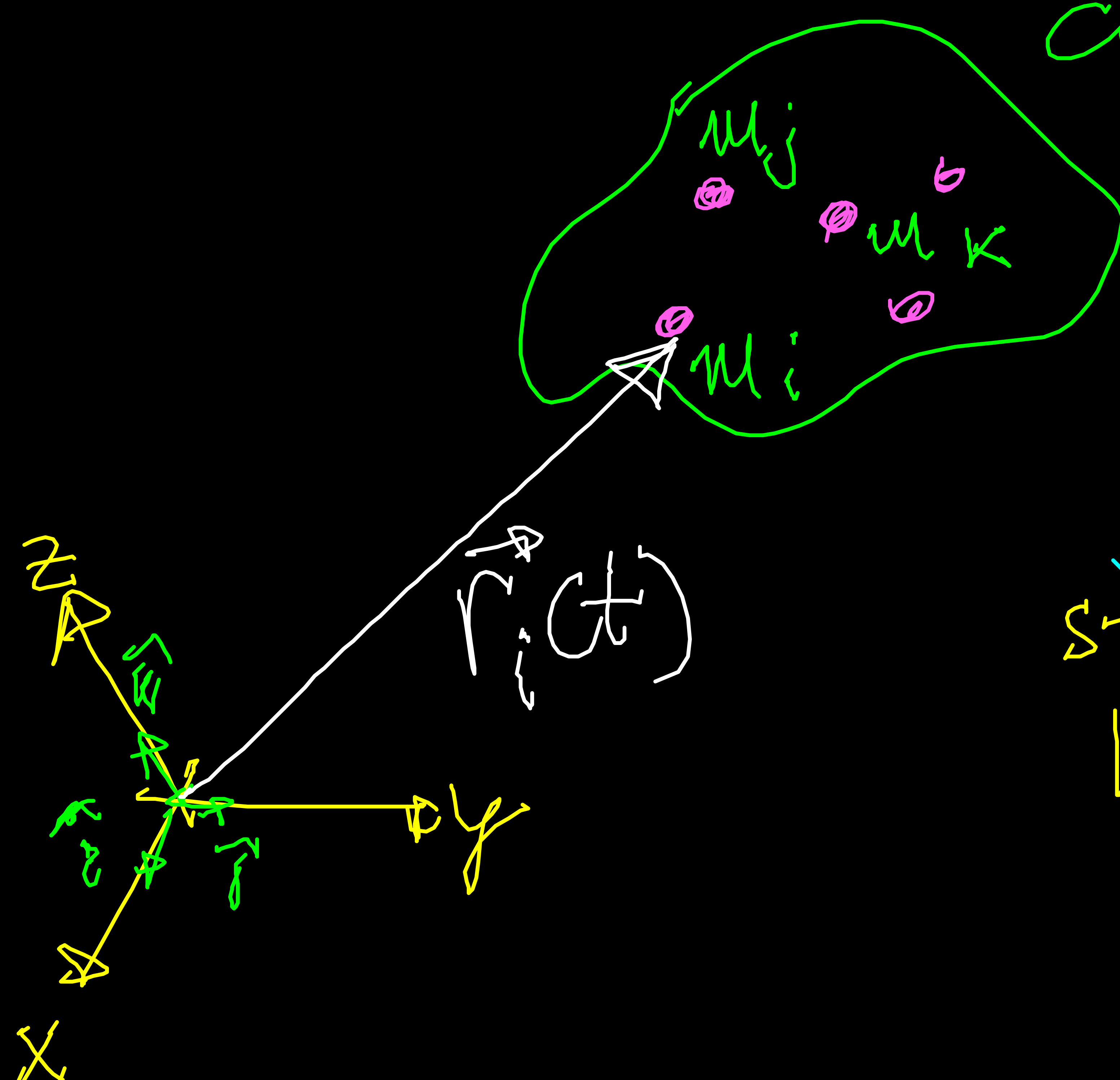
MOTO DEI SISTEMI



CORPO ESTESO

$i = 1, \dots, N$ CORPUSCOLI

$N \approx N_A \approx 6{,}023 \times 10^{23}$

~~STUDIO DEL MOTO INDIVIDUALE DEGLI~~ ~~$m_i \Rightarrow 6 \cdot N_A$ VARIABILI~~

