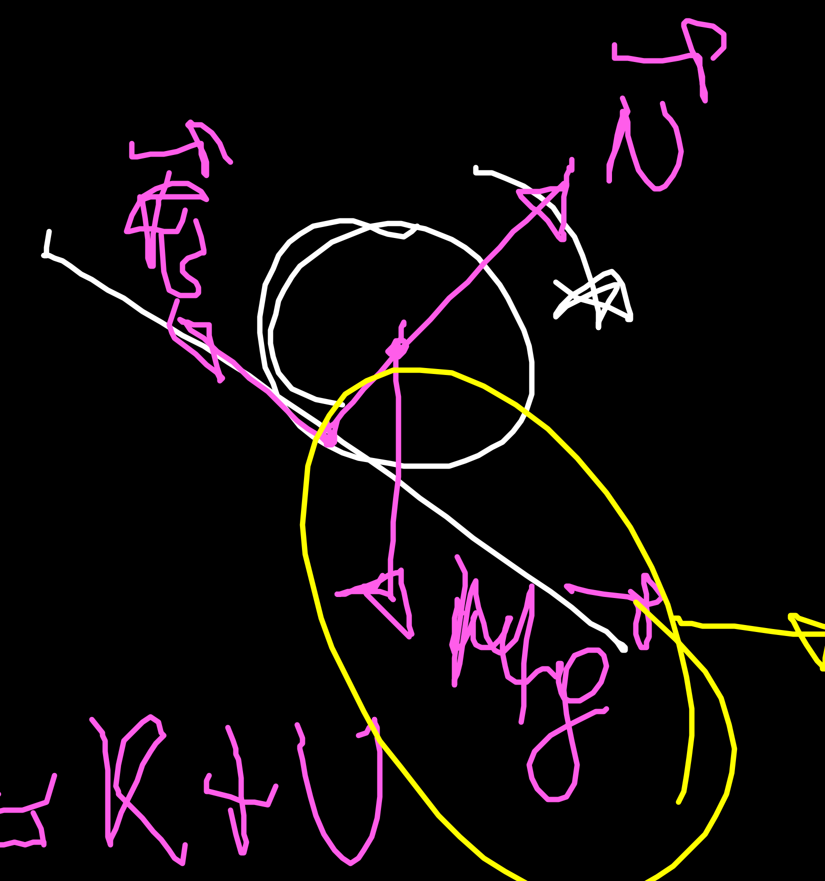
$$v_{cf}^2 + \frac{2}{2}\frac{I_{cm}}{M}\frac{v_{cm}^2}{R^2} = \frac{2Mg(y_f - y_i)}{M}$$

