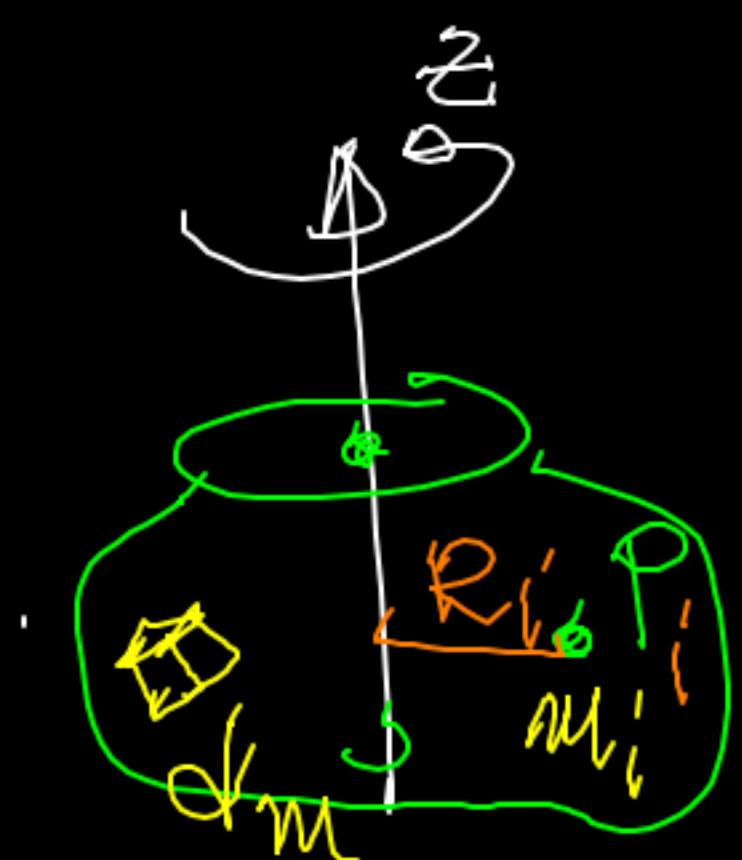
MOMENTO DI INERZIA

$K = \frac{1}{2} I_z \omega_z^2$

ROT. DI CORPO RIGIDO INTORNO A Z FISSO

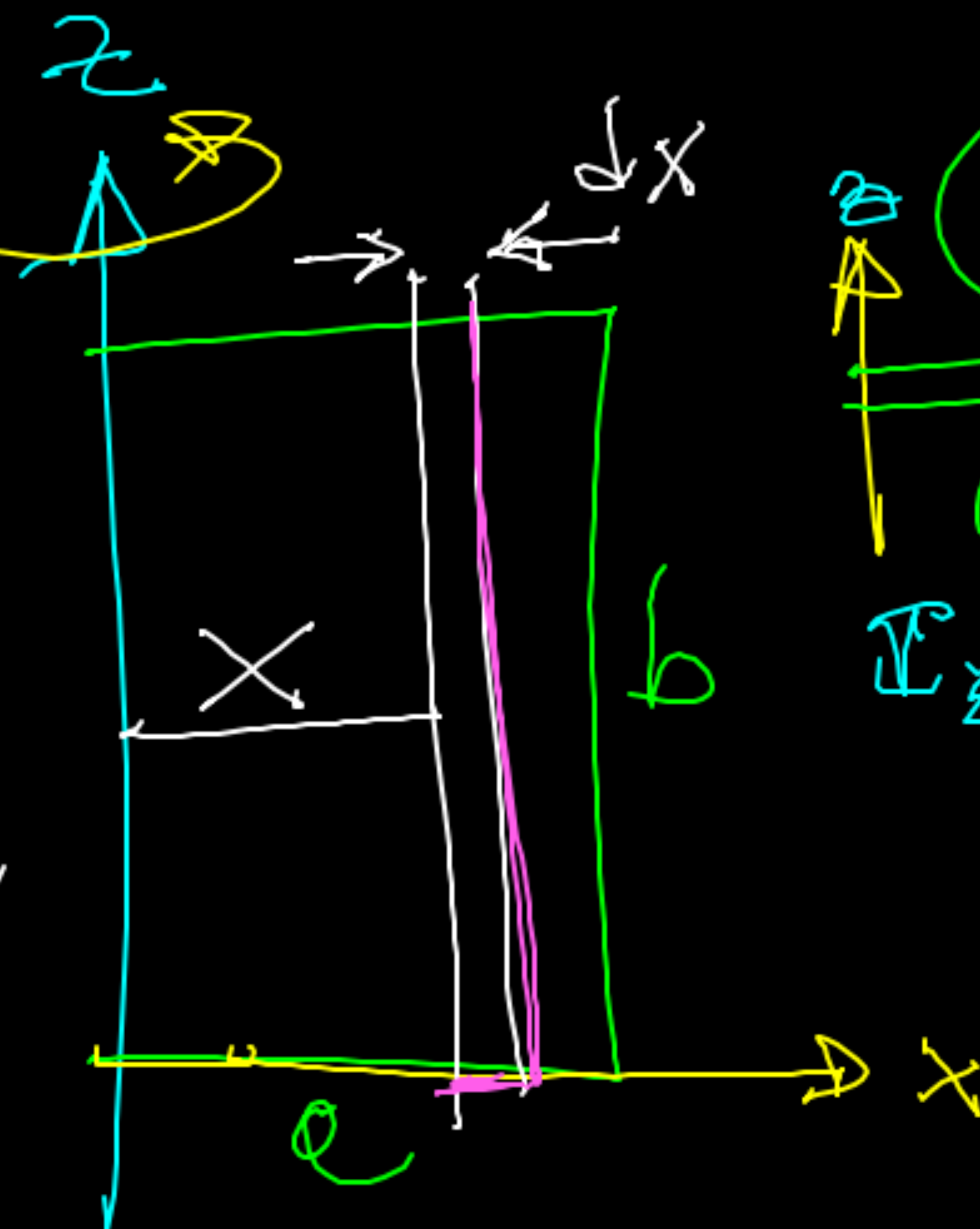


$$I_z = \sum_{i=1}^{N} m_i R_i^2$$

CASO DISCRETO

$$I_z = \int_{\text{CORPO}} R^2 dm = \int_{\text{Vol. del corpo}} \rho R^2 dV$$

ESEMPIO DI CALCOLO



M

ρ

$V = ebd$

$\rho = \frac{M}{ebd}$

$I_z = \frac{Me^2}{3}$

I_z della porta (omogenea) sottile

$I_z = \int \rho R^2 \, dV$

vol del corpo

$dV = b\,d\,dx$

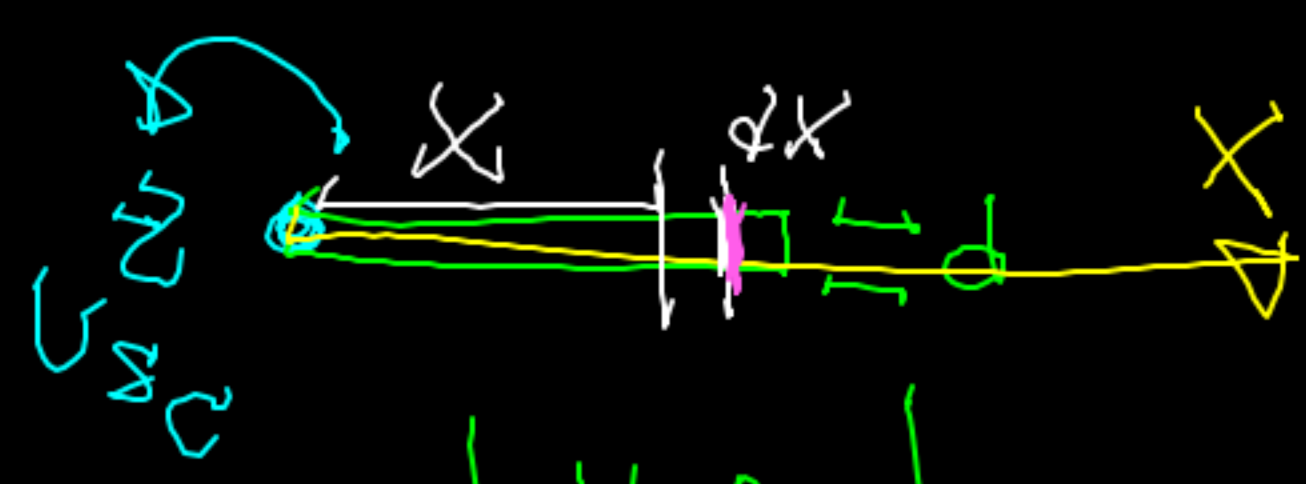