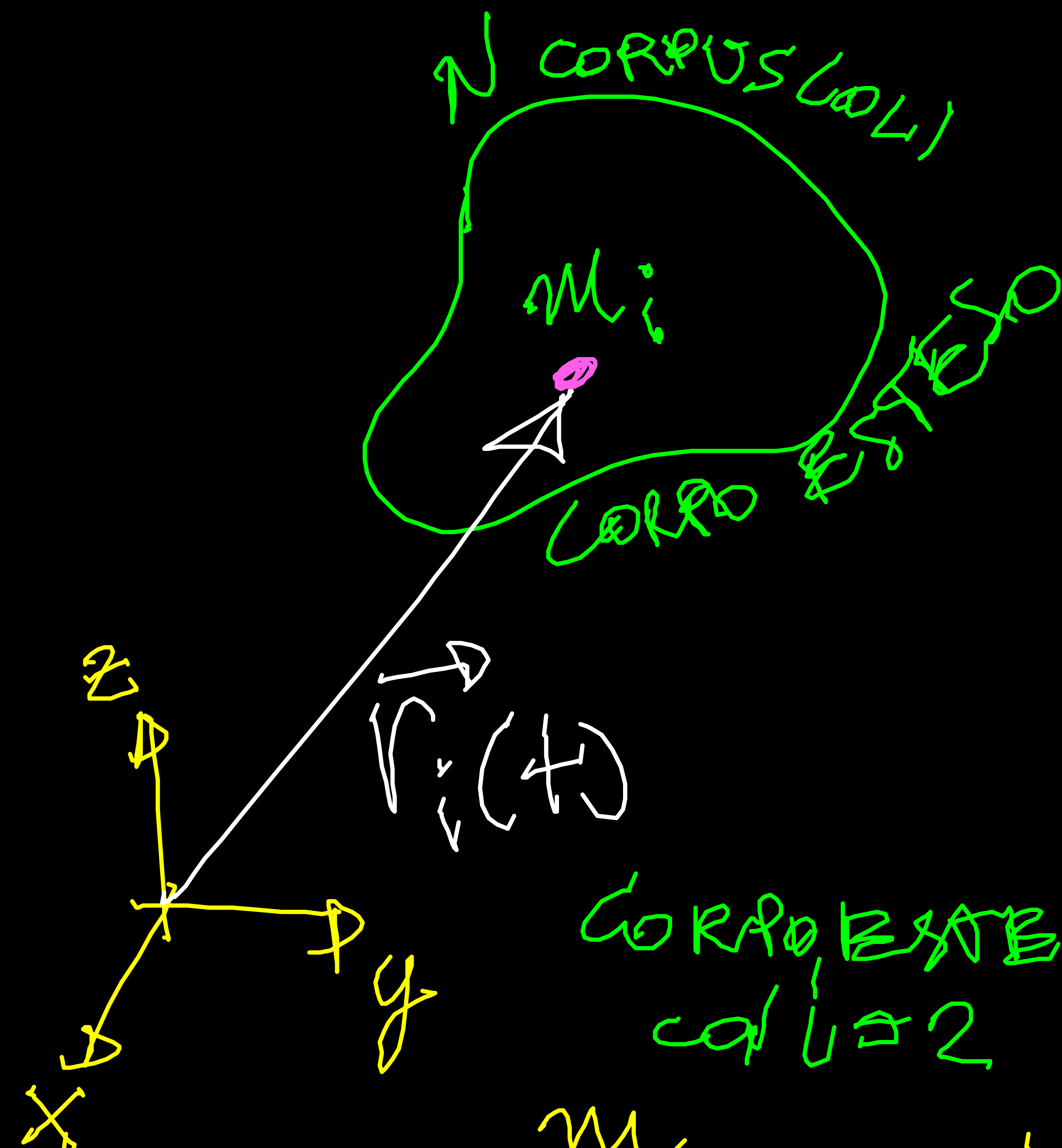
$\Downarrow$

CENTRO DI MASSA $\Leftarrow$ PROPRIETÀ MEDIE

$\Downarrow$

RIDURRE IL NUM. DELLE VARIABILI

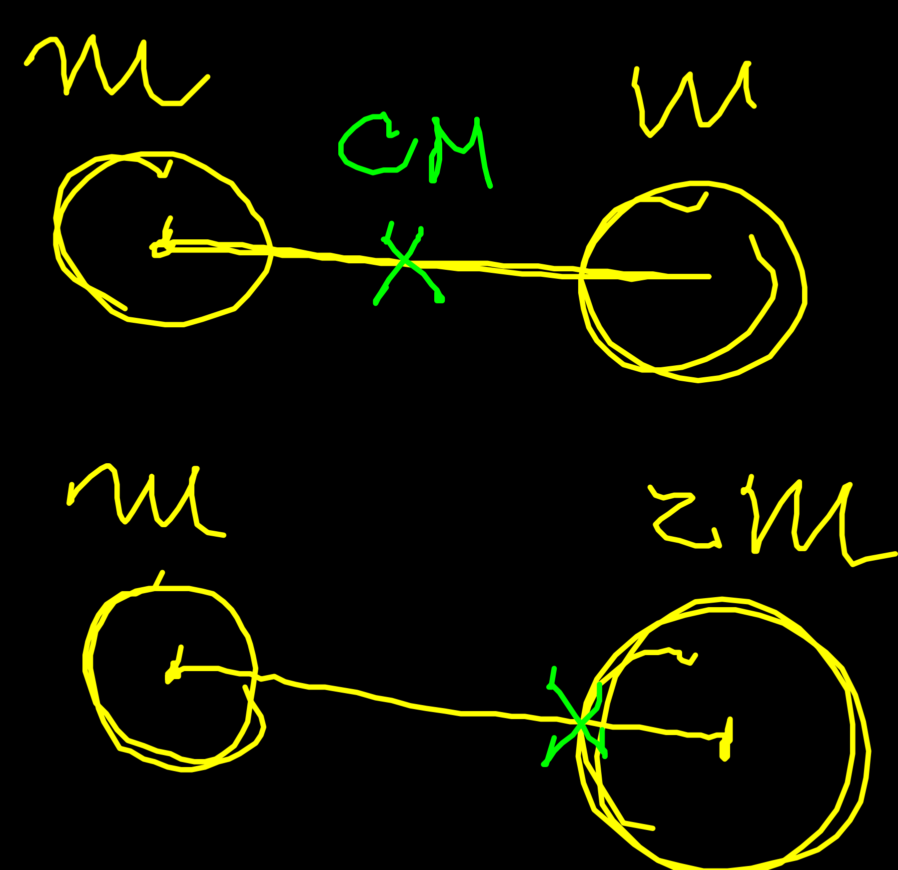
CENTRO DI MASSA