$$v_{cf}^2\left(1 + \frac{I_{cm}}{MR^2}\right) = 2g(y_f - y_i)$$

$$v_{cf} = \sqrt{2gh \Big/ \left(1 + \frac{I_{cm}}{MR^2}\right)}$$

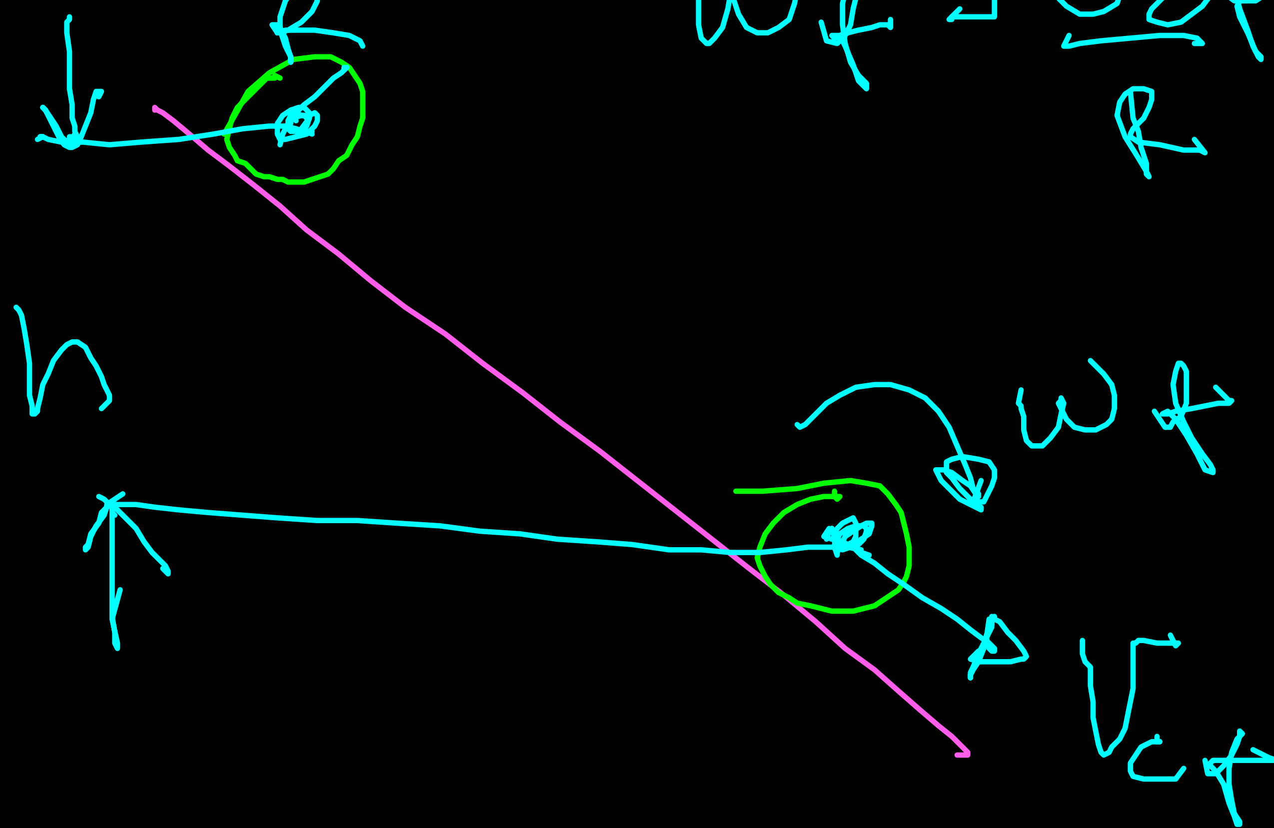
puro rotolamento senza strisciamento

CECM



$$\mathcal{L}_{attr.} = 0 \Rightarrow E = K + U \text{ è cost.}$$

