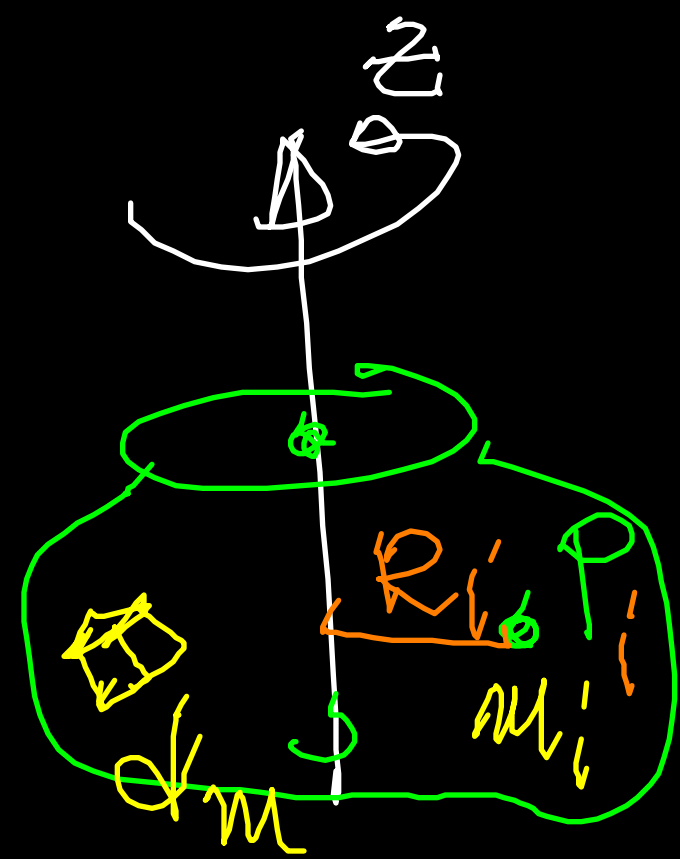


MOMENTO DI INERZIA

$$K = \frac{1}{2} I_z \omega_z^2$$



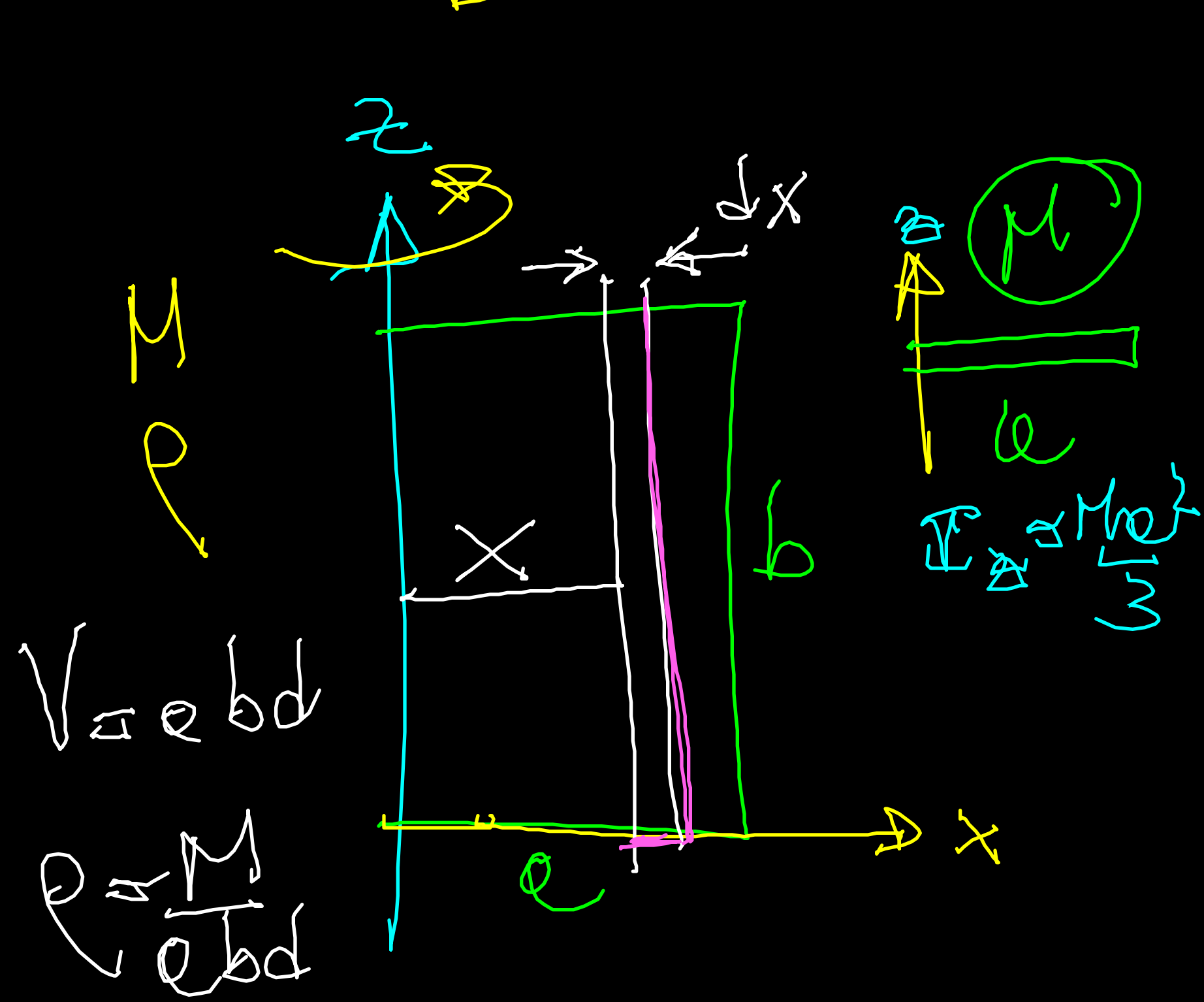
ROT. DI CORPO RIGIDO
INTORNO A Z FISSO

$$I_z = \sum_{i=1}^N m_i R_i^2$$

CASO
DISCRETO

$$I_z = \int_{\text{CORPO}} R^2 dm = \int_{\text{Vol. del corpo}} \rho R^2 dV$$