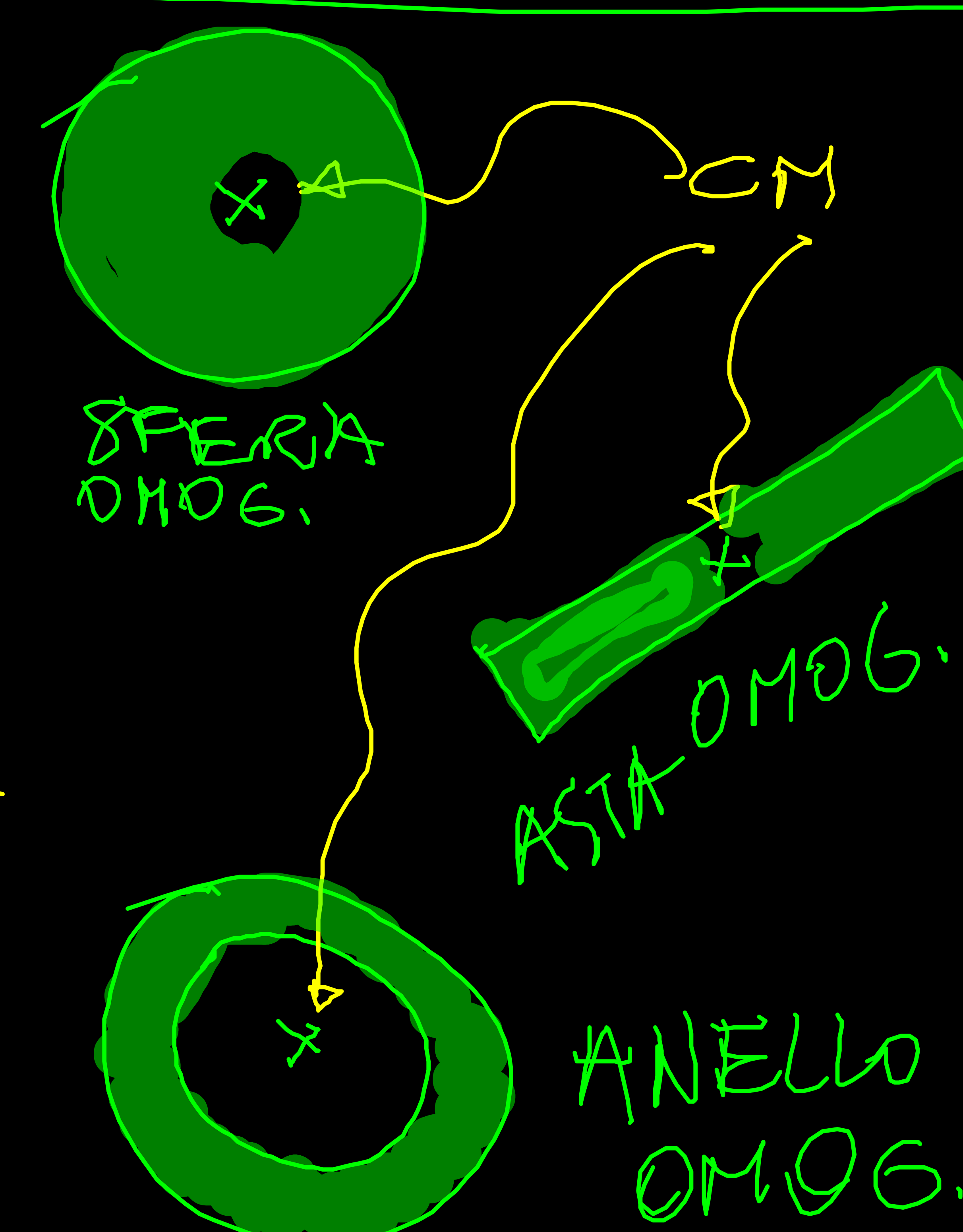
VETTORE POSIZIONE DEL "CENTRO DI MASSA"

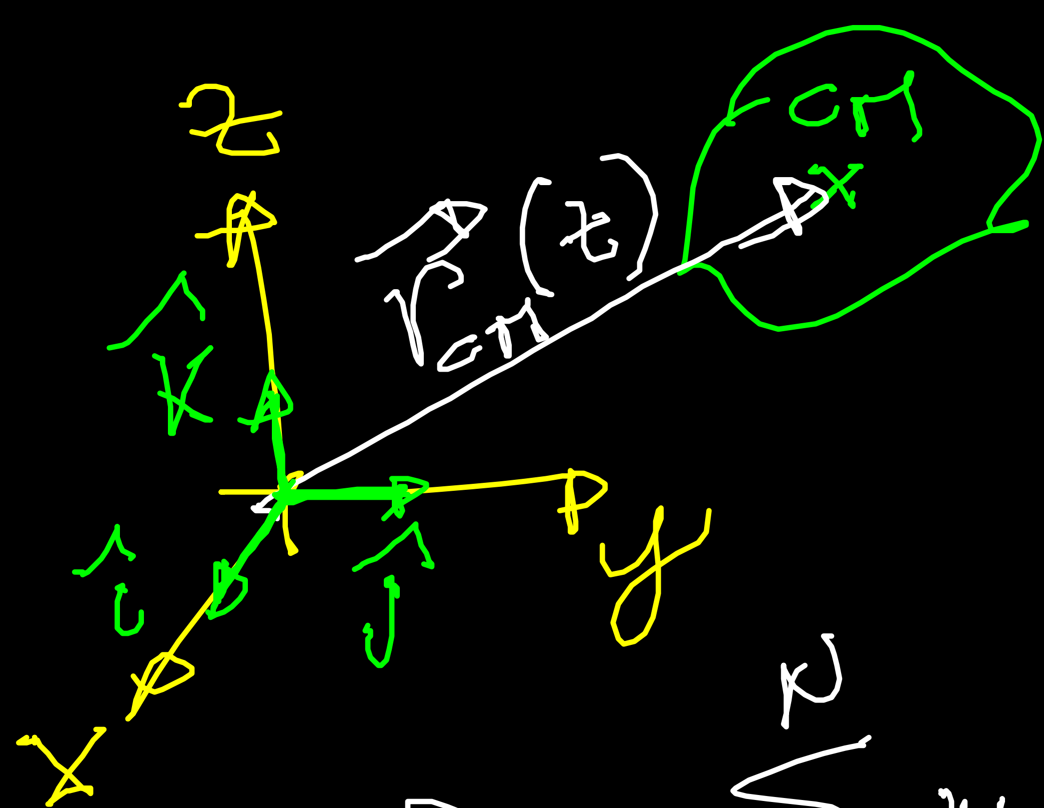
$$\vec{r}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i(t)}{\sum_{i=1}^{N} m_i}$$

