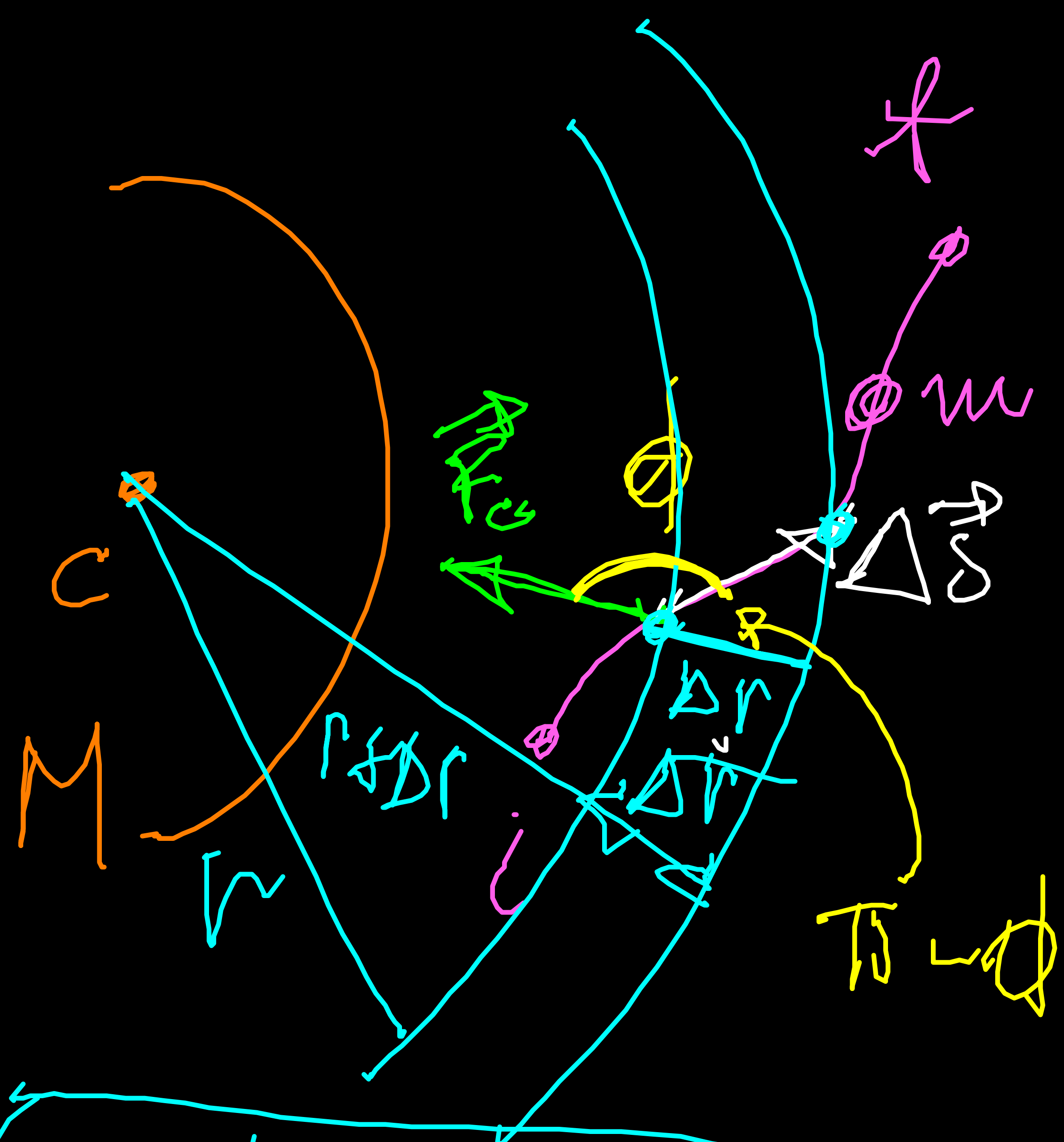
FORZA GRAVITAZIONALE $\longrightarrow$ EN. POTENZIALE

$\mathcal{L}_{i\to f} = \int_i^f \vec{F}_G \cdot d\vec{s}$ è indip. dal percorso? SI!