$R = x$

$\rightarrow I_z = \int_0^e \rho\, b d\, x^2\, dx$

$I_z = \rho\, bd \int_0^e x^2\, dx =$

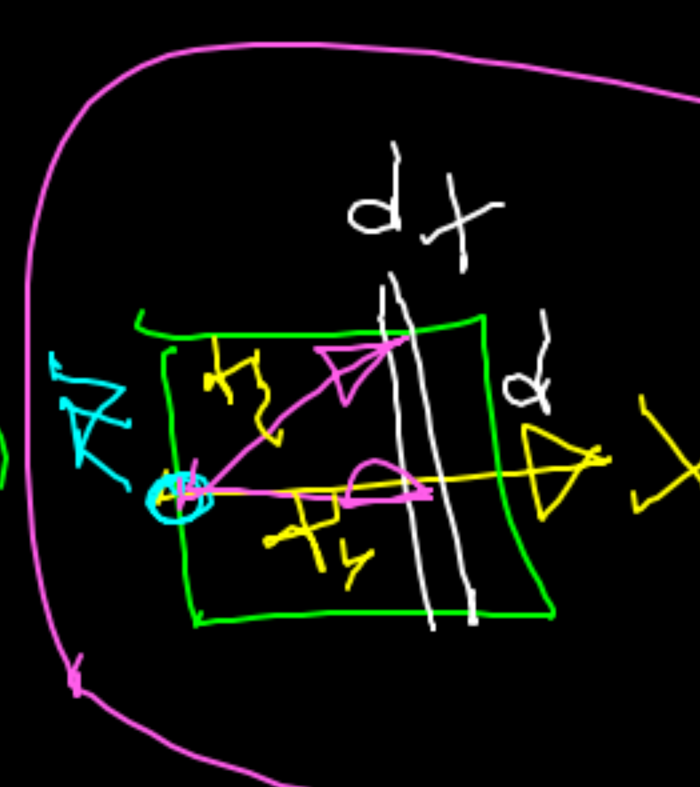
$= \rho\, bd \frac{e^3}{3} =$

$I_z = \frac{M}{ebd}\, bd \frac{e^3}{3} = \frac{Me^2}{3}$



$d \ll e, b$

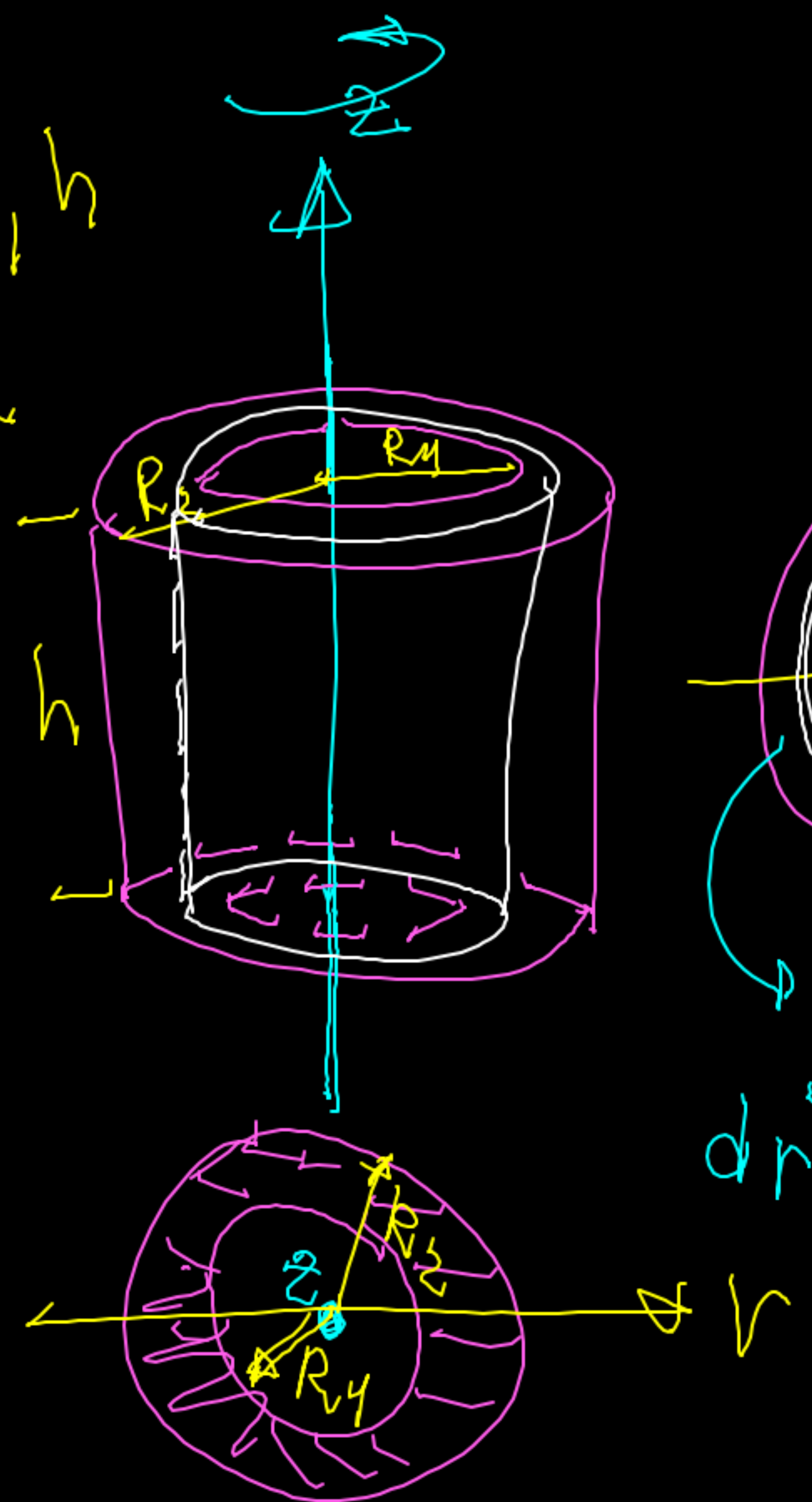
porta sottile

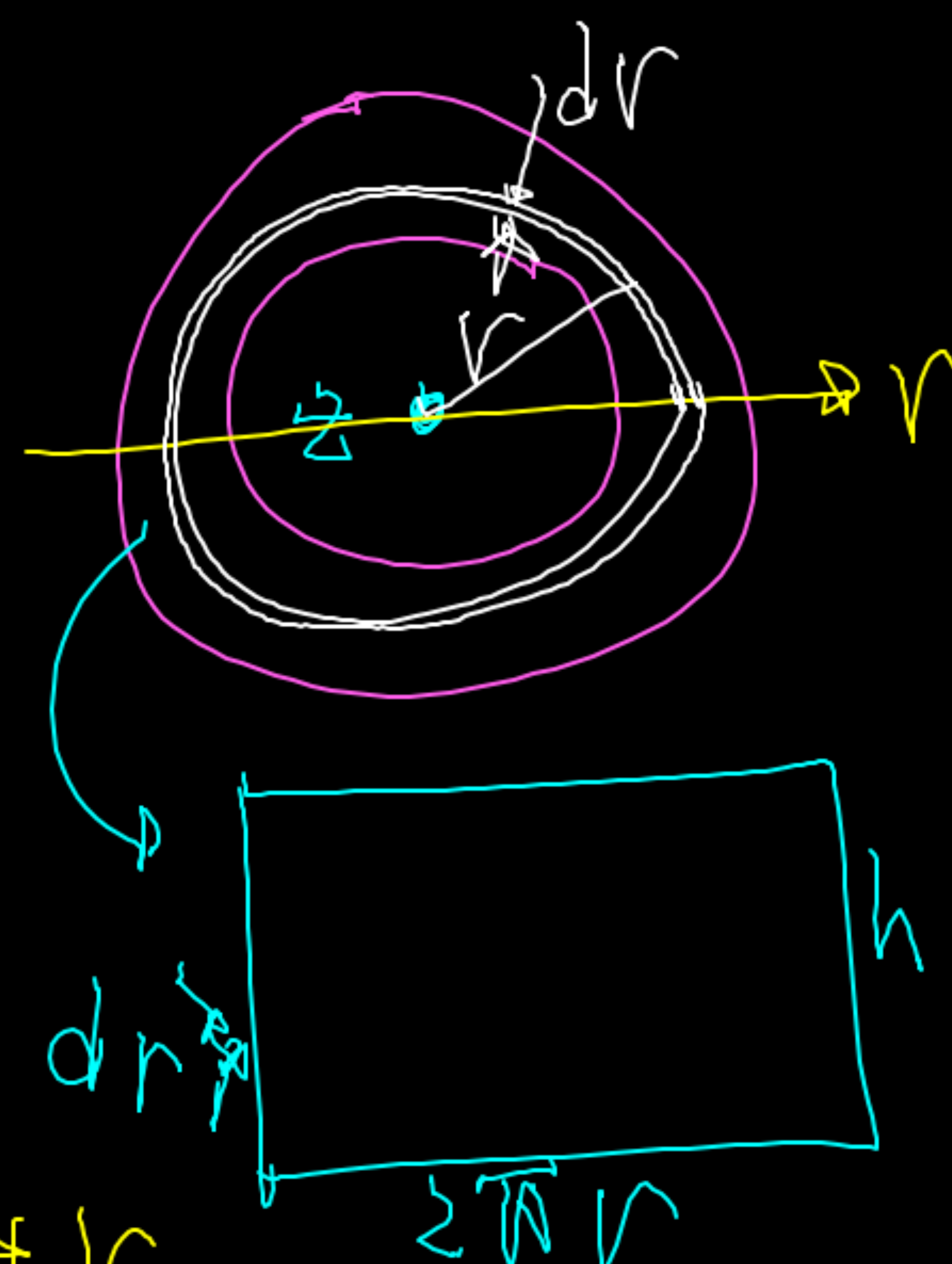


Porta non sottile

ESEMPIO

MOM. DI INERZIA DEL CILINDRO (omogeneo)





$$I_z = \int_{Vol} \rho R^2 dV$$

$$dV = 2\pi h r dr$$

$$R^2 = r^2$$

$$I_z = \int_{Vol} \rho 2\pi h r^3 dr =$$