ESEMPIO DI CALCOLO



I_z della parte (omogenea) rettale

$$I_z = \int \rho R^2 dV \rightarrow I_z = \int_0^e \rho b d x^2 dx$$

Vol del corpo

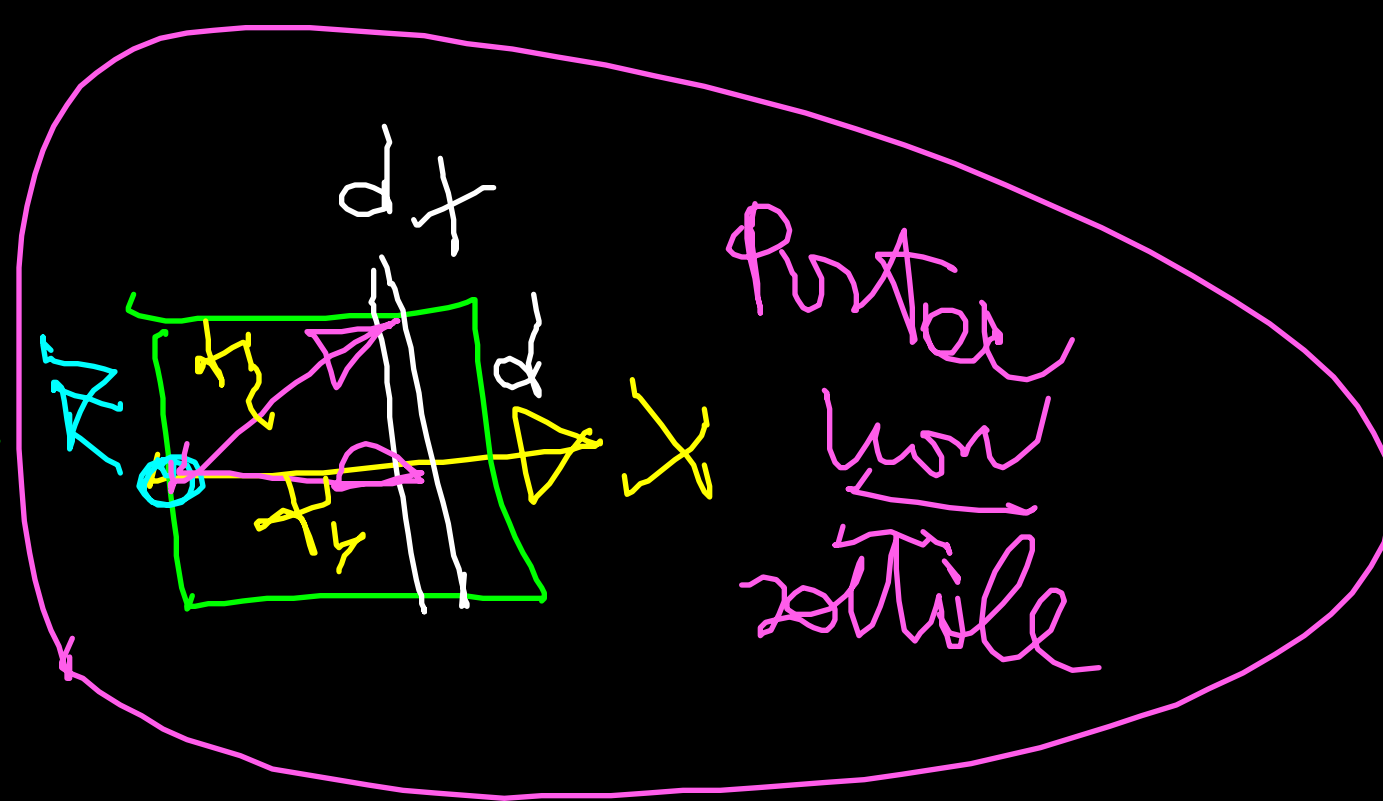
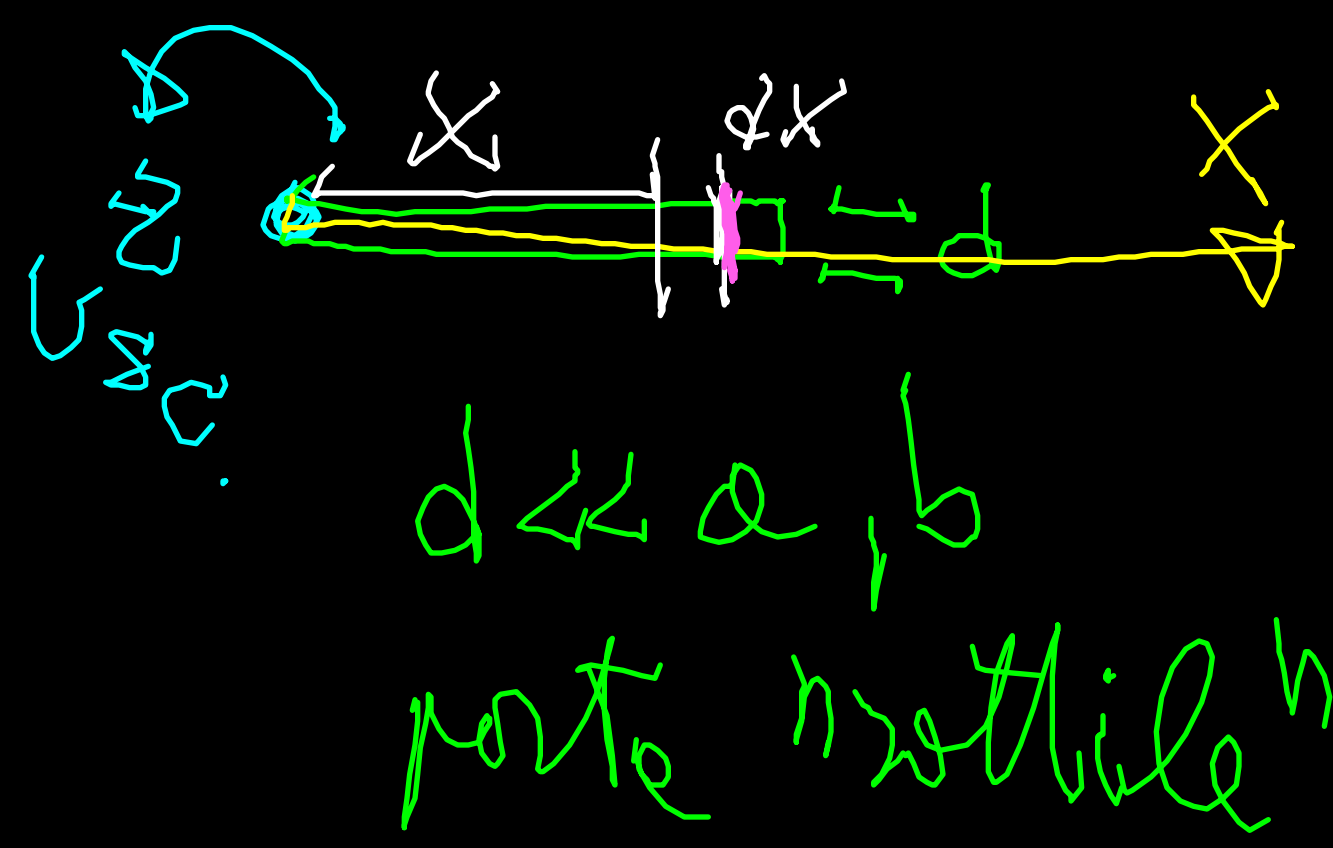
$$dV = b d dx$$