$|\Delta\vec{s}|\cos(\pi-\phi) = \Delta r = -|\Delta\vec{s}|\cos\phi$

$\Delta\vec{s} \longrightarrow d\vec{s}$

$\Delta r \longrightarrow dr$

$dr = -|d\vec{s}|\cos\phi$

$\vec{F}_G \cdot d\vec{s} = |\vec{F}_G||d\vec{s}|\cos(\pi-\phi) = -|\vec{F}_G|\,dr$

$|\vec{F}_G| = \dfrac{GMm}{r^2}$

$|\vec{F}_G| \to 0$ per $r \to \infty$

$$\mathcal{L}_{i\to f} = \int_i^f \vec{F}_G \cdot d\vec{s} = -\int_{r_i}^{r_f} \frac{GMm}{r^2}\,dr$$

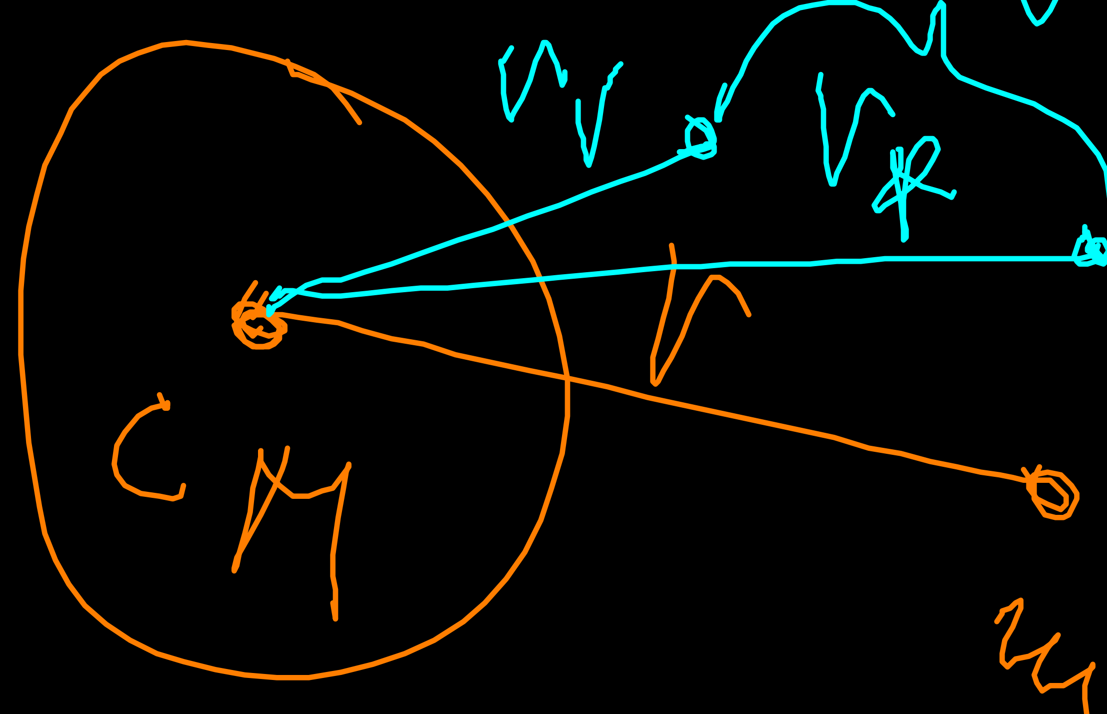
$$(U_{G,f} - U_{G,i}) = -W_{i\to f} = \int_{r_i}^{r_f} \frac{GMm}{r^2}\, dr = GMm \int_{r_i}^{r_f} \frac{dr}{r^2}$$

$$\boxed{\frac{d}{dr}\left(-\frac{1}{r}\right) = \frac{1}{r^2}} \qquad = GMm\left[-\frac{1}{r}\right]_{r_i}^{r_f} = -GMm\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

$$\underbrace{U_{G,f} - U_{G,i}}_{\Delta U} = -\left(\frac{1}{r_f} - \frac{1}{r_i}\right) GMm \to U(r) = -\frac{GMm}{r}$$