$$= 2\pi h \rho \int_{R_1}^{R_2} r^3 dr = \frac{2\pi h \rho}{4}\left(R_2^4 - R_1^4\right)$$

$$\rho = \frac{M}{Vol} = \frac{M}{\pi R_2^2 h - \pi R_1^2 h}$$

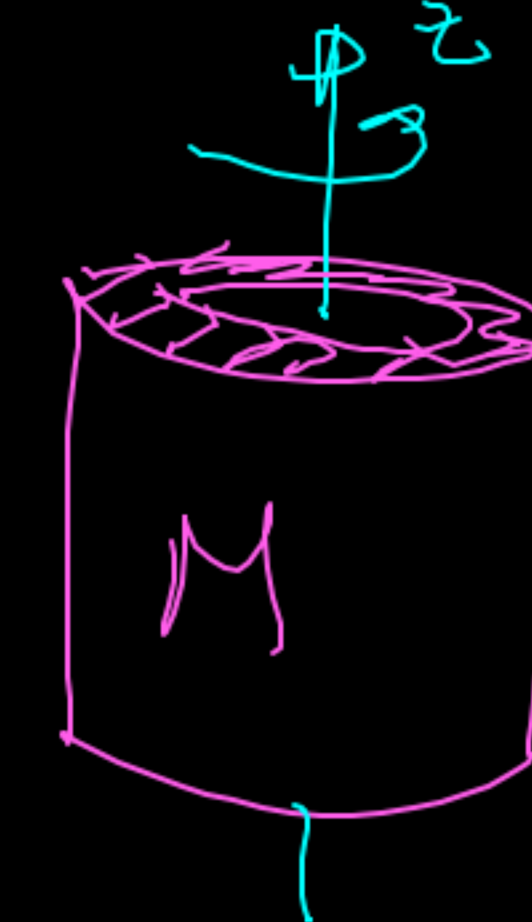
$$I_z = \frac{\pi h \rho}{2}\left(R_2^4 - R_1^4\right) = \frac{\pi h M \left(R_2^4 - R_1^4\right)}{2\left(R_2^2 - R_1^2\right)\pi h} =$$

$$\rho = \frac{M}{\pi h \left(R_2^2 - R_1^2\right)}$$

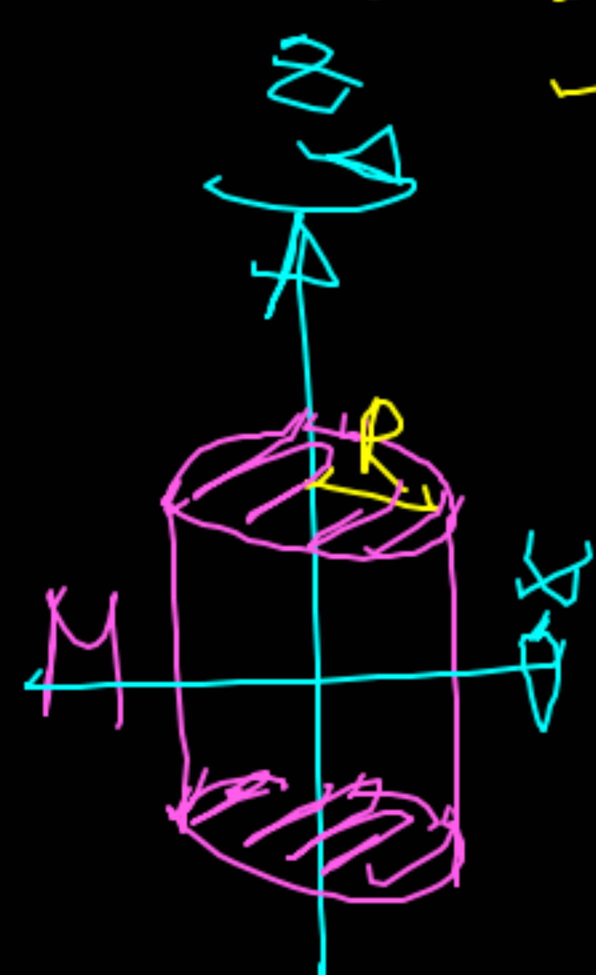
$$= \frac{M}{2}\frac{\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)}{\left(R_2^2 - R_1^2\right)} =$$