CORPO ESTESO con $i=2$



ESEMPI DI CORPI OMOG. DI FORMA REGOLARE





CORPO CON ELEM. DISCRETI

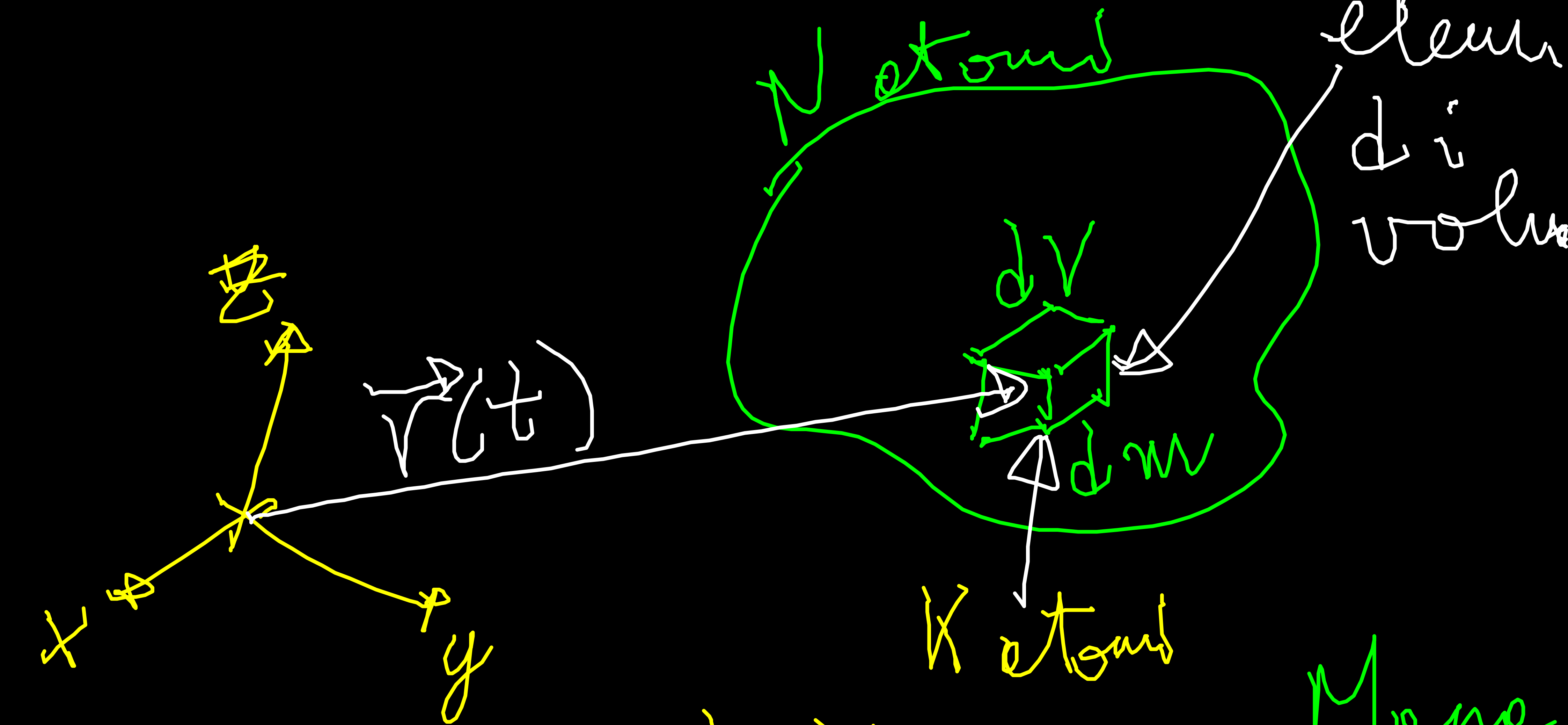
$$\vec r_{CM} = \frac{\sum_{i=1}^{N} m_i \vec r_i}{\underbrace{\sum_{i=1}^{N} m_i}_{\text{MASSA TOT. DEL CORPO}}} = \frac{\sum_{i=1}^{N} m_i \vec r_i}{M}$$

$$\vec r_{CM} \cdot \hat i = X_{CM} = \frac{\sum_{i=1}^{N} m_i \vec r_i \cdot \hat i}{M} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

$$\vec r_{CM} \cdot \hat j = y_{CM} = \frac{\sum m_i y_i}{M}$$

$$\vec r_{CM} \cdot \hat k = z_{CM} = \frac{\sum m_i z_i}{M}$$

CORPO "CONTINUO"