$$R = x$$

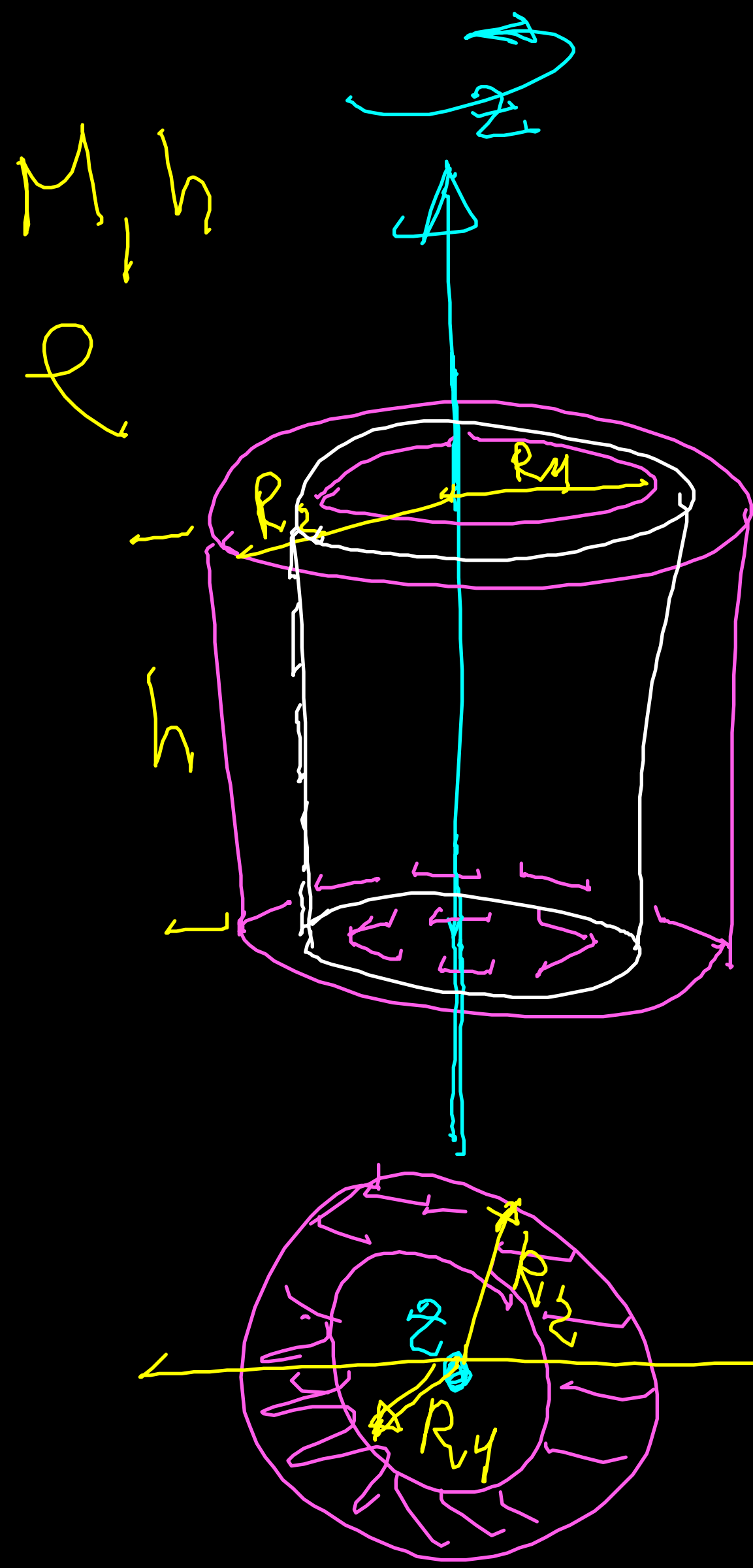
$$I_z = \rho b d \int_0^e x^2 dx =$$

$$= \rho b d \frac{e^3}{3} =$$

$$I_z = \frac{M}{e b d} b d \frac{e^3}{3} = \frac{M e^2}{3}$$



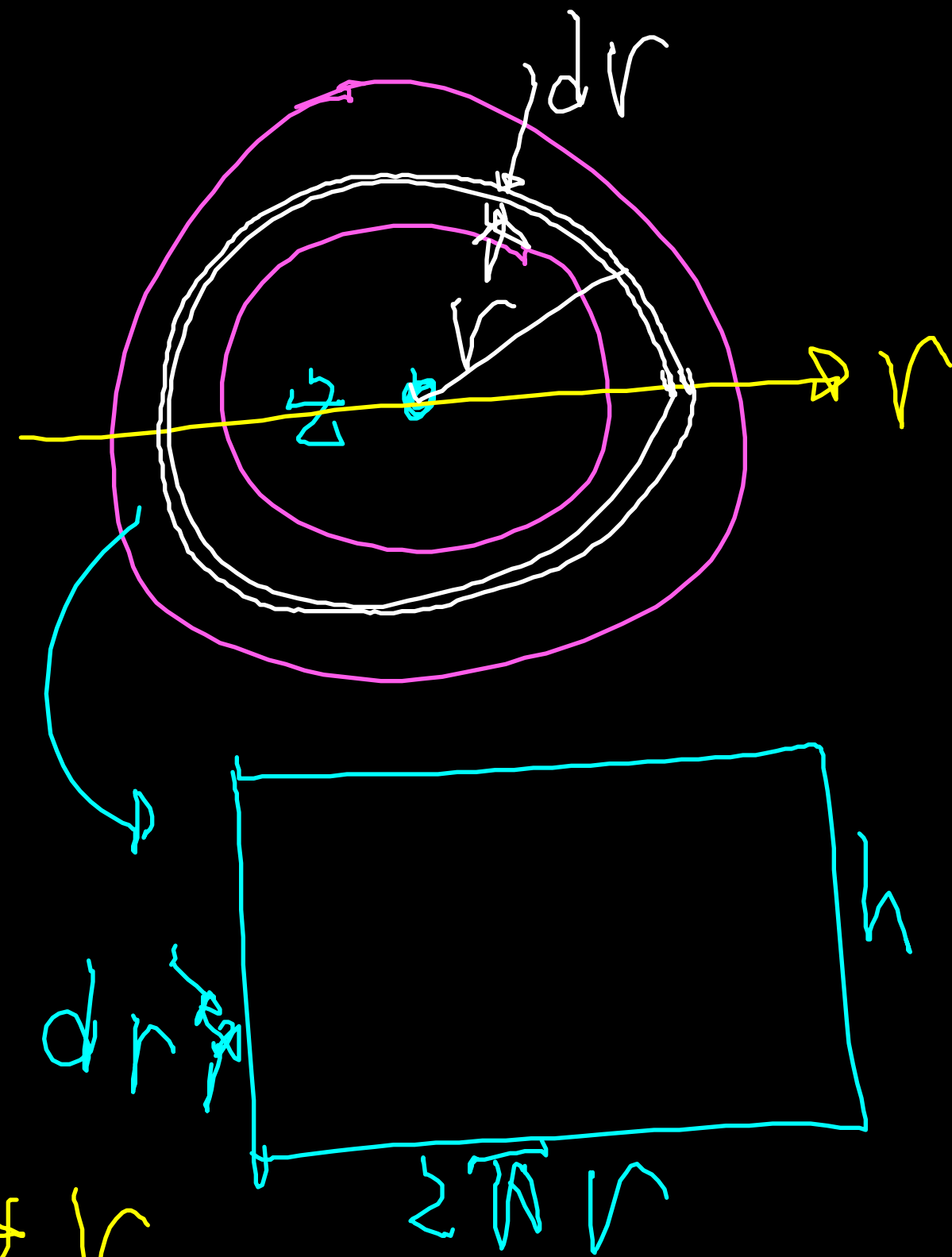
ESEMPIO MOM. DI INERZIA DEL CILINDRO (omogeneo)



