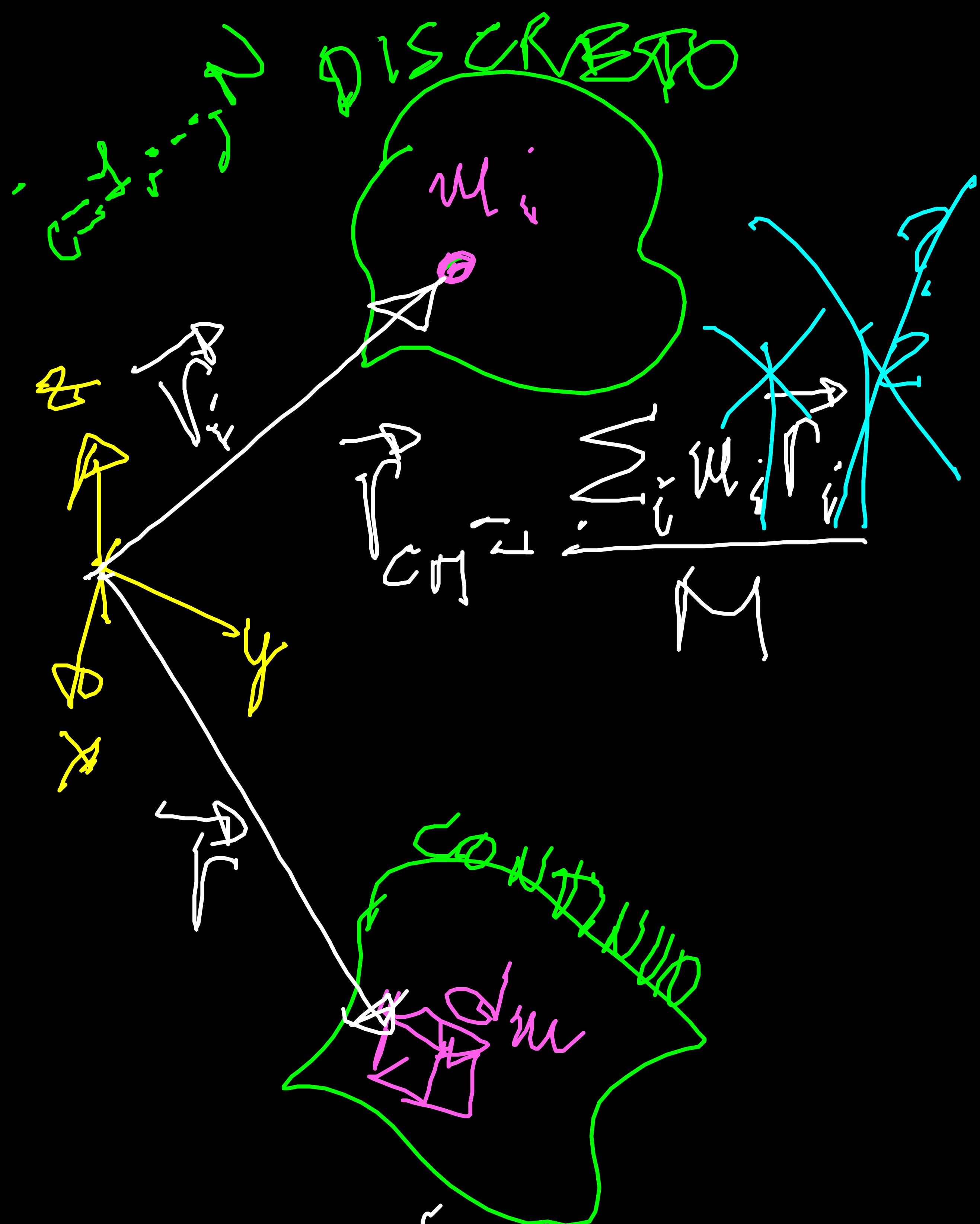
CENTRO DI MASSA → $\boxed{\text{MOTO}}$



$$\vec r_{CM} = \frac{\sum_i m_i \vec r_i}{M}$$

$$\vec V_{CM} = \frac{\int_{CORPO} \vec r\, dm}{M}$$

$$\vec r_{CM} = \frac{\sum_{i=1}^{N} m_i \vec r_i(t)}{M} \qquad M = \sum_{i=1}^{N} m_i$$

$$\vec V_{CM} = \frac{d}{dt}\vec r_{CM} = \frac{d}{dt}\left(\frac{\sum_{i=1}^{N} m_i \vec r_i(t)}{M}\right) =$$