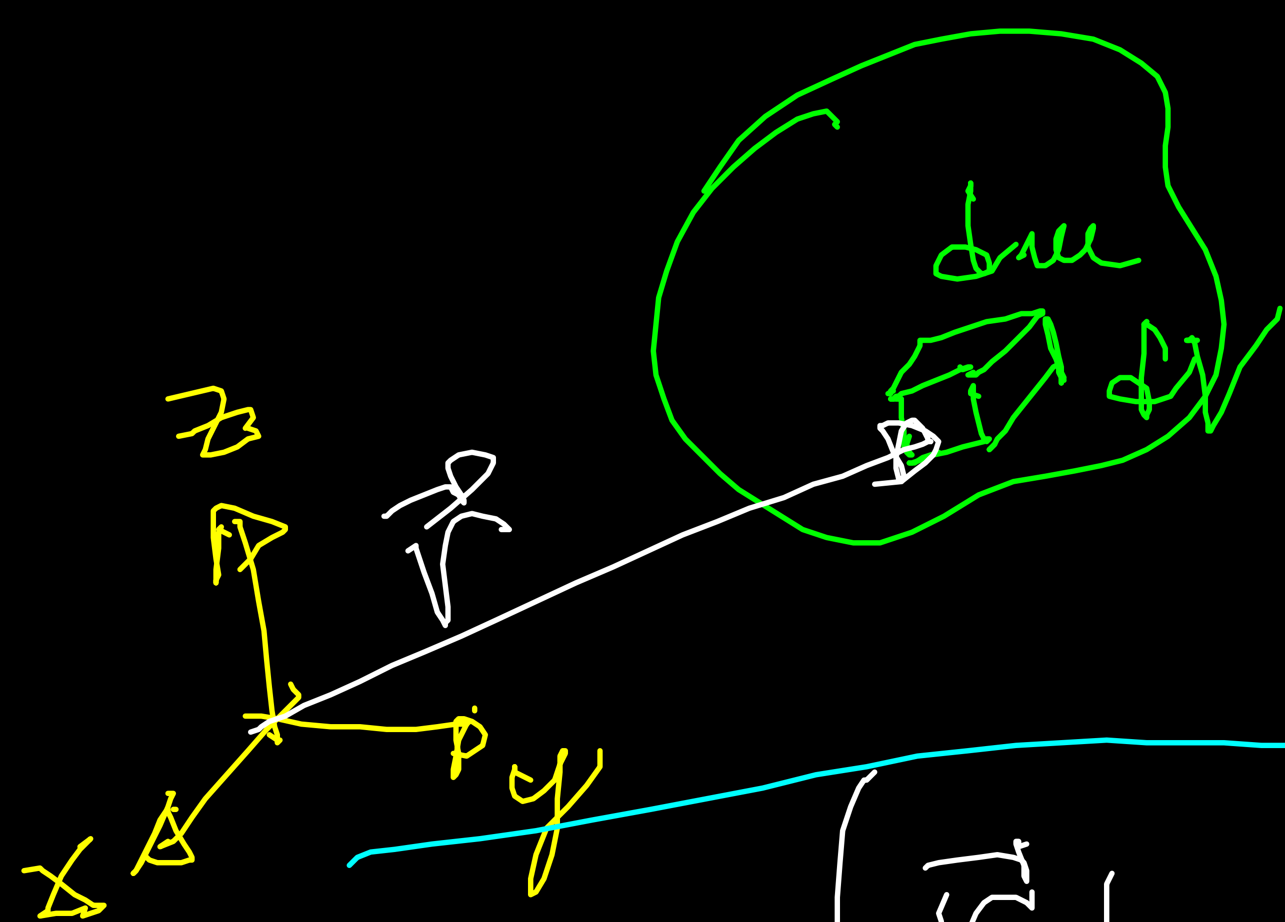


K è >> 1

ma K << N

Massa Tot M

$$\vec r_{CM} = \frac{\int_{CORPO} \vec r \, dm}{\underbrace{\int_{CORPO} dm}_{\text{massa totale}}} = \frac{\int_{CORPO} \vec r \, dm}{M}$$

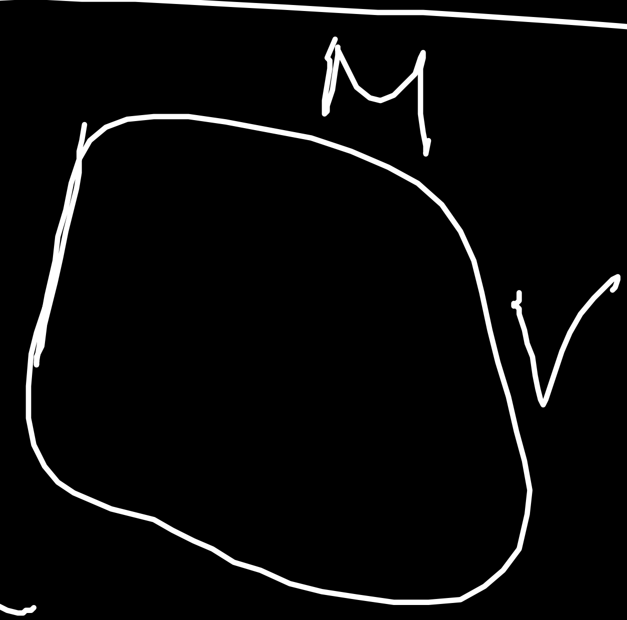


$$dm = \rho(\vec{r})\, dV$$

$$\boxed{\vec{r}_{CM} = \frac{\int_{CORPO} \vec{r}\, dm}{M}} = \frac{\int_{CORPO} \vec{r}\rho(\vec{r})\, dV}{\int_{CORPO} dm} = \frac{\int_{VOL.} \vec{r}\rho(\vec{r})\, dV}{\int_{VOL} \rho(\vec{r})\, dV}$$

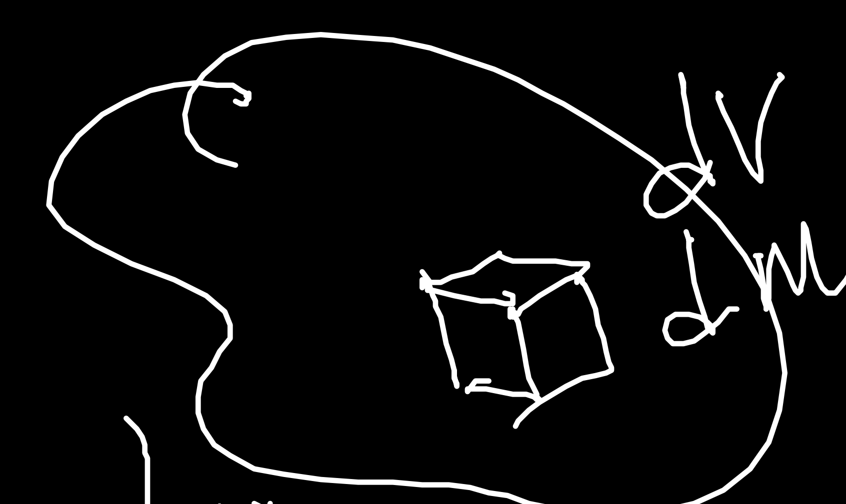
DENSITÀ (massa/vol)

CORPO OMOG.



$\rho = \frac{M}{V}$ = costante

CORPO NON OMOG.