$$I_z = \int_{Vol} \rho R^2 dV$$

$$dV = 2\pi h r dr$$

$$R^2 = r^2$$

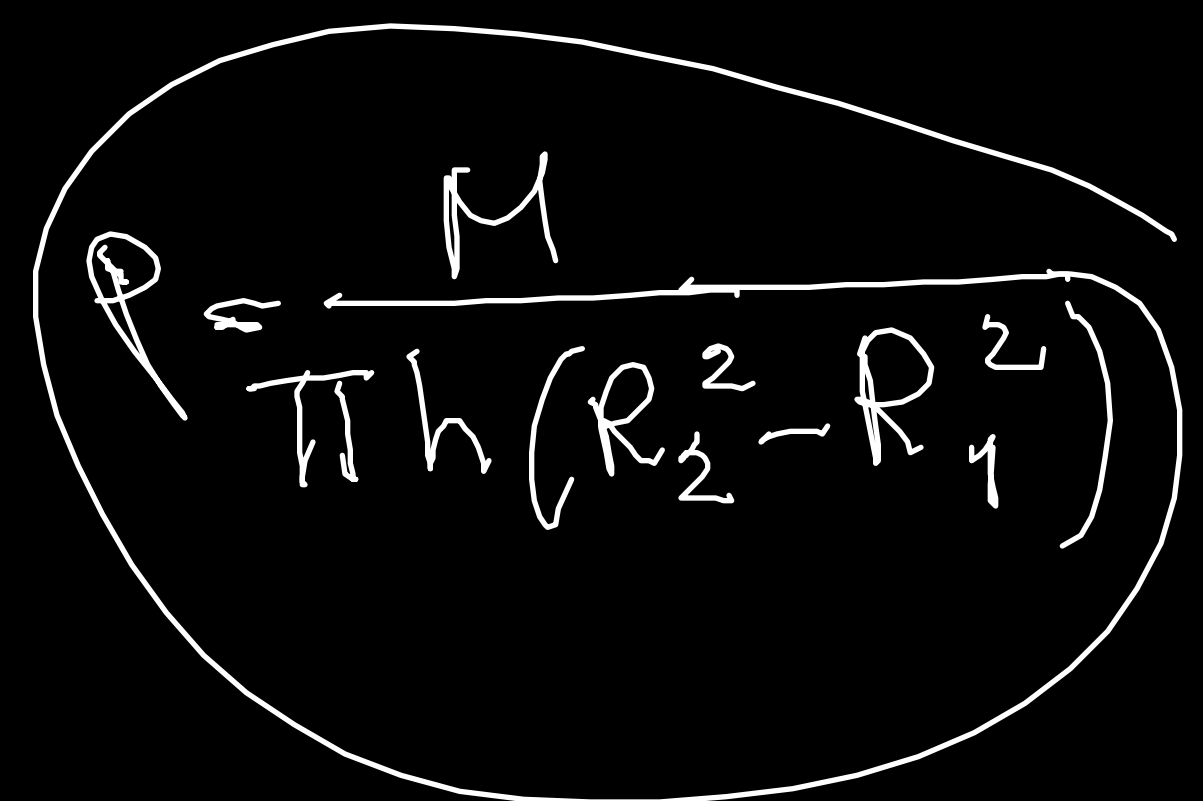


$$I_z = \int_{Vol} \rho 2\pi h r^3 dr =$$

$$= 2\pi h \rho \int_{R_1}^{R_2} r^3 dr = \frac{2\pi h \rho}{4} (R_2^4 - R_1^4)$$

$$\rho = \frac{M}{Vol} = \frac{M}{\pi R_2^2 h - \pi R_1^2 h}$$

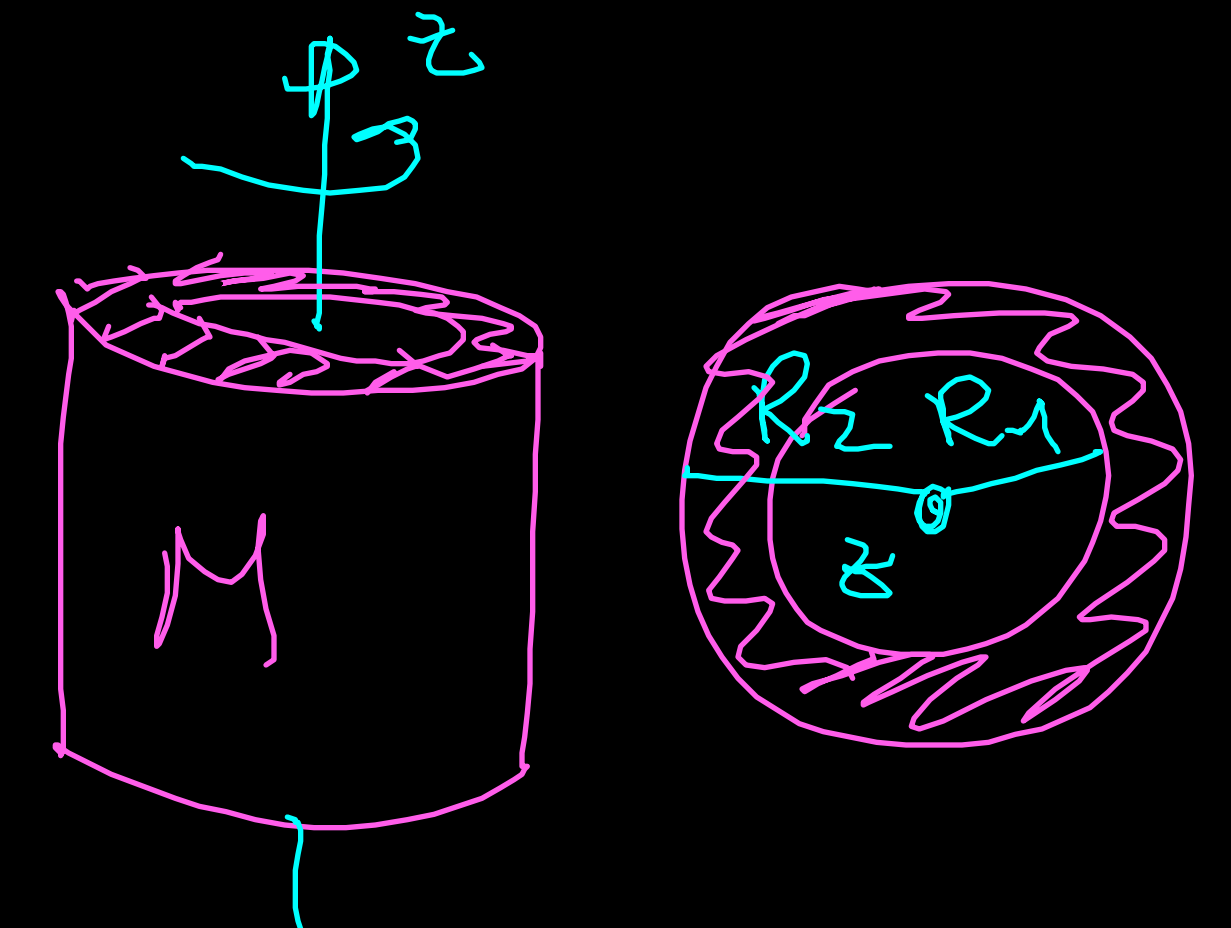
$$I_z = \frac{\pi h \rho}{2} (R_2^4 - R_1^4) = \frac{\pi h M (R_2^4 - R_1^4)}{2 (R_2^2 - R_1^2) \pi h} =$$