CIL. CAVO

$$I_z = \frac{M}{2}\left(R_2^2 + R_1^2\right)$$



NB $I_z \neq I_x$



$R_1 \to 0$

$R_2 \to R$

$$I_z = \frac{1}{2} M R^2$$

CIL. PIENO (DISCO)

$R_1 \to R$

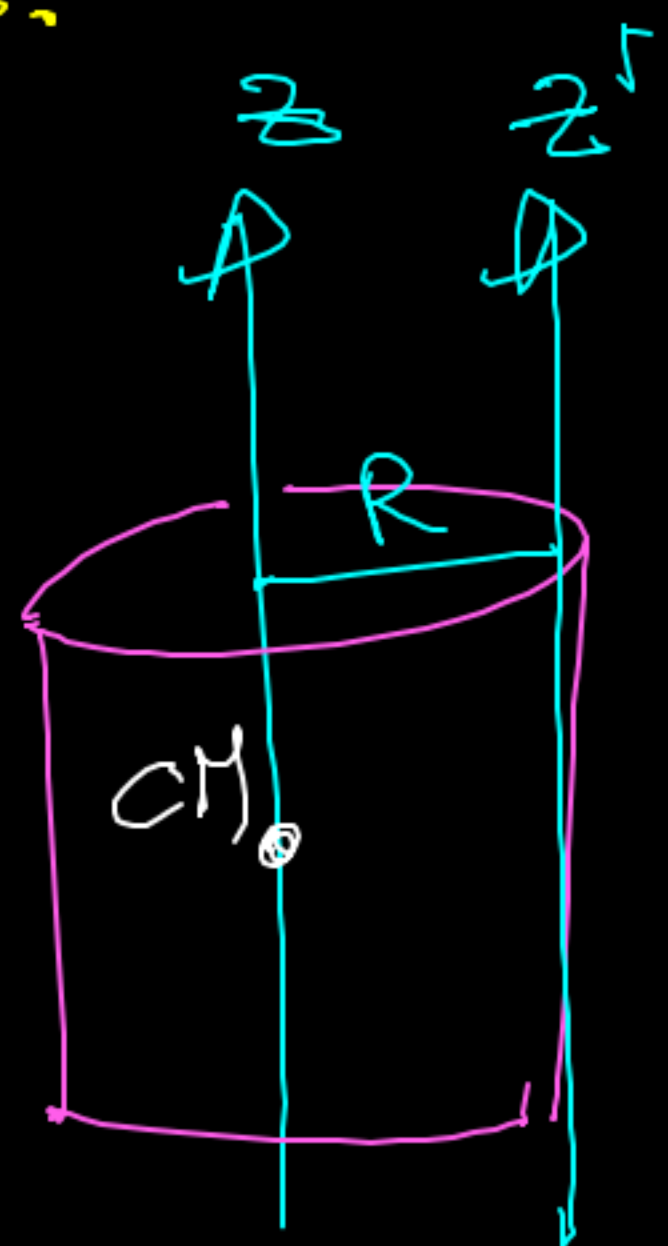
$R_2 \to R$

$$I_z = M R^2$$

STRATO CILIND. SOTTILE

TEOREMA DI HUYGENS-STEINER O DEGLI ASSI PARALLELI

Es.



$$I_z \neq I_{z'}$$

$$I_z = \frac{MR^2}{2}$$

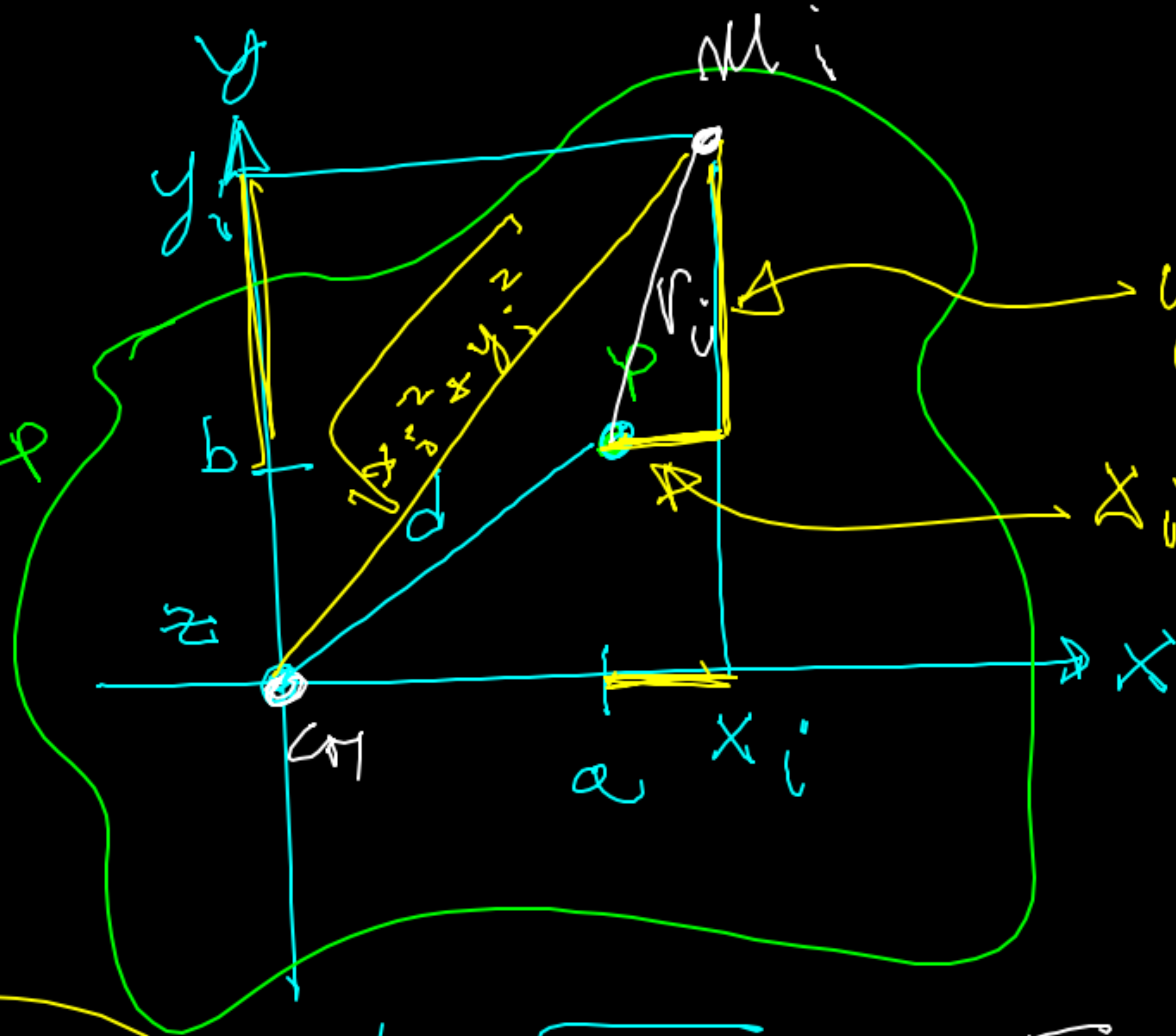
$$I_{z'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$z' \parallel z$, distanza d