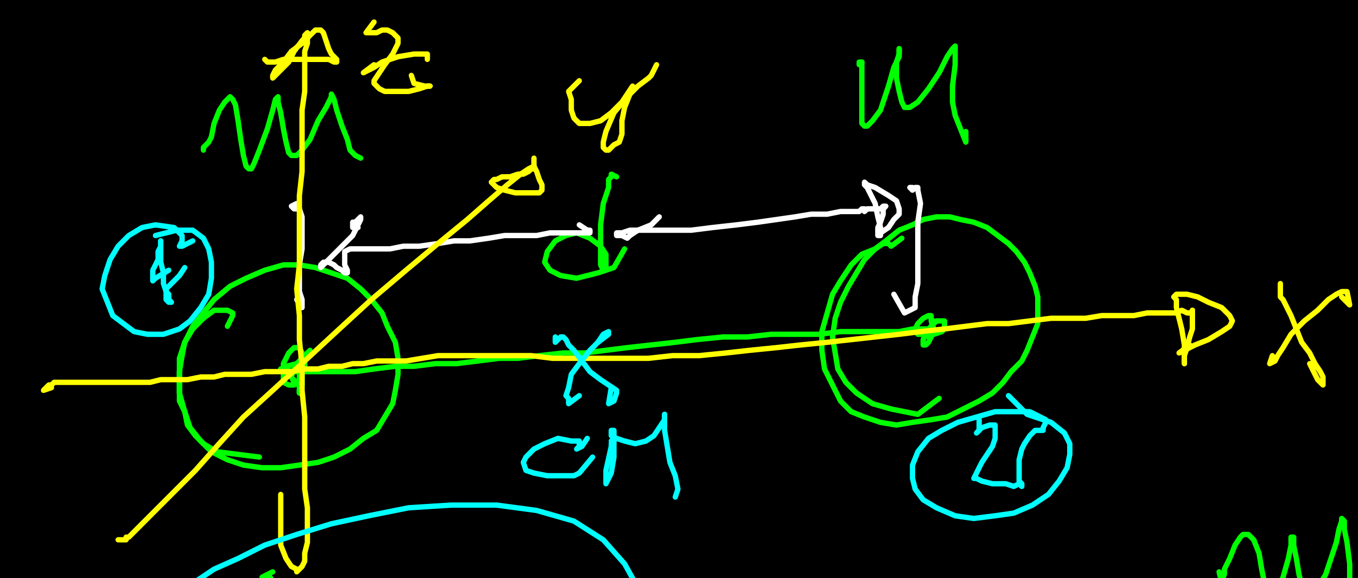


$\rho = \frac{dm}{dV}$ = funzione della posizione

ESEMPI

MANUBRIO RIGIDO

d è cost

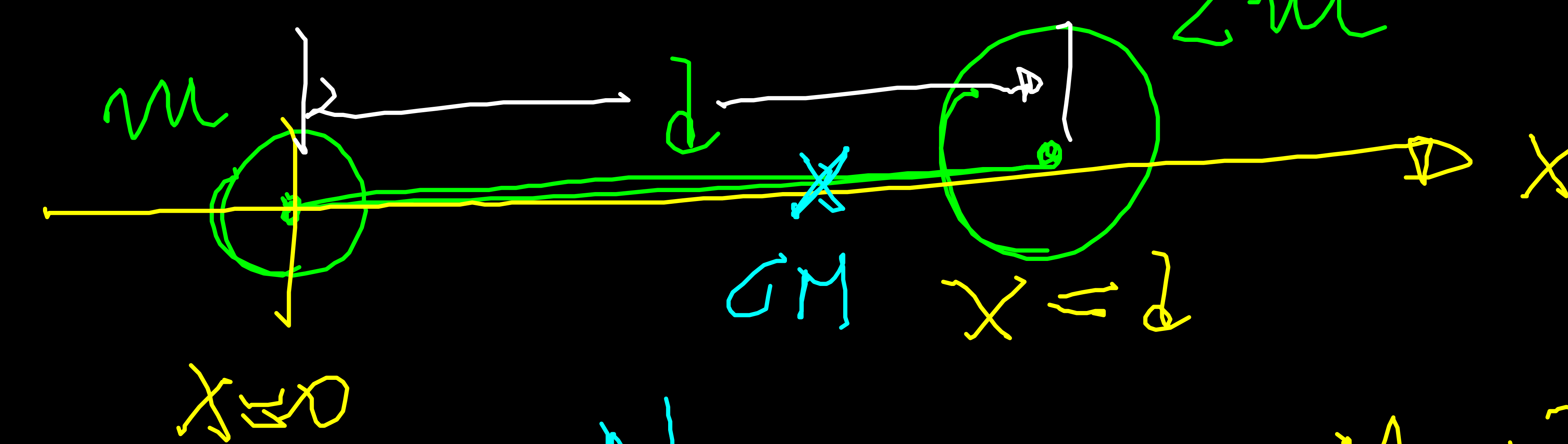


$N = 2$

$m_{asta} \simeq 0$

$z_{CM} = 0 \; ; \; y_{CM} = 0$

$$X_{CM} = \frac{\sum_{i=1}^{N} m_i X_i}{\sum_i m_i} = \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2} = \frac{md}{2m} = \frac{d}{2}$$