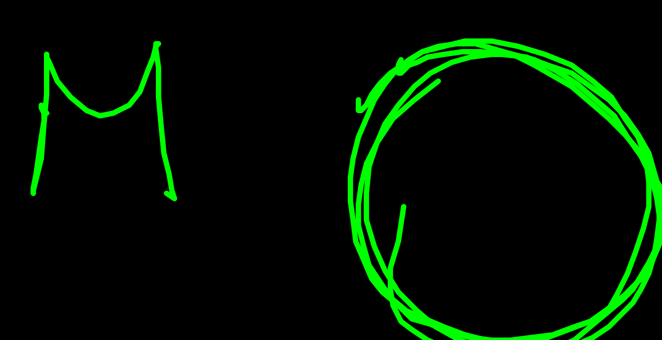
$$\omega_f = \frac{v_{cf}}{R}$$



$$v_{cf} = \sqrt{\frac{2gh}{1+\frac{I_{cm}}{MR^2}}}$$

CILINDRO $\rightarrow$ $I_{cm} = \frac{1}{2}MR^2$

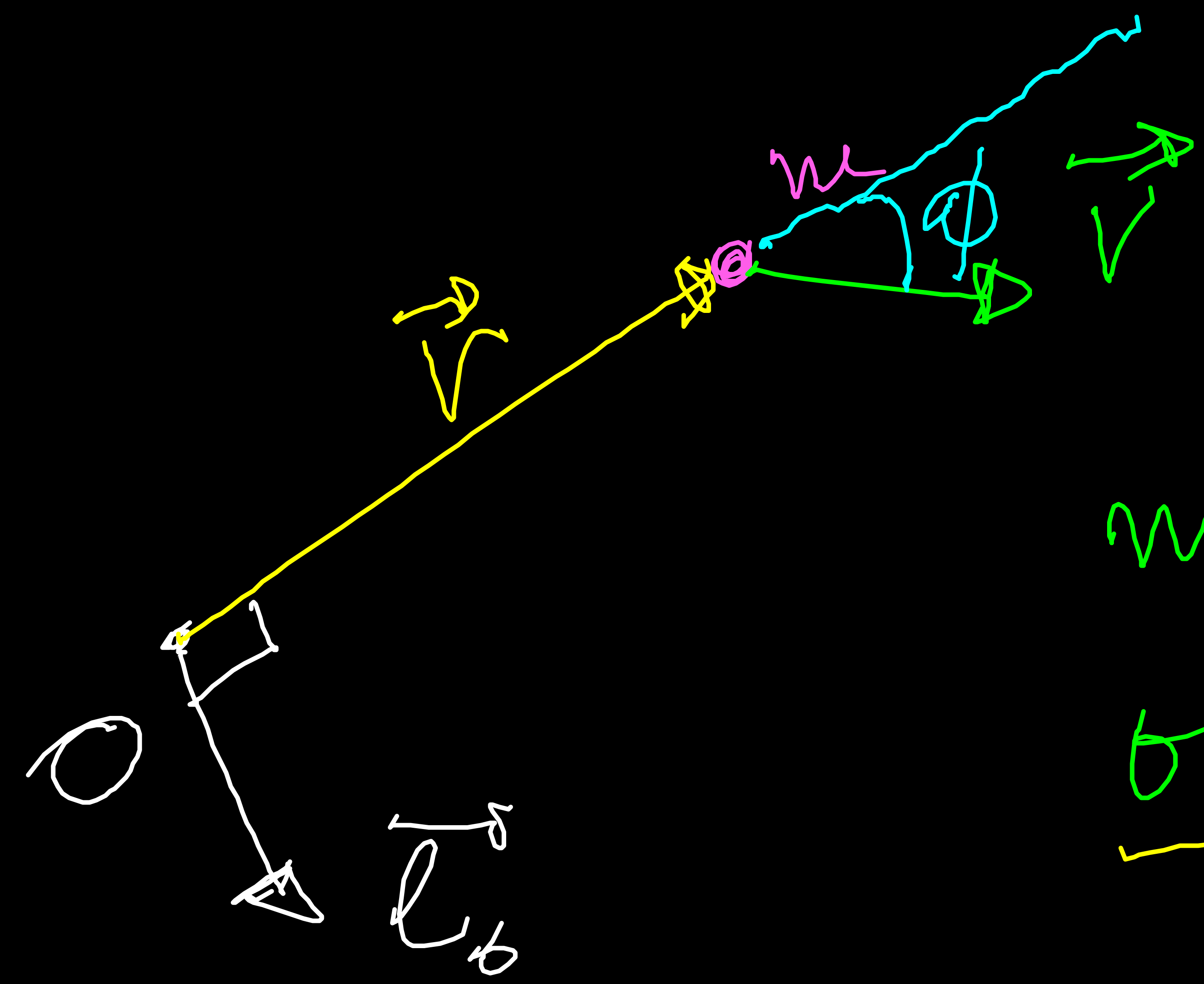
$$v_{cil} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{4gh}{3}}$$