$$= \frac{M}{2} \frac{(R_2^2 - R_1^2) (R_2^2 + R_1^2)}{(R_2^2 - R_1^2)} =$$

CIL. CAVO

$$I_z = \frac{M}{2} (R_2^2 + R_1^2)$$



NB

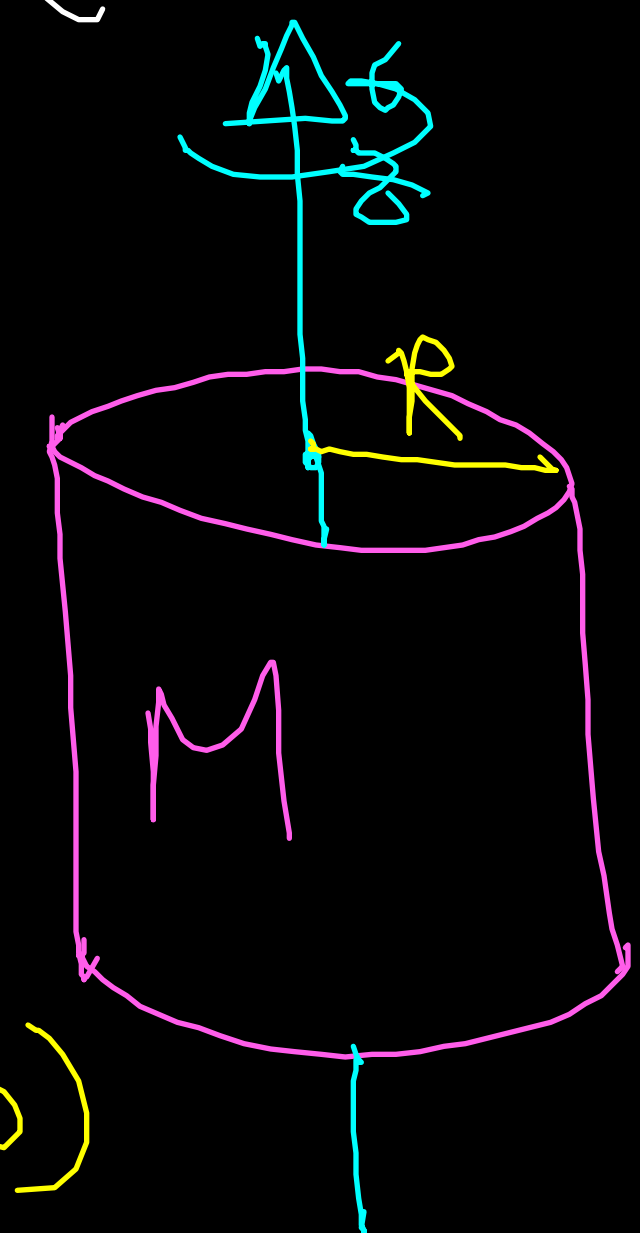
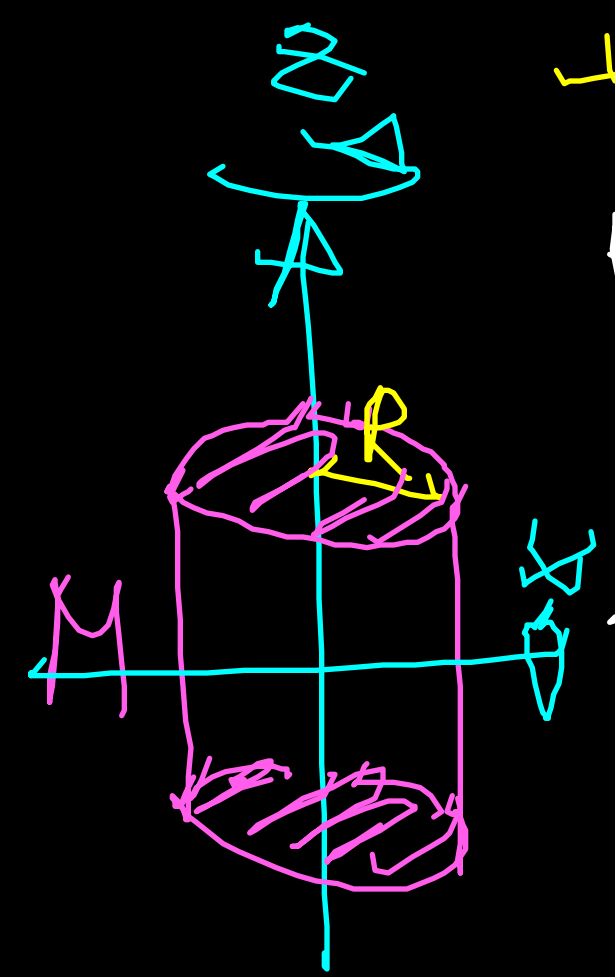
$$I_z \neq I_x$$

$$R_1 \rightarrow 0$$

$$R_2 \rightarrow R$$

$$I_z \approx \frac{1}{2} M R^2$$

CIL. PIENO (DISCO)



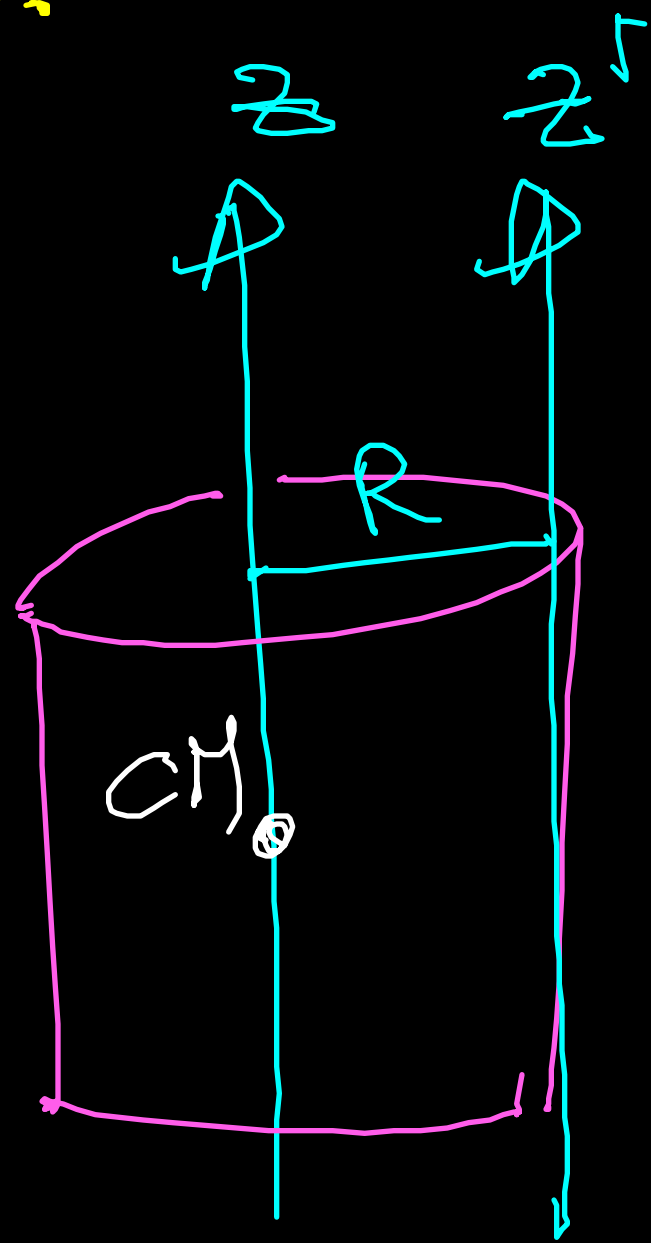
$$R_1 \rightarrow R$$

$$R_2 \rightarrow R$$

$$I_z = M R^2$$

STRATO CILIND. SOTTILE

ES.