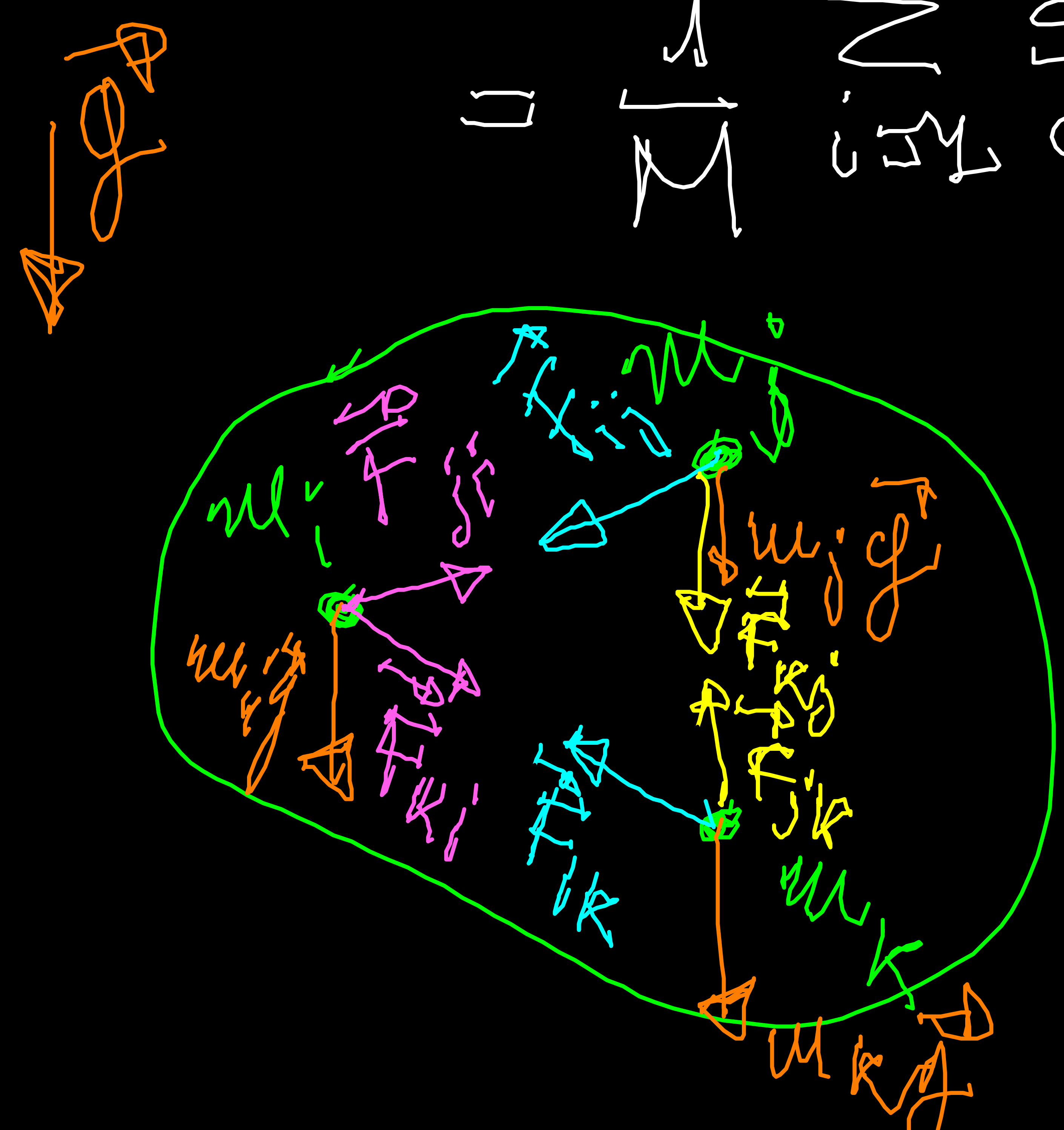
$$= \frac{1}{M}\sum_{i=1}^{N} \frac{d}{dt}\left[m_i \vec r_i(t)\right] =$$

$$= \frac{1}{M}\sum_{i=1}^{N} m_i \left(\frac{d\vec r_i(t)}{dt}\right) = \frac{1}{M}\sum_{i=1}^{N} m_i \underbrace{\vec V_i} = \vec V_{CM}$$

$$\vec{V}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{V}_i(t)$$

$$\vec{a}_{CM} = \frac{d}{dt}\vec{V}_{CM}(t) = \frac{d}{dt}\left[\frac{1}{M}\sum_{i=1}^{N} m_i \vec{V}_i(t)\right] =$$

$$= \frac{1}{M}\sum_{i=1}^{N}\frac{d}{dt}\left[m_i \vec{V}_i(t)\right] = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{d\vec{V}_i}{dt} = \boxed{\frac{1}{M}\sum_{i=1}^{N} m_i \vec{a}_i = \vec{a}_{CM}}$$