$$X_{CM} = \frac{\sum_{i=1}^{N} m_i X_i}{\sum_i m_i}$$

$$X_{CM} = \frac{2md}{m+2m} = \frac{2}{3} d$$

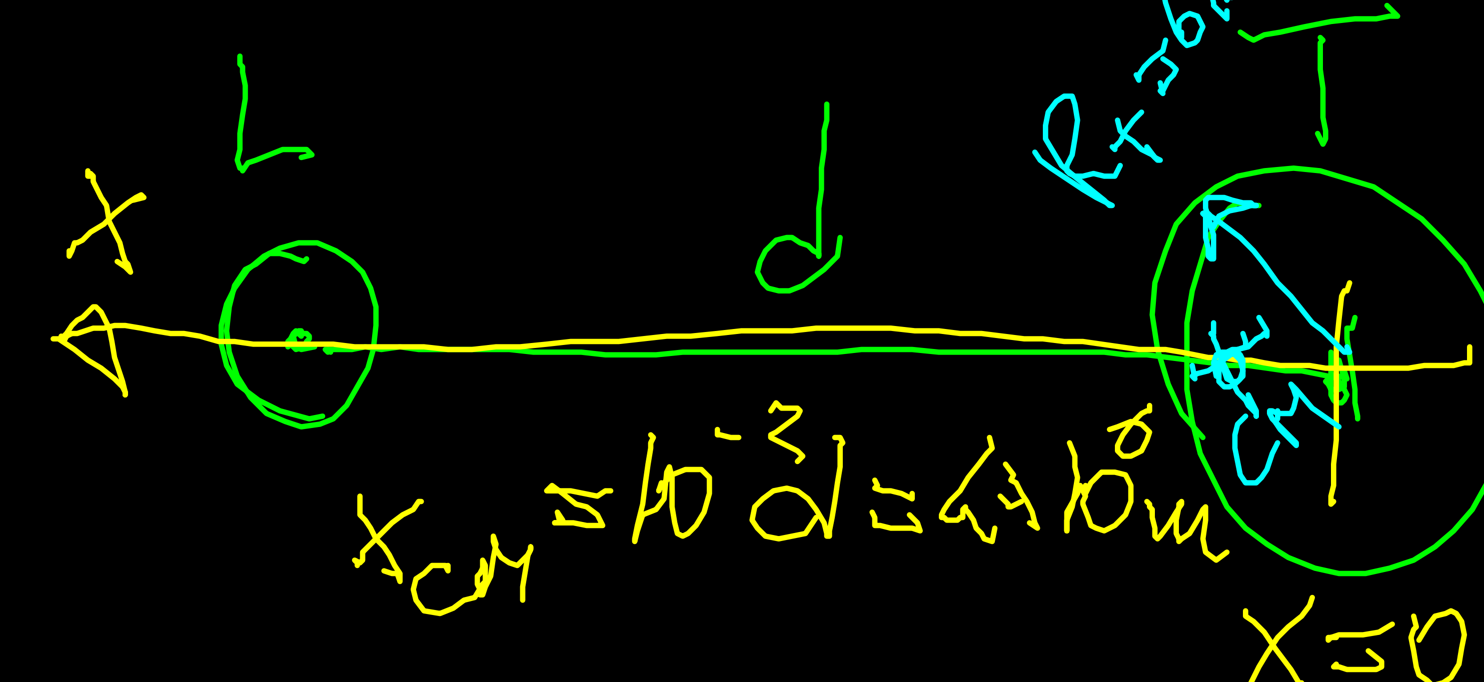
$M_L = 7,35 \times 10^{22}$ kg

$M_T = 7 \times 10^{24}$ kg

$$\frac{7 \times 10^{22}}{7,07 \times 10^{24}} \simeq 10^{-2}$$

R_T

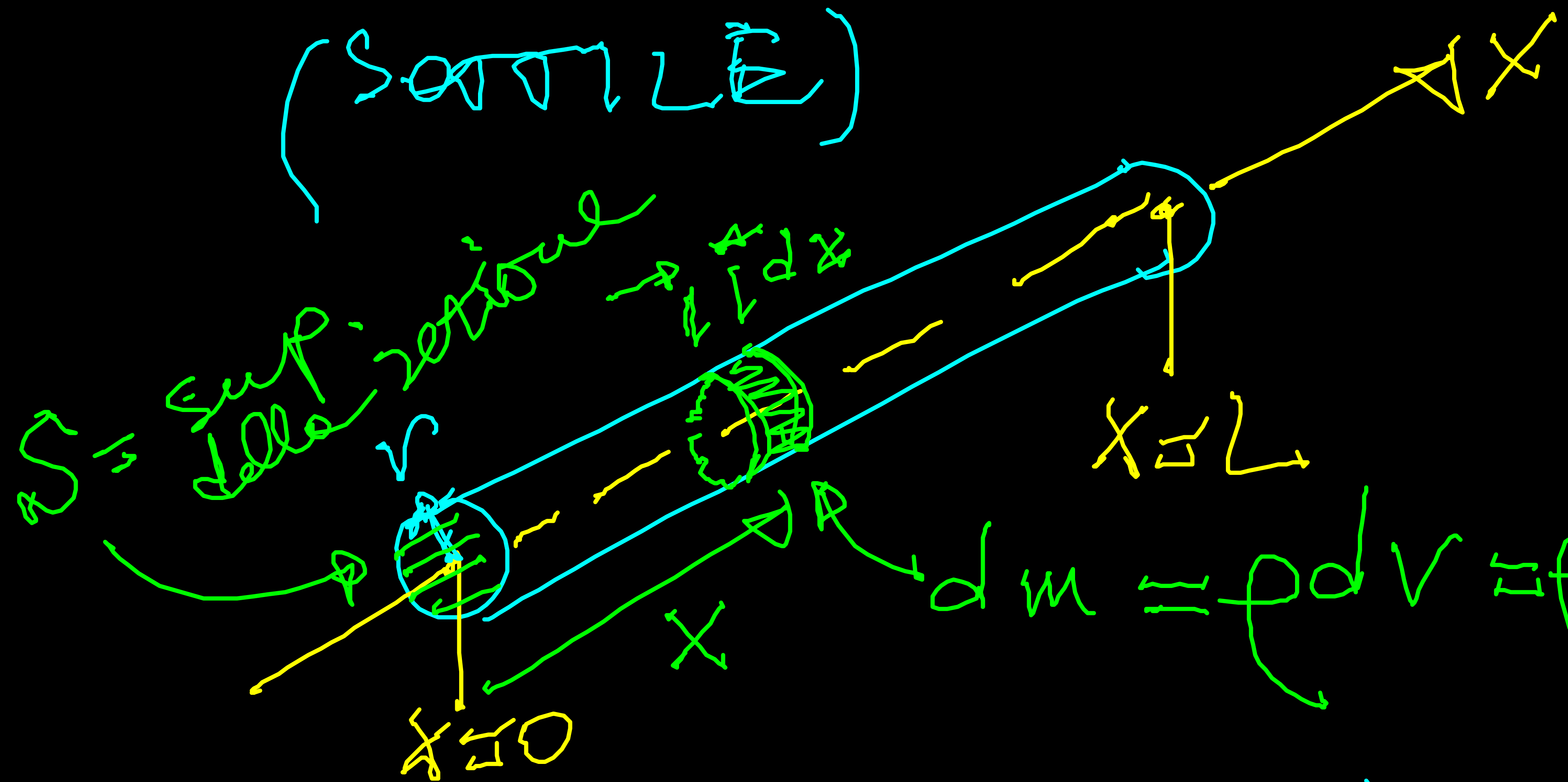
$d = 4 \times 10^{8}$ m



$$X_{CM} = \frac{M_L d}{M_T + M_L} = 10^{-2}$$

$X_{CM} = 10^{-2} d = 4 \times 10^{6}$ m

BARRA CILINDRICA (SOTTILE)