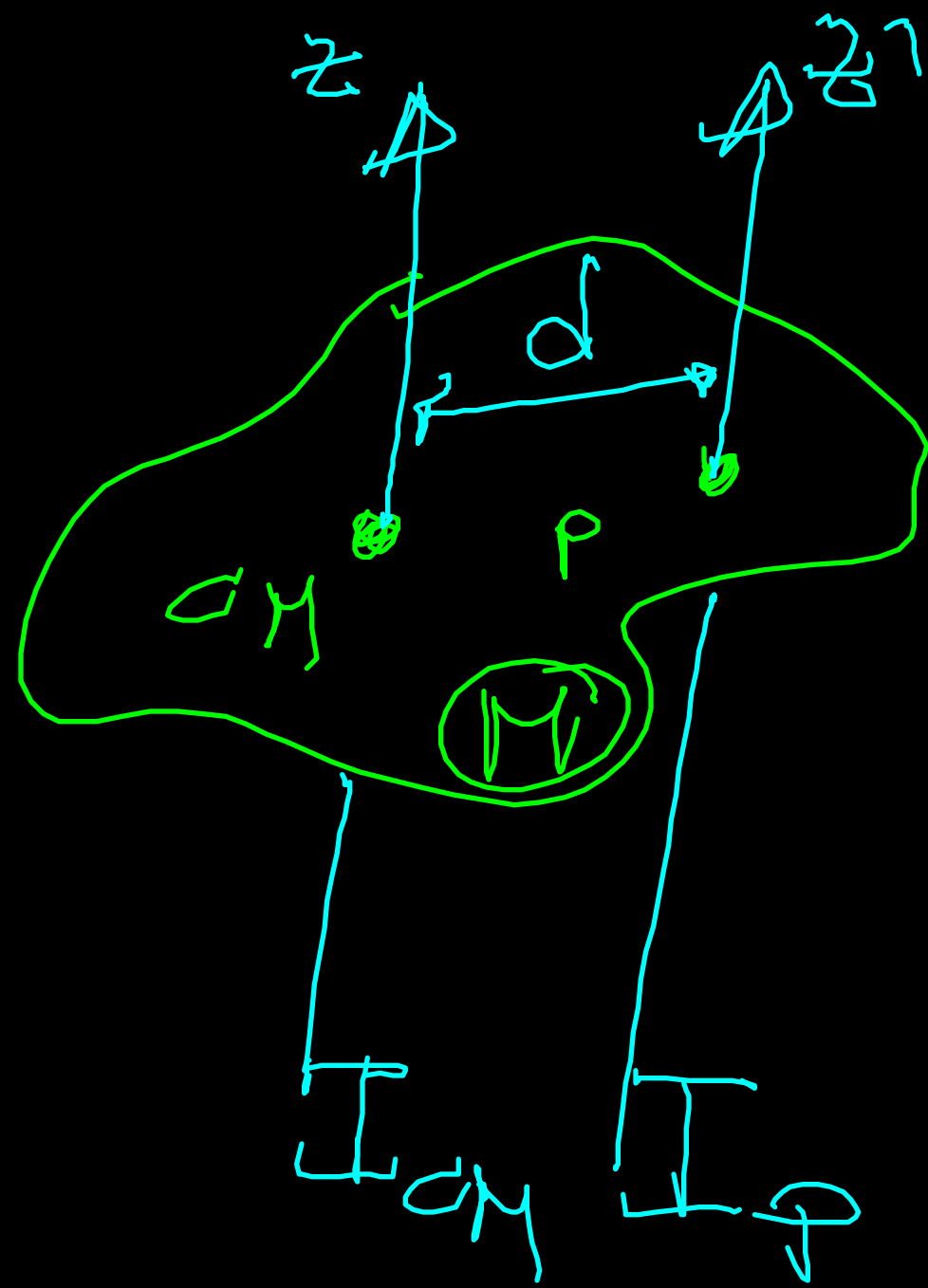


$$I_z \neq I_{z'}$$

$$I_z = \frac{MR^2}{2}$$

$$I_{z'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

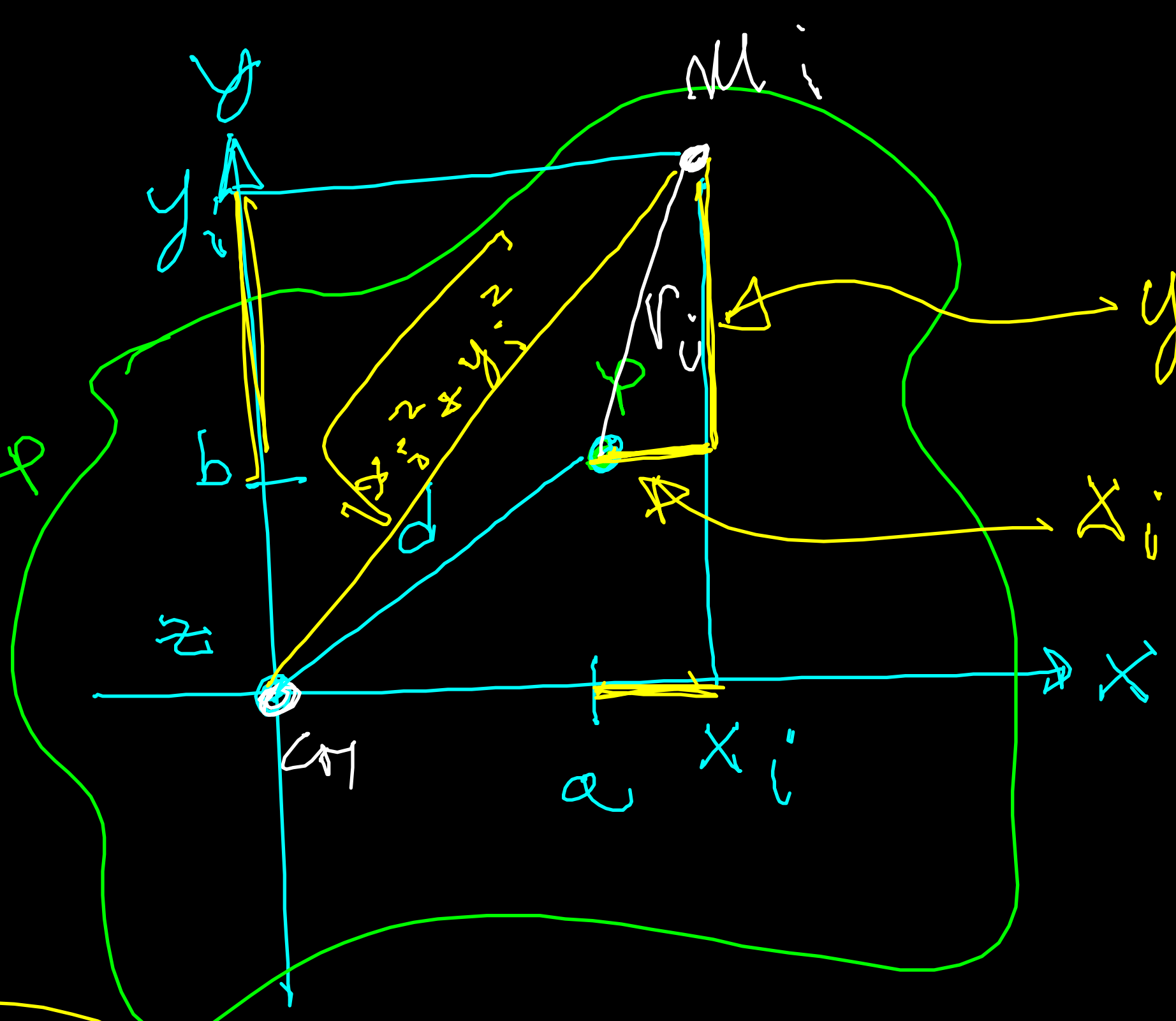
TEOREMA DI HUYGENS-STEINER O DEGLI ASSI PARALLELI



$z' \parallel z$, distanza d

$$I_P = I_{CM} + Md^2$$

Achse
 z-Masse
 D1 Rot
 $I_{cm} + Md^2 = I_p$



$$I_p = \sum_{i=1}^N m_i r_i^2$$

$$r_i^2 = (x_i - a)^2 + (y_i - b)^2$$

$$r_i^2 = x_i^2 - 2ax_i + a^2 + y_i^2 - 2by_i + b^2$$