ANELLO $\rightarrow$ $I_{cm} = MR^2$

$$\boxed{v_{cil} > v_{anello}}$$

$$v_{anello} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$



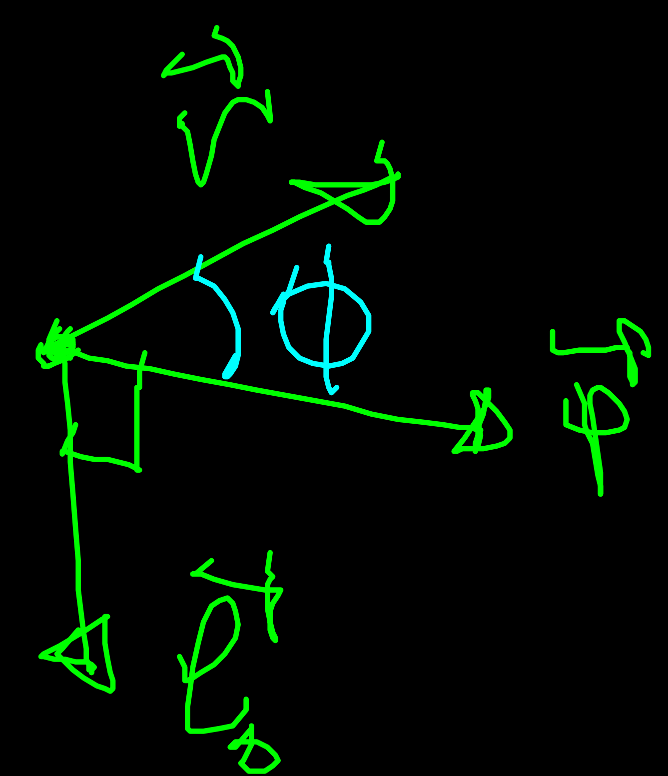
$\vec{p} = m\vec{v}$

momento della quantità di moto

o momento angolare

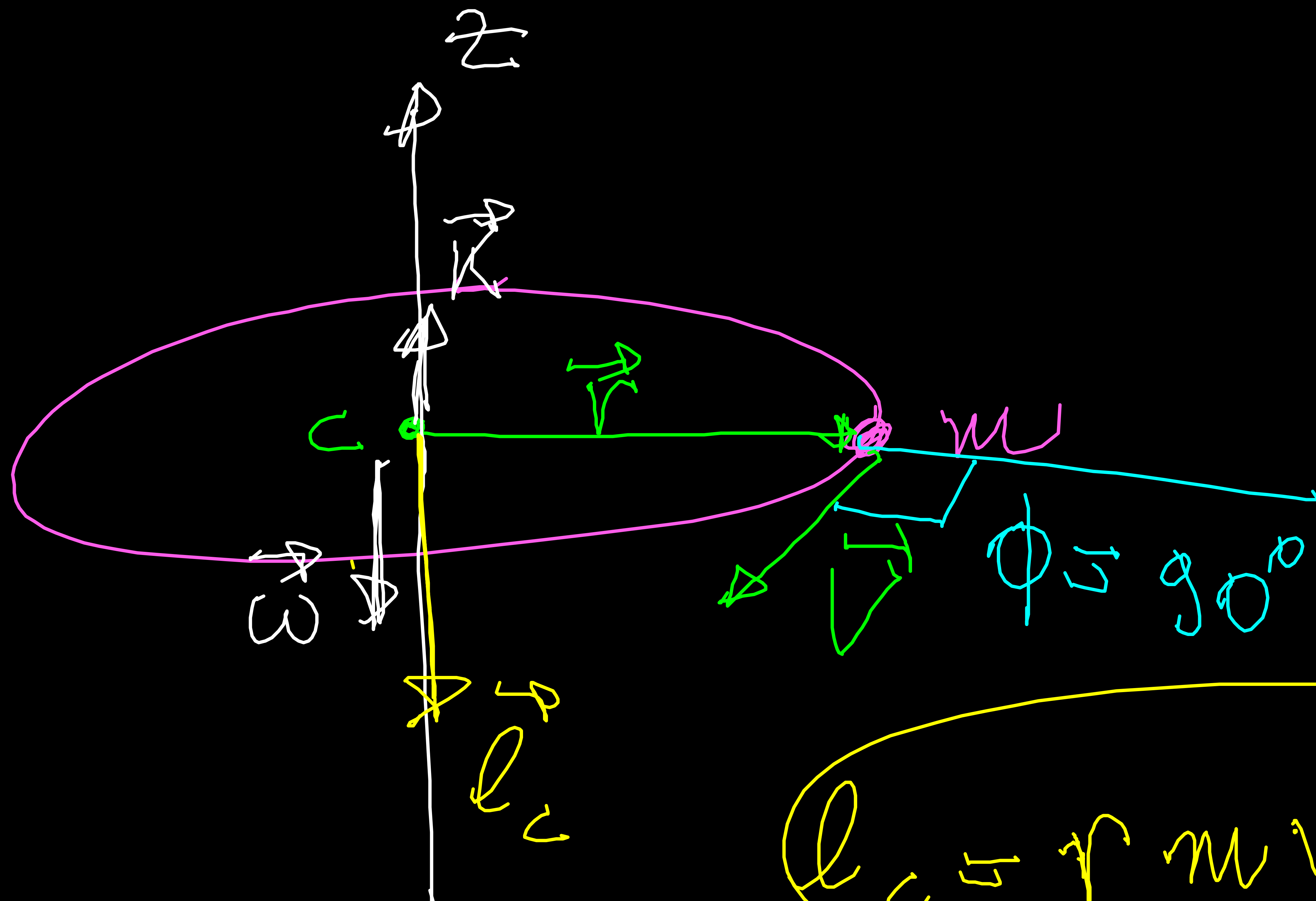
$\vec{\ell}_O = \vec{r} \times \vec{p}$

O ← polo



$\ell_O = r p \sin\phi$

MOTO CIRCOLARE $\longrightarrow$ MOM. ANGOLA



$v = \omega r$

Se moto circ. unif.