$dm = \rho\, dV = \rho S\, dx$

$$x_{CM} = \frac{\int_{BARRA} x\, dm}{\int_{BARRA} dm} = \frac{\int_{BARRA} x \rho S\, dx}{\int_{BARRA} \rho S\, dx}$$

$$\boxed{x_{CM} = \frac{\int_0^L x \rho S\, dx}{\int_0^L \rho S\, dx}} \quad \text{BARRA}$$

OMOGENEA ($\rho = \text{cost}$)

massa M

lunghezza L

$$\rho = \frac{M}{V} = \frac{M}{SL}$$

SOTTILE $r \ll L$

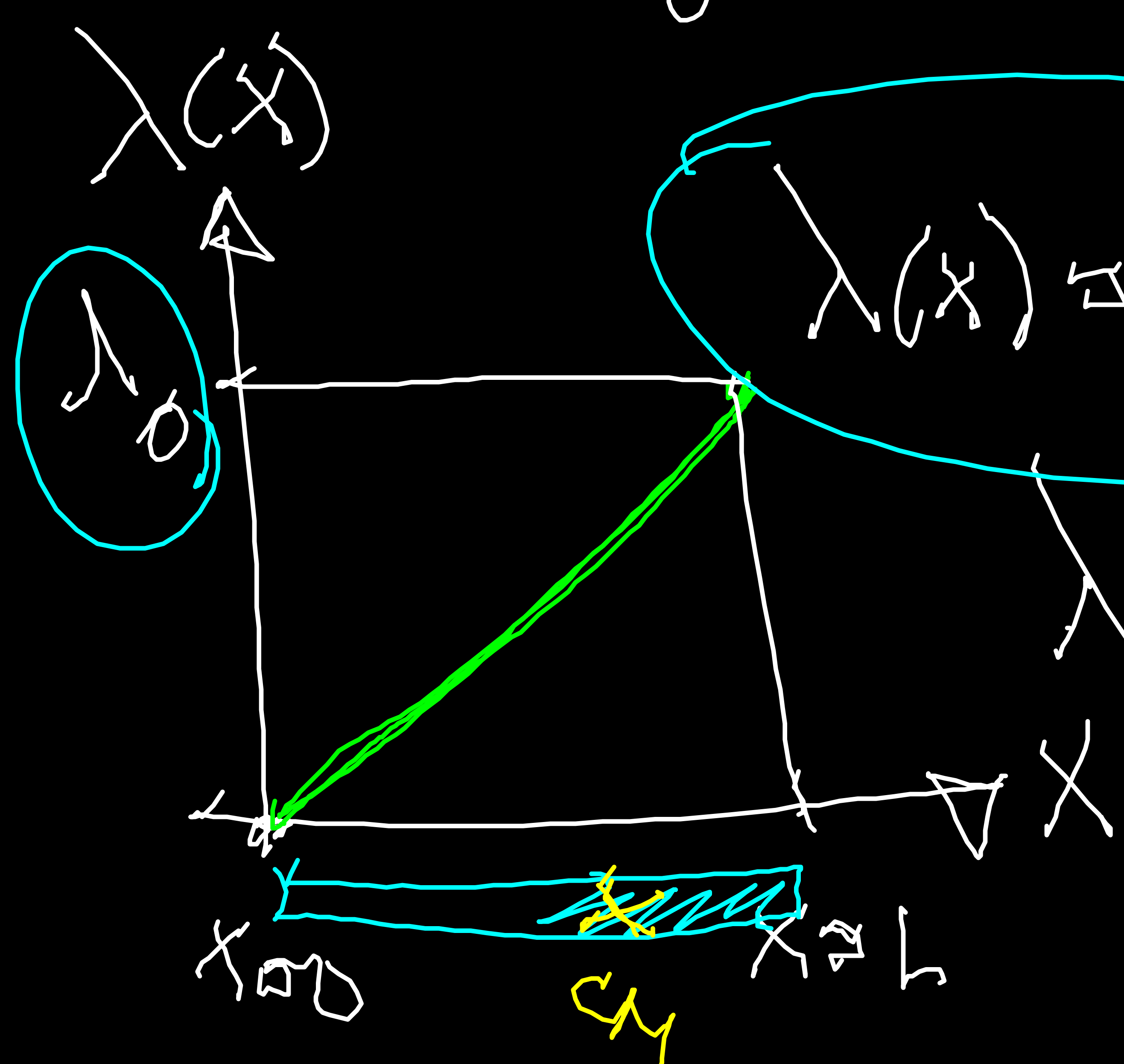
SE OMOGENEA

$$x_{CM} = \frac{\cancel{\rho S} \int_0^L x\, dx}{\cancel{\rho S} \int_0^L dx}$$

$$x_{CM} = \frac{\int_0^L x\, dx}{L} = \frac{\left[\frac{x^2}{2}\right]_0^L}{L} = \frac{L}{2}$$

BARRA NON OMOGENEA

$$x_{CM} = \frac{\int_0^L \rho \cancel{S} x\,dx}{\int_0^L \rho \cancel{S}\,dx} = \frac{\int_0^L S\rho(x)\,x\,dx}{\int_0^L S\rho(x)\,dx} = \frac{\int_0^L \lambda(x)\,x\,dx}{\int_0^L \lambda(x)\,dx} = \frac{\lambda_0 L^2}{3M} = \frac{2}{3}L$$



$$\lambda(x) = \frac{x}{L}\lambda_0$$

$$\rho(x)\cdot S = \lambda(x)$$

$$\lambda(x)\ \left[\frac{kg}{m}\right]$$

DENSITÀ LINEARE

$$\int_0^L \lambda(x)\,dx = M$$

$$\int_0^L \frac{x}{L}\lambda_0\,dx = \frac{\lambda_0}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{\lambda_0 L}{2} = M$$ DEN.

$$\lambda_0 = \frac{2M}{L}$$

$$\int_0^L \frac{x}{L}\lambda_0\,x\,dx = \frac{\lambda_0}{L}\int_0^L x^2\,dx = \frac{\lambda_0 L^2}{3}$$ NUM.