$$= (x_i^2 + y_i^2) - 2ax_i - 2by_i + a^2 + b^2$$

$x_{cm} = 0$
 $y_{cm} = 0$

$$d = \sqrt{a^2 + b^2}$$

$$x_{cm} = \frac{\sum m_i x_i}{M}$$

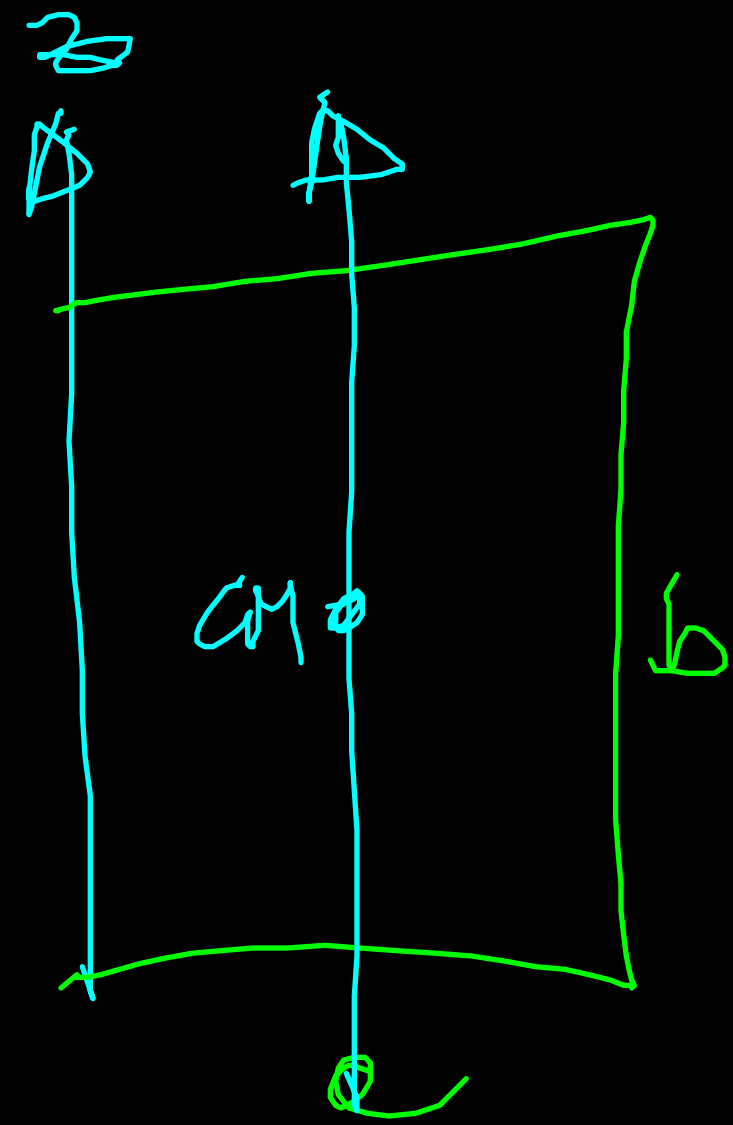
$$M x_{cm} = \sum m_i x_i = 0$$

$$I_p = \sum m_i (x_i^2 + y_i^2) + m_i (a^2 + b^2) - 2a m_i x_i - 2b m_i y_i$$

$$= \sum m_i \underbrace{(x_i^2 + y_i^2)}_{I_{cm}} + \sum m_i \underbrace{(a^2 + b^2)}_{Md^2} - 2a \sum m_i x_i - 2b \sum m_i y_i$$