$$\boxed{I_P = I_{CM} + Md^2}$$

ASSE Z uscente DI ROT

$I_{CM} + Md^2 = I_p$



$$I_p = \sum_{i=1}^{N} m_i r_i^2$$

$y_i - b$, $x_i - a$ $\Rightarrow$ $r_i^2 = (x_i - a)^2 + (y_i - b)^2 =$

$$r_i^2 = x_i^2 - 2ax_i + a^2 + y_i^2 - 2y_i b + b^2$$

$$= (x_i^2 + y_i^2) - 2ax_i - 2by_i + a^2 + b^2$$

$X_{CM} = 0$

$Y_{CM} = 0$

$$d = \sqrt{a^2 + b^2}$$

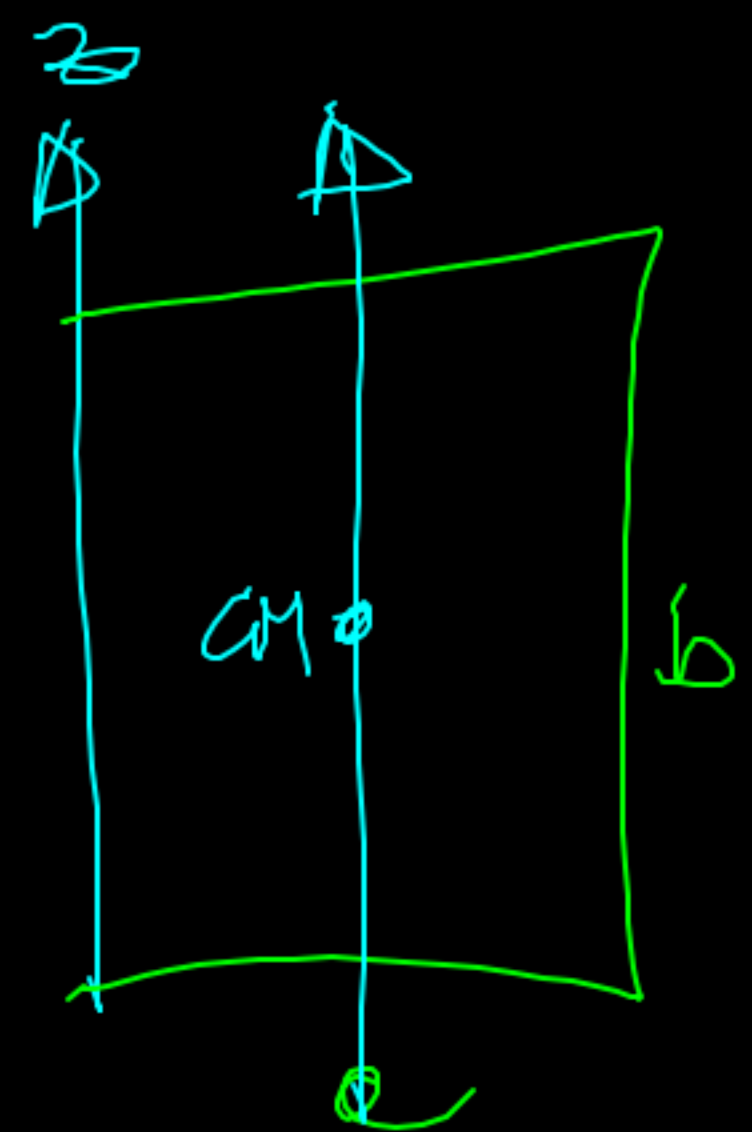
$$I_p = \sum m_i(x_i^2 + y_i^2) + m_i(a^2 + b^2) - 2am_i x_i - 2bm_i y_i$$

$$X_{CM} = \frac{\sum m_i x_i}{M}$$

$$= \underbrace{\sum m_i(x_i^2 + y_i^2)}_{I_{CM}} + \underbrace{\sum m_i(a^2 + b^2)}_{Md^2}$$

$$\underline{MX_{CM} = \sum m_i x_i = 0}$$

$$-2a\underbrace{\cancel{\sum m_i x_i}}_{=0} - 2b\underbrace{\cancel{\sum m_i y_i}}_{=0}$$



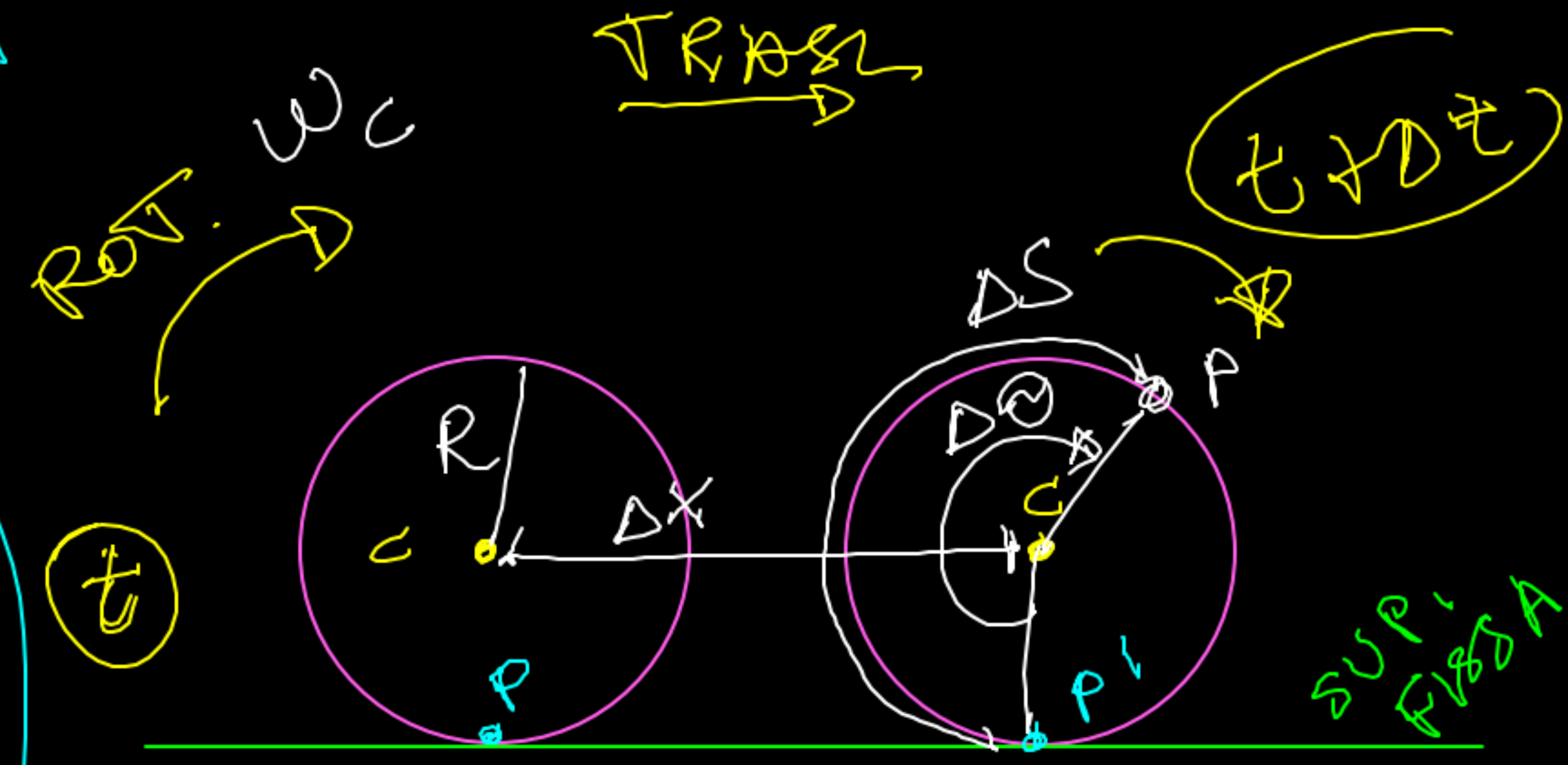
$$I_z = \frac{Ma^2}{3}$$

$$I_{cm} = ?$$

$$I_z = I_{cm} + \frac{Ma^2}{4}$$

$$I_{cm} = I_z - \frac{Ma^2}{4} = \frac{Ma^2}{3} - \frac{Ma^2}{4} = Ma^2 \frac{1}{12}$$