$m_i \vec{a}_i = \vec{F}_i$ ← forza risultante sull' i-esimo corpuscolo dovuta a:

$\vec{F}_i = m_i \vec{g} + \vec{F}_{ji} + \vec{F}_{ki}$

- altri corpuscoli del corpo stesso
- altri corpi

forza risultante su "i"

$$\vec{F}_i = m_i\vec{g} + \vec{F}_{ji} + \vec{F}_{ki} + \dots$$

$+$

$$\vec{F}_j = m_j\vec{g} + \vec{F}_{ij} + \vec{F}_{kj} + \dots$$

$+$

$$\vec{F}_k = m_k\vec{g} + \vec{F}_{ik} + \vec{F}_{jk} + \dots$$

$+$

$\dots$

N

$\vec{F}_{ji}$ = forza su "i" dovuta a "j"

$$\vec{F}_{ji} = -\vec{F}_{ij}$$

forza su "j" dovuta a "i"

$$\vec{a}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{a}_i = \frac{1}{M}\sum_{i=1}^{N}\vec{F}_i$$

$$\vec{F}_i + \vec{F}_j + \vec{F}_k + \dots = (m_i\vec{g} + m_j\vec{g} + m_k\vec{g} + \dots) + \vec{F}_{ji} + \vec{F}_{ki} + \dots + \vec{F}_{ij} + \vec{F}_{kj} + \dots +$$

$\cancel{}$

$$\sum m_i \vec{a}_i = \left(m_i \vec{g} + m_j \vec{g} + m_k \vec{g} + \dots \right) + \sum \vec{F}_i$$

II P.

$$+ \vec{F}_{ji} + \vec{F}_{xj} + \dots + \cancel{\vec{F}_{ij}} + \cancel{\vec{F}_{kj}} + \dots + \cancel{\vec{F}_{ik}} + \vec{F}_{jk} + \dots =$$

$$\vec{F}_{ji} = -\vec{F}_{ij}$$

III P.