$r_i \to \infty$

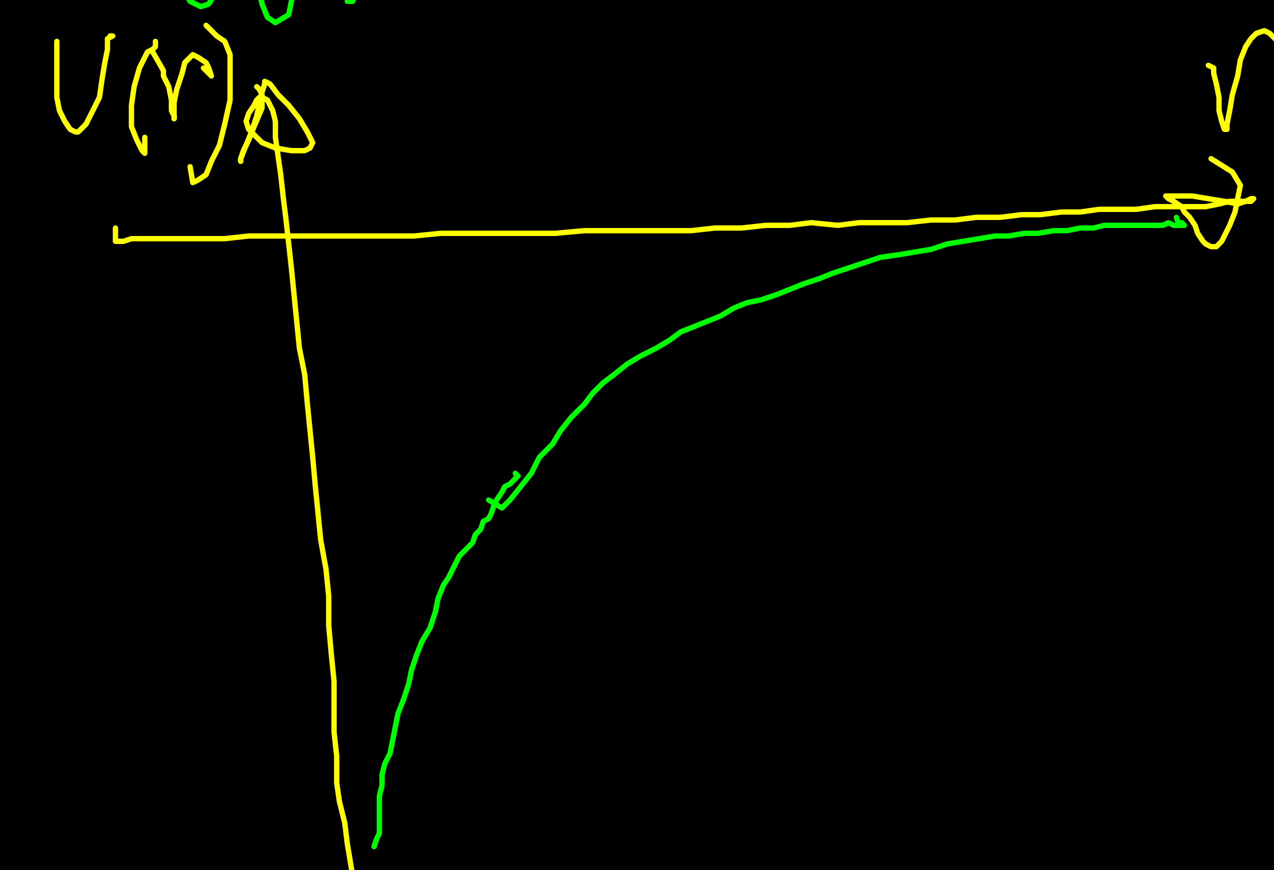
$U(r_i) \to 0$