$$= I_{cm} + Md^2 - 2a \sum m_i x_i - 2b \sum m_i y_i$$

$\sum m_i x_i = 0$ $\sum m_i y_i = 0$



$$I_z = \frac{M a^2}{3}$$

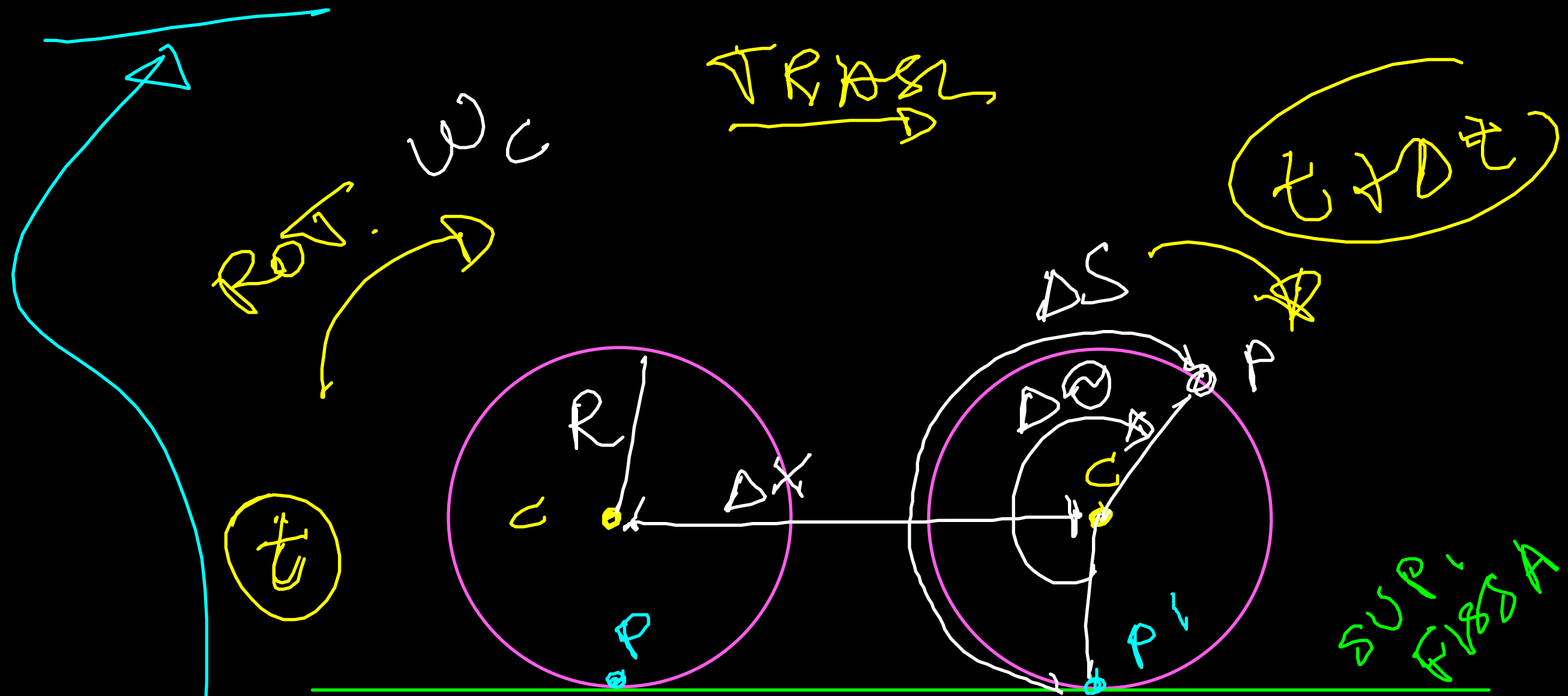
$$I_{cm} = ?$$

$$I_z = I_{cm} + M \frac{a^2}{4}$$

$$I_{cm} = I_z - M \frac{a^2}{4} =$$

$$= \frac{M a^2}{3} - M \frac{a^2}{4} = \frac{M a^2}{12}$$

PURO ROTOLAMENTO DI UN CORPO RIGIDO



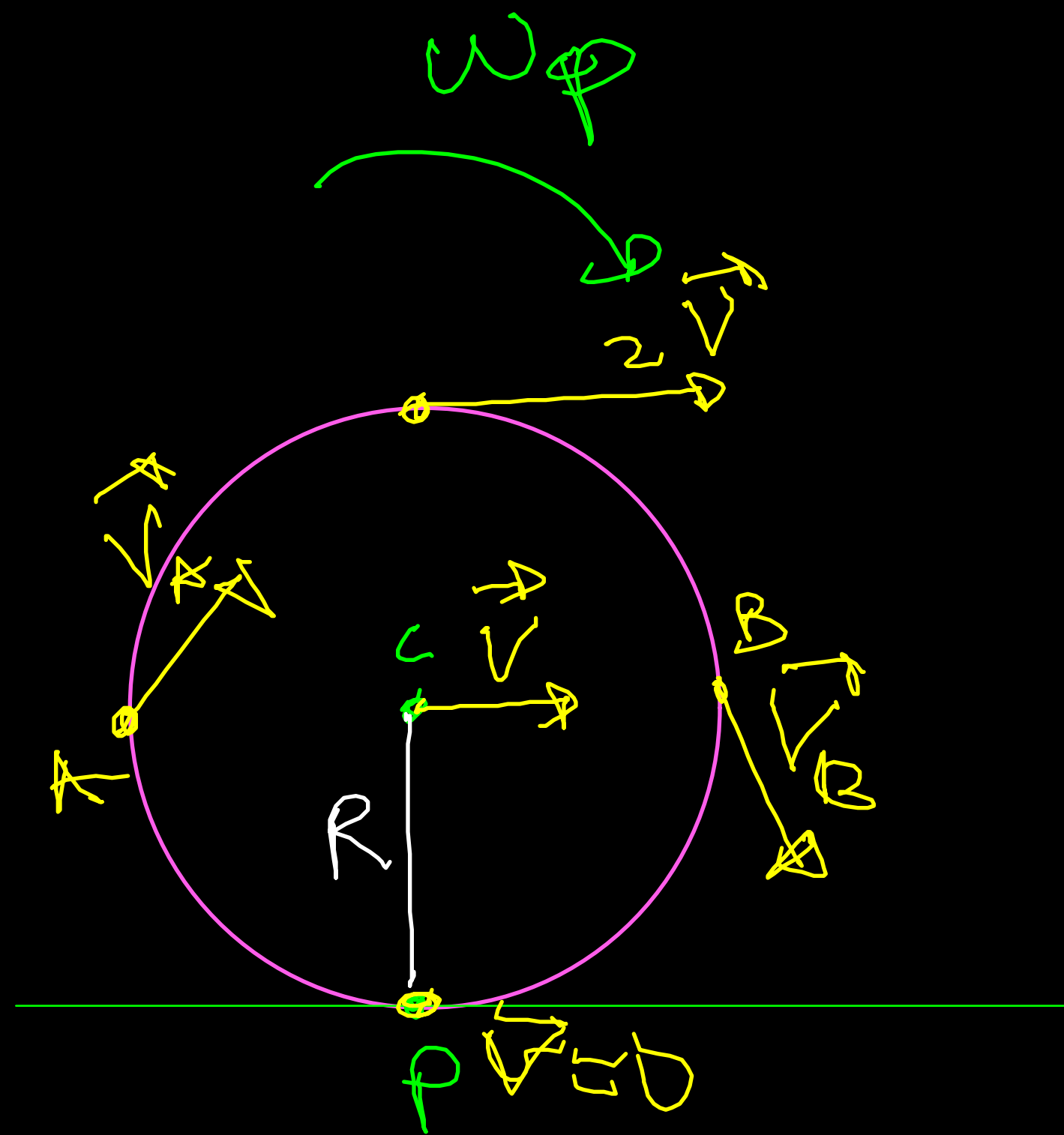
$v_C = \omega_C R$

Condizione di
 puro rotolamento
 (l'axe poutante per C
 rimane parallelo
 e x stesso durante
 il moto)

$v_P(t) = 0$
 " ROTOLARE
 SENZA
 STRISCIARE "

$$\frac{\Delta S}{\Delta t} = \frac{\Delta X}{\Delta t} = v_C$$

$$\frac{\Delta S}{R} = \Delta \theta \Rightarrow \frac{R \Delta \theta}{\Delta t} \rightarrow R \omega_C$$



ROTAZIONE
 ISTANTANEA
 INTORNO A P

$V = \omega_P R$ " C " punto da " P " $\Rightarrow \omega_C = \omega_P = \omega$

$V = \omega_C R$ " P " punto da " C "