$-\vec{F}_{ji}$ $\quad -\vec{F}_{jk}$ $\quad -\vec{F}_{ki}$

somma = 0

somma = 0

somma = 0

$$= \sum_{i=1}^{N} m_i \vec{g} = \sum_{i=1}^{N} m_i \vec{a}_i$$

$$\vec{a}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{a}_i$$

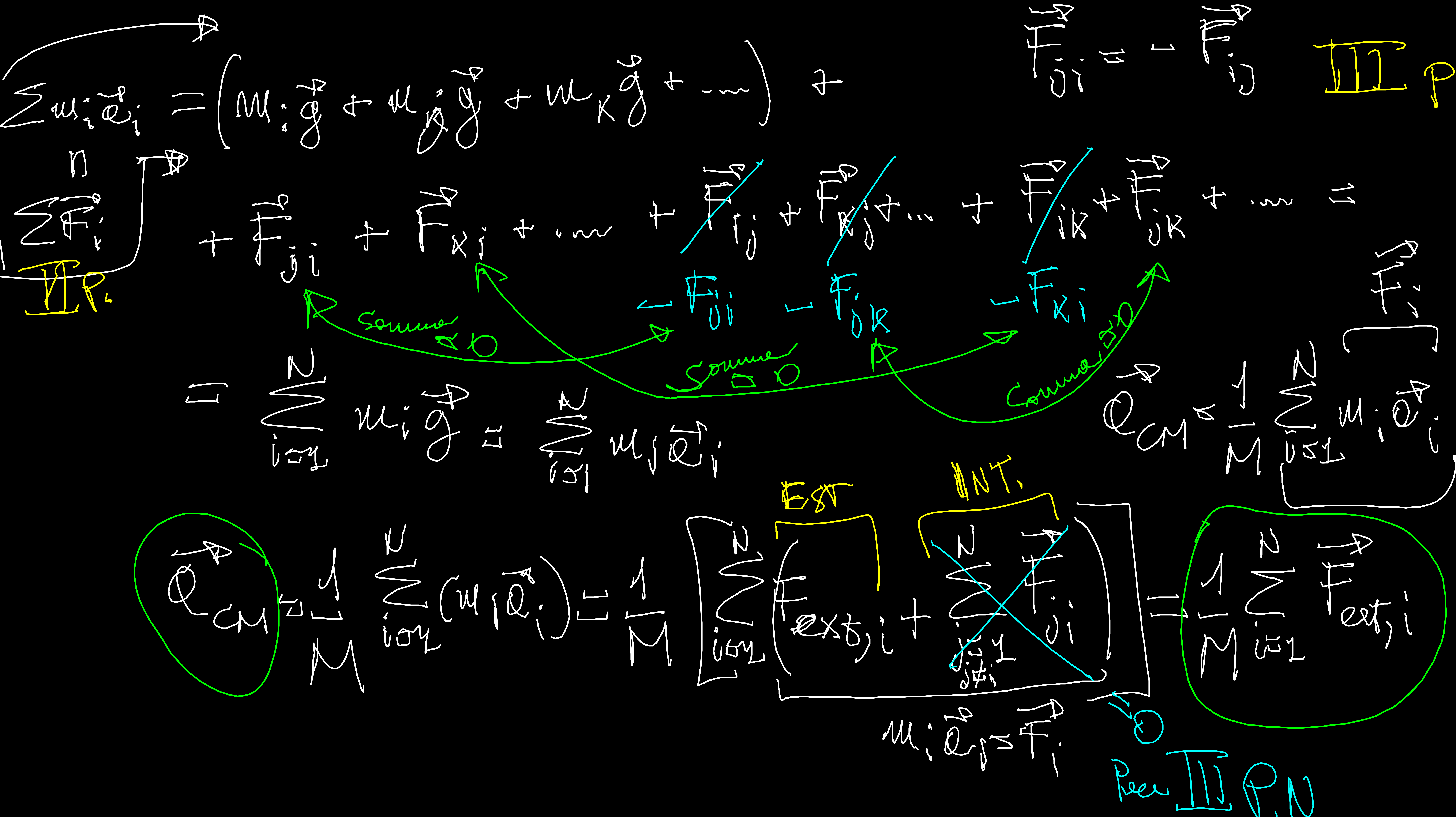
EST INT.

$$\vec{a}_{CM} = \frac{1}{M} \sum_{i=1}^{N} (m_i \vec{a}_i) = \frac{1}{M} \left[\sum_{i=1}^{N} \left(\vec{F}_{ext,i} + \cancel{\sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{F}_{ji}} \right) \right] = \frac{1}{M} \sum_{i=1}^{N} \vec{F}_{ext,i}$$

$$m_i \vec{a}_i = \vec{F}_i$$

$= 0$

Per III P.N.