PURO ROTOLAMENTO DI UN CORPO RIGIDO



$\vec{v}_P(t) = 0$

"ROTOLARE SENZA STRISCIARE"

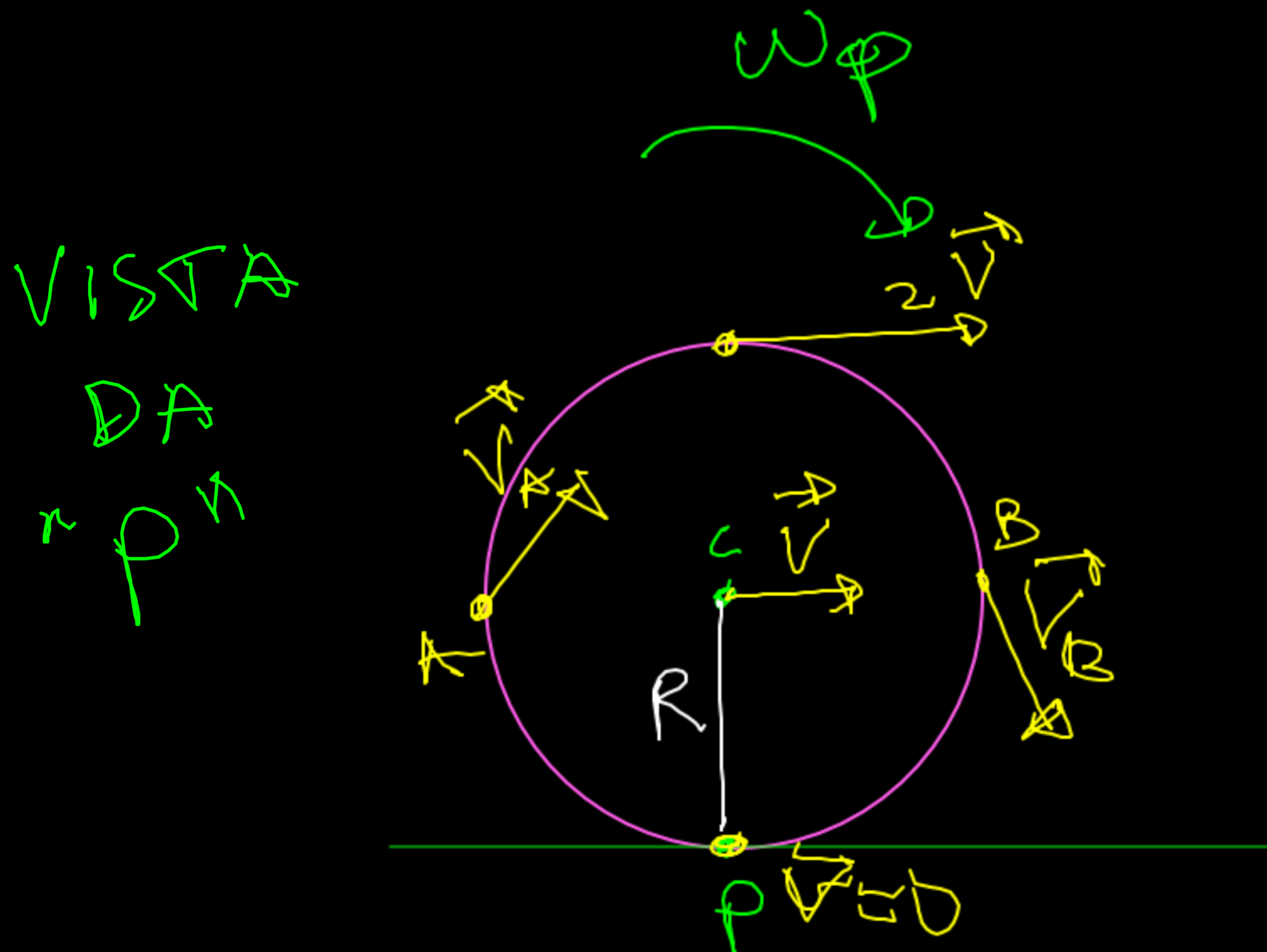
$$\frac{\Delta S}{\Delta t} = \frac{\Delta X}{\Delta t} = v_c$$

$$\frac{\Delta S}{R} = \Delta\theta \Rightarrow \frac{R\,\Delta\theta}{\Delta t} = R\,\omega_c$$

$$\boxed{v_c = \omega_c R}$$

condizione di puro rotolamento

(l'asse passante per C rimane parallelo a se stesso durante il moto)



ROTAZIONE ISTANTANEA INTORNO A P

$V = \omega_P R$ "C" visto da "P"

$V = \omega_C R$ "P" visto da "C"

$\Rightarrow \omega_C = \omega_P = \omega$

Se puro rotolamento

ENERGIA CINETICA (puro rotolamento)

$$K = \frac{1}{2} I_P \omega^2 = \frac{1}{2}\left(I_{CM} + MR^2\right)\omega^2 =$$

CECM $= \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M \underbrace{R^2\omega^2} = \frac{1}{2} I_{CM}\omega^2 + \frac{1}{2} M v_{CM}^2$

$v_{CM} = \omega R$

$$K = \underbrace{\frac{1}{2} I_{CM}\omega^2}_{\text{E.c. rot.}} + \underbrace{\frac{1}{2} M v_{CM}^2}_{\text{E.c. trasl.}}$$

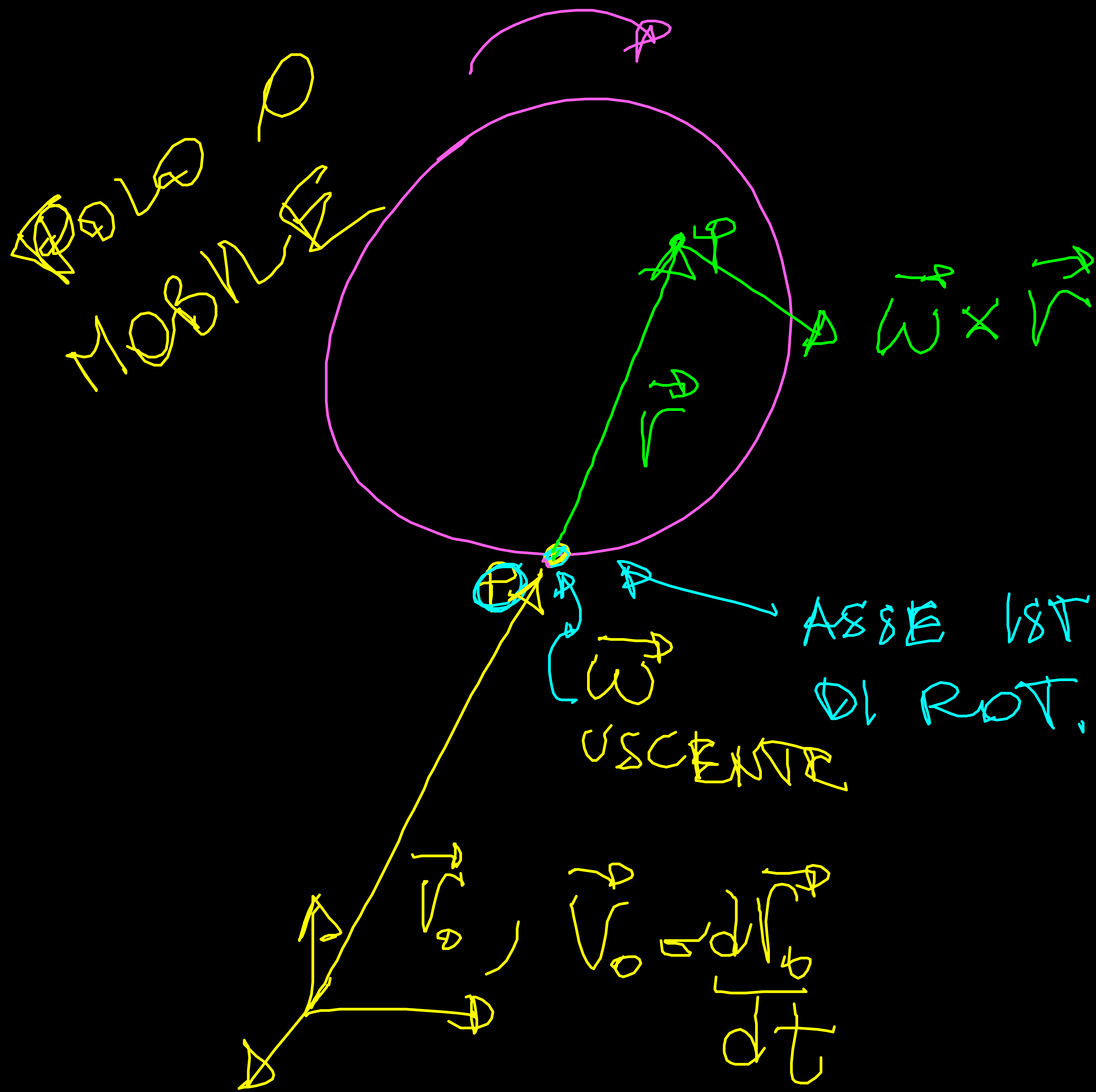
Se no puro rot.