$\vec{\omega}$ è costante

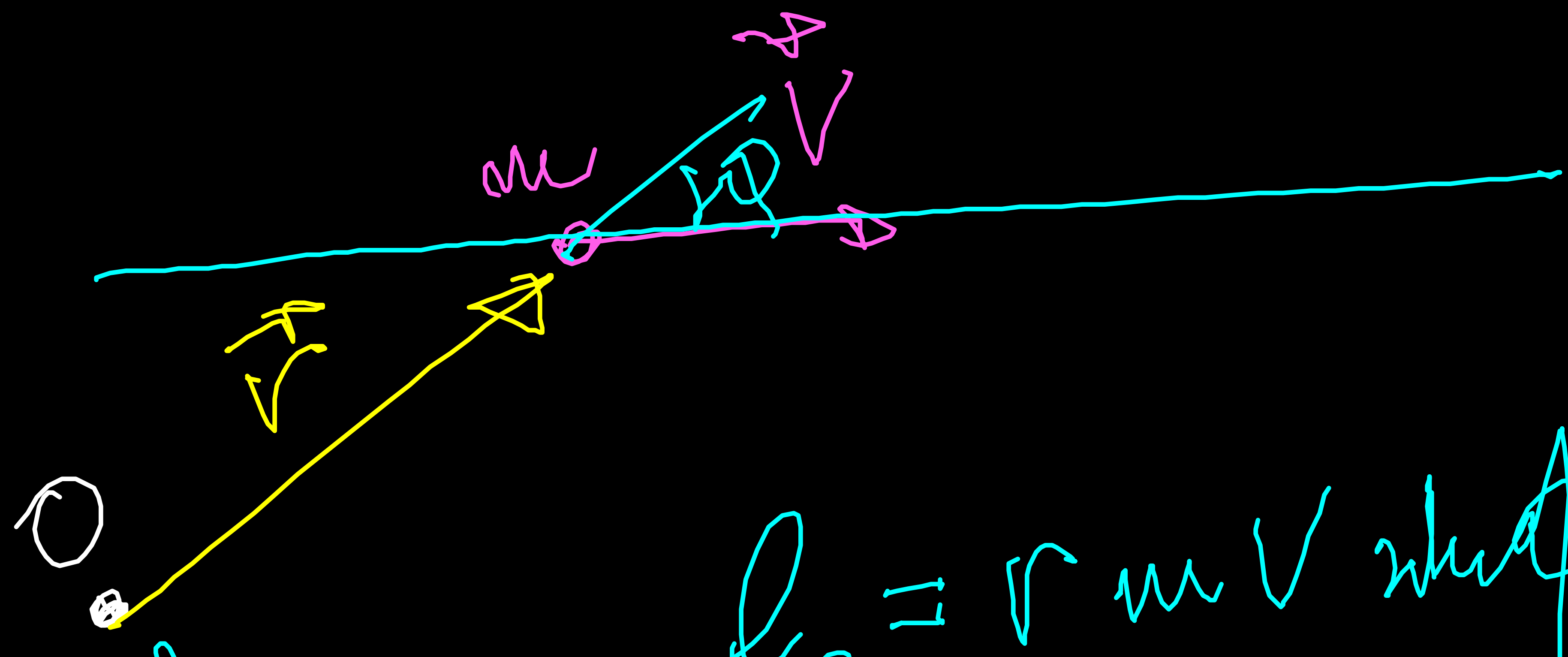
$\Downarrow$

$\vec{\ell}_c$ è costante

$$\ell_c = r m v = m \omega r^2$$

$$\vec{\ell}_c = \ell_z \hat{k} = (-m\omega r^2)\hat{k}$$

$$\vec{\omega} = \omega_z \hat{k} = \left(-\frac{v}{r}\right)\hat{k}$$



$\ell_O = r m v \sin\phi$

$\vec{\ell}_O$ è costante

$$\frac{d\vec{\ell}_O}{dt} = \frac{d}{dt}(\vec r \times \vec p)$$

Vettore posizione

O ≡ origine

O è fisso

$$\frac{d\vec{\ell}_O}{dt} = \frac{d\vec r}{dt}\times\vec p + \vec r\times\frac{d\vec p}{dt} =$$