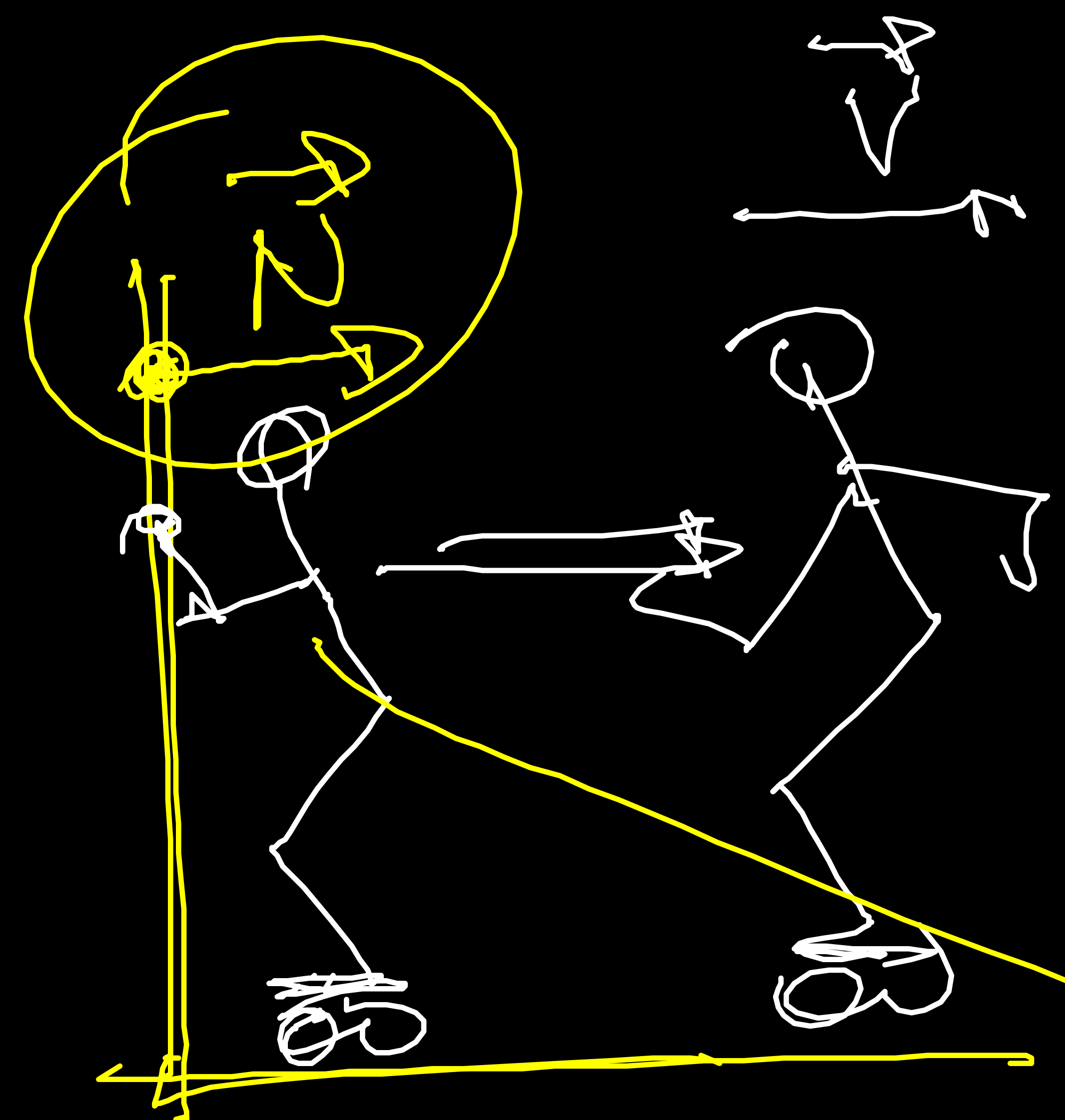
MOTO DEL CENTRO DI MASSA

$$\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

$$\Rightarrow \vec{a}_{CM} = \frac{1}{M}\sum \left(\vec{F}_{ext,i} + \cancel{\vec{F}_{int,i}}\right) = \frac{1}{M}\sum_{i=1}^{N} \vec{F}_{ext,i}$$

III P.N.

Il CM di massa si muove come un punto materiale di massa M soggetto alle sole forze esterne



$$\Delta K = \mathcal{L}_{ext} + \mathcal{L}_{int} \neq 0$$

Forze muscolari

$$\Delta K \neq 0$$
$$= \frac{1}{2} m V^2$$

$\vec{N}$ è $\underline{ext}$

ma $\mathcal{L}_N = 0$

?

ma $\sum_{i=1}^{N} \vec{F}_{int,i} = 0$

$$\boxed{M \vec{a}_{CM} = \sum_{i=1}^{N} \vec{F}_{ext,i}}$$

QUANTITÀ DI MOTO

Per un punto materiale $\vec{p} = m\vec{v}$

II P.N. $\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$

VALE ANCHE CON m NON COSTANTE

se m è costante

$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$

Per un sistema di N punti materiali

$\vec{P} = \sum_{i=1}^{N} \vec{p}_i$ q. di m. totale

$$\frac{d}{dt}\left(\vec{P}\right) = \frac{d}{dt}\sum_{i=1}^{N}\vec{p}_i = \frac{d}{dt}\underbrace{\sum_{i=1}^{N} m_i\vec{v}_i}_{=M\vec{v}_{CM}} = \frac{d}{dt}\left(M\vec{v}_{CM}\right) = M\frac{d\vec{v}_{CM}}{dt}$$

$$= M\vec{a}_{CM}$$

$$\vec{v}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i$$

$$M\vec{a}_{CM} = \boxed{\frac{d\vec{P}}{dt} = \sum_{i=1}^{N}\vec{F}_{ext,i}}$$

I EQ. CARDINALE DELLA DINAMICA

Supponiamo che

$$\sum_{i=1}^{N}\vec{F}_{ext,i} = 0$$

$$\Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} \text{ SI "CONSERVA"}$$

FINORA DUE QUANTITÀ CONSERVATE

SCALARE

E MECC. $\xrightarrow{\text{CONS.}}$ SE SOLO FORZE CONS.

$\vec{P}$ TOT $\xrightarrow{\text{CONS.}}$ SE $\sum \vec{F}_{est} = 0$

VETTORE