$v \neq \omega R$ $\Rightarrow K = \frac{1}{2} I_{CM}\omega^2 + \frac{M}{2} v^2$

$\neq v_{CM}$

TEOREMA DI KÖNIG

(ASSE DI ROT. SI MANT. ∥)

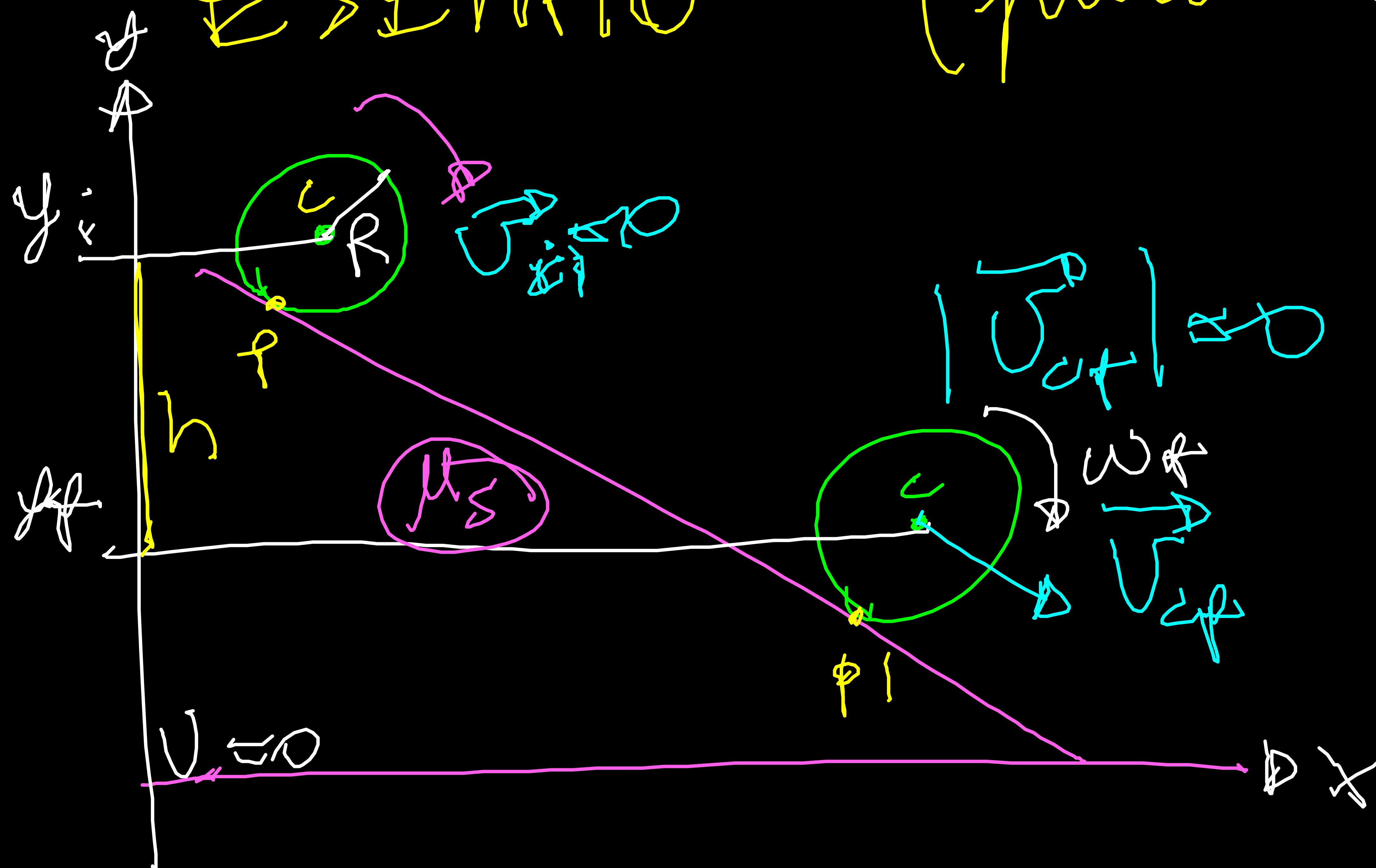


$$\boxed{\vec{v}_P = \vec{v}_O + \vec{\omega} \times \vec{r}}$$

Vel. di P vista da un sistema fisso

$\vec{v}_O$ Vel. del polo mobile vista dal sistema fisso

ESEMPIO (puro rotolamento)



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} M v_{cf}^2 + \frac{1}{2} I_{cm} \omega_f^2 + M g y_f = M g y_i$$

$$\omega_f = \frac{v_{cf}}{R}$$

$$\frac{1}{2} M v_{cf}^2 = Mg(y_f - y_i) - \frac{1}{2} I_{cm} \omega_f^2$$

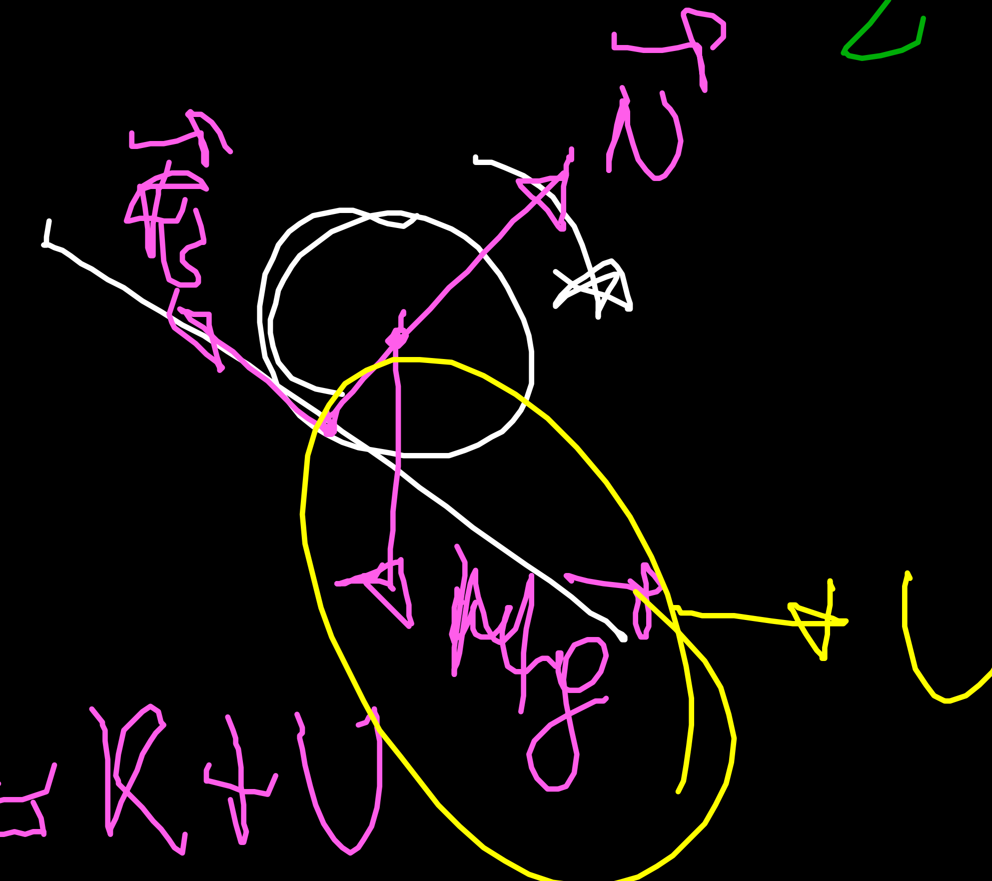
$$v_{cf}^2 + \frac{2}{2}\frac{I_{cm}}{M}\frac{v_{cm}^2}{R^2} = \frac{2 Mg(y_f - y_i)}{M}$$