$$= \cancel{\vec v\times(m\vec v)} + \vec r\times\left(\sum\vec F\right)$$

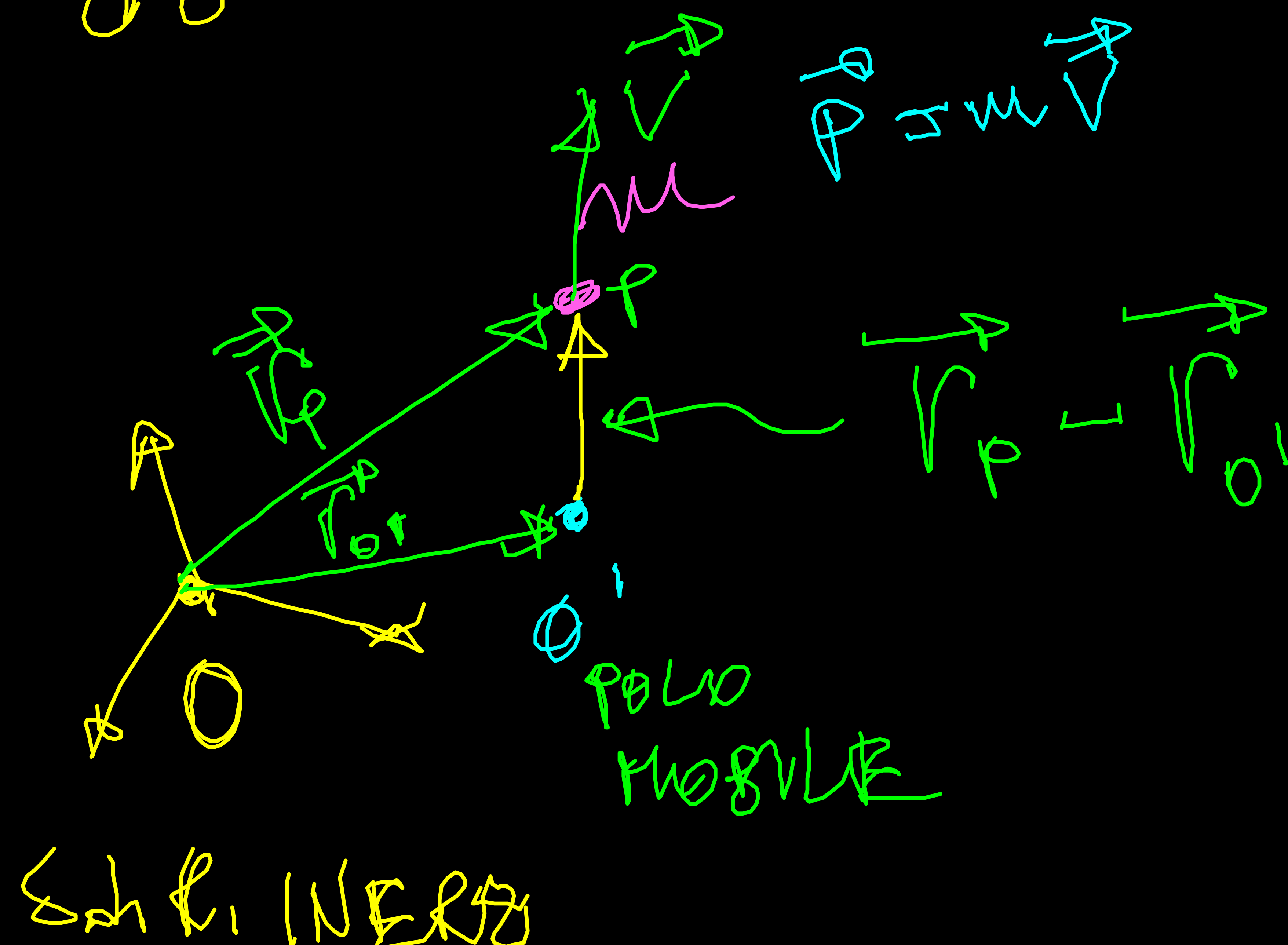
$$\sum \vec F = \frac{d\vec p}{dt}$$

(punto materiale)

$$\frac{d\vec{\ell}_O}{dt} = \underbrace{\vec r\times\left(\sum\vec F\right)}_{\text{momento delle forze applicate}}$$