$$v_{cf}^2 \left(1 + \frac{I_{cm}}{MR^2}\right) = 2g(y_f - y_i)$$

$$v_{cf} = \sqrt{2gh \Big/ \left(1 + \frac{I_{cm}}{MR^2}\right)}$$

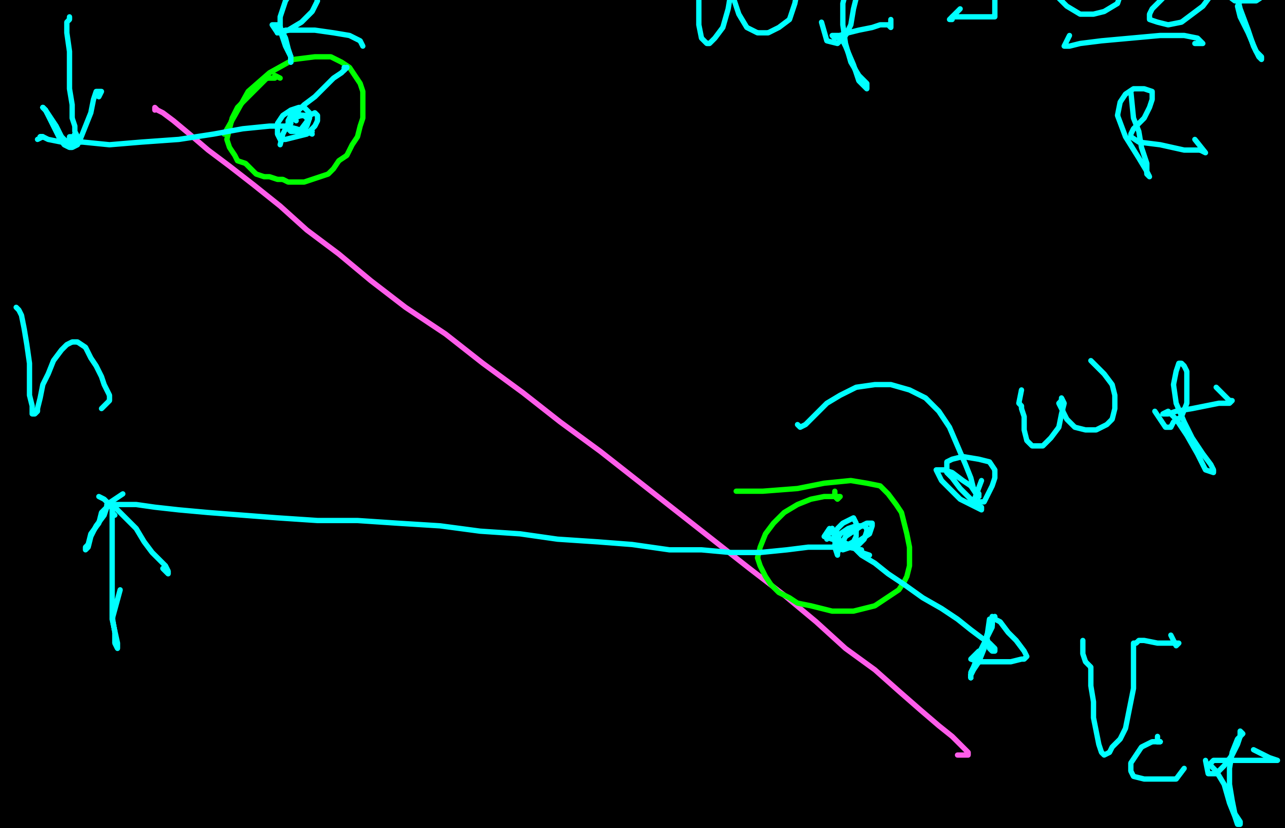
puro rotolamento senza strisciamento

CECM



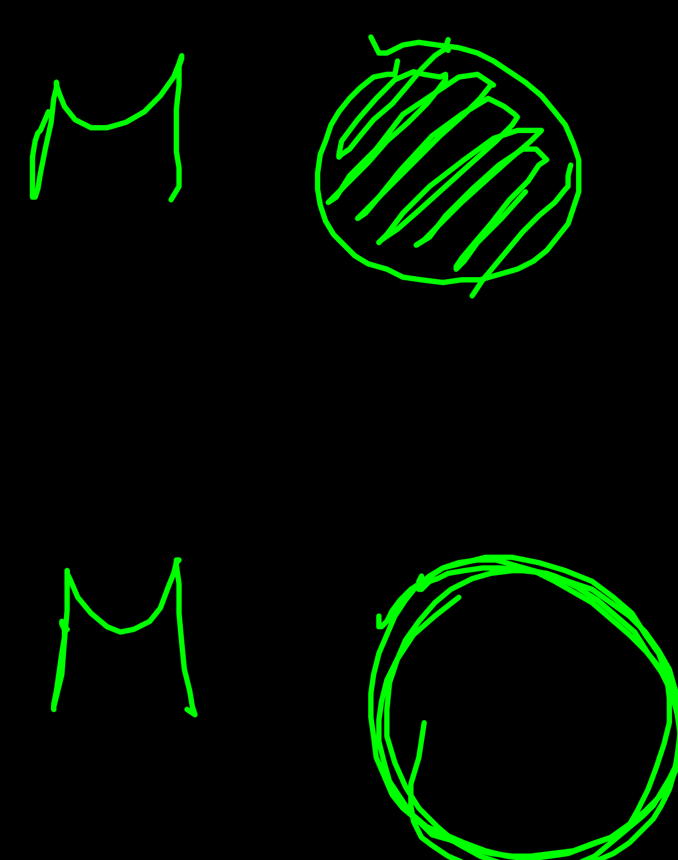
$\mathcal{L}_{attr.} = 0 \Rightarrow E = K + U$ è cost.

$$\omega_f = \frac{v_{cf}}{R}$$



$$v_{cf} = \sqrt{\frac{2gh}{1+\frac{I_{cm}}{MR^2}}}$$

CILINDRO $\longrightarrow$ $I_{cm} = \frac{1}{2}MR^2$



$$v_{cil} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{4gh}{3}}$$

ANELLO $\longrightarrow$ $I_{cm} = MR^2$

$$\boxed{v_{cil} > v_{anello}}$$

$$v_{anello} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$