$$\frac{d\vec{l}_O}{dt} = \vec{r} \times \left(\sum \vec{F}\right)$$

– polo O fisso e nell'origine
– s.d.R. inerziale



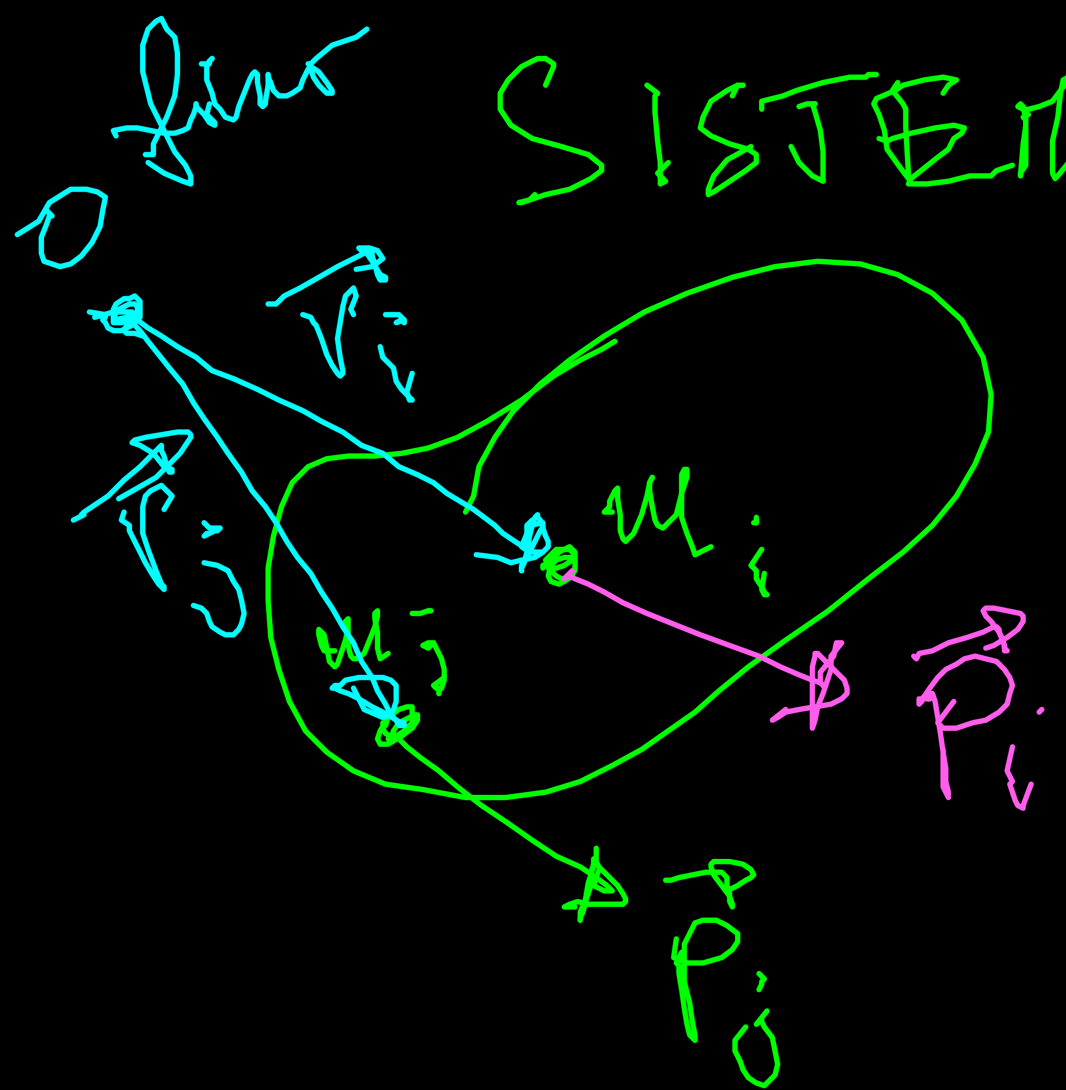
$$\vec{l}_{O'} = (\vec{r}_P - \vec{r}_{O'}) \times \vec{P}$$

$$\frac{d\vec{l}_{O'}}{dt} = \frac{d}{dt}\left(\vec{r}_P \times \vec{P} + \vec{r}_{O'} \times \vec{P}\right)$$

$$\boxed{\frac{d\vec{l}_{O'}}{dt} = \vec{r}_P \times \left(\sum \vec{F}\right) - \frac{d\vec{r}_{O'}}{dt} \times \vec{P}}$$

POLO O' MOBILE

SISTEMA DI PUNTI MATERIALI



$$\vec{l}_i = \vec{r}_i \times \vec{p}_i \Rightarrow \vec{L}_O = \sum_{i=1}^{n} \vec{l}_i$$

MOM. ANGOLARE TOTALE (rispetto ad O)

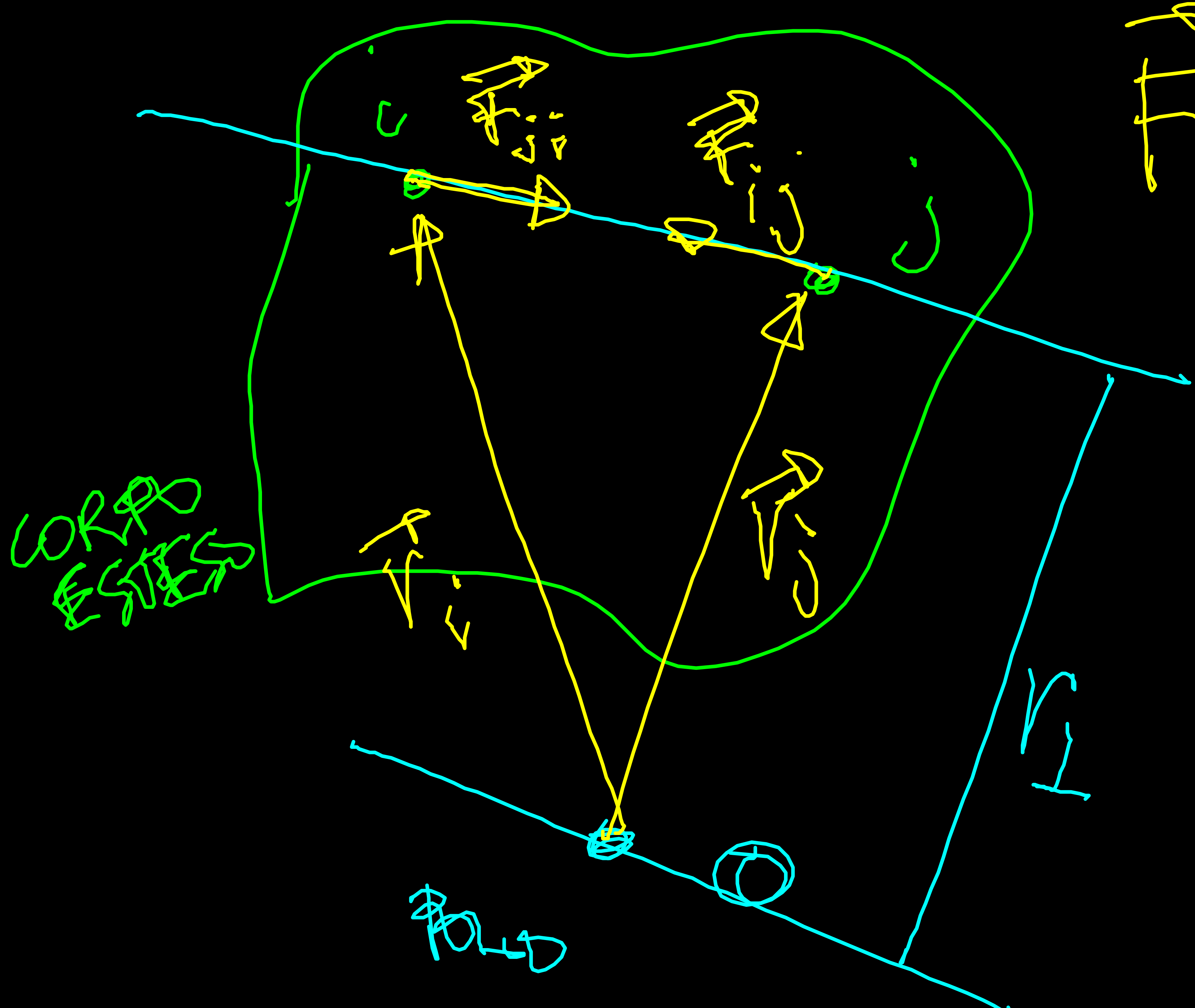
$$\frac{d\vec{L}_O}{dt} = \frac{d}{dt}\left(\sum \vec{l}_i\right) = \sum \frac{d\vec{l}_i}{dt} = \sum \vec{\tau}_i = \frac{d\vec{L}_O}{dt}$$

nel punto "i" agiscono più forze

$$\frac{d\vec{l}_i}{dt} = \vec{r}_i \times \left(\sum \vec{F}\right)_i = \vec{r}_i \times \vec{F}_i = \vec{\tau}_i = \vec{\tau}_{i,est} + \vec{\tau}_{i,INT}$$

momento della forza risultante applicata ad "i"

FORZE INTERNE



$$\vec F_{ij} = -\vec F_{ji}$$

$$\vec r_i \times \vec F_{ji} = \vec\tau_{ji} \Rightarrow |\vec\tau_{ji}| = r_\perp |\vec F_{ji}|$$

$$\vec r_j \times \vec F_{ij} = \vec\tau_{ij} \qquad |\vec\tau_{ij}| = r_\perp |\vec F_{ij}|$$

Versi opposti $\Rightarrow \vec\tau_{ji} = -\vec\tau_{ij}$

$\forall$ coppia di punti i e j $\vec\tau_{ij} + \vec\tau_{ji} = 0$

$$\Rightarrow \sum \vec\tau_{i,INT} = 0$$

$$\frac{d\vec{L}_o}{dt} = \sum\left(\cancel{\vec{\tau}_{i,int}} + \vec{\tau}_{i,ext}\right) = \sum \vec{\tau}_{i,ext}$$