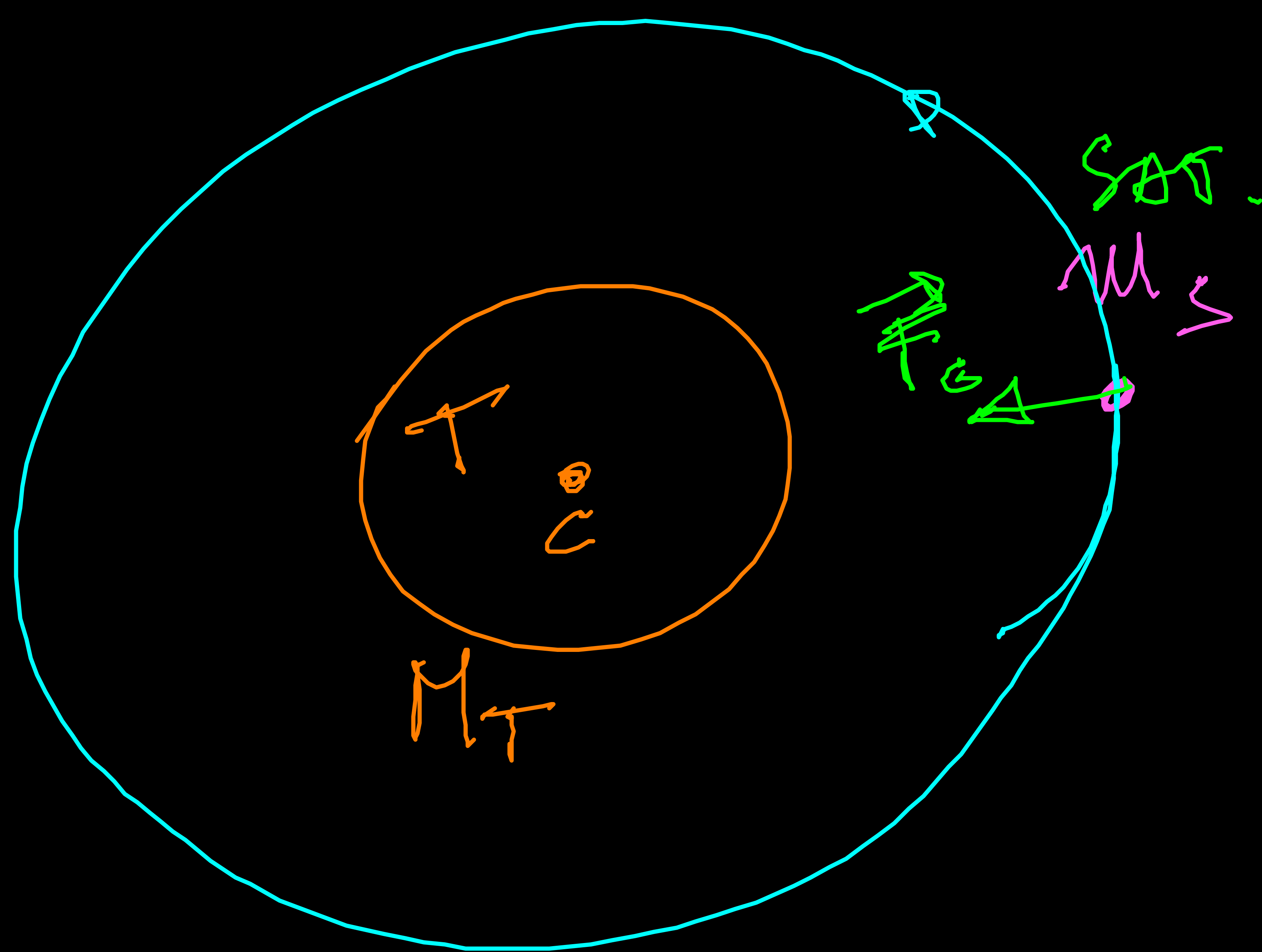



$U(\infty) = 0$

$r \to \infty$

$U(r) \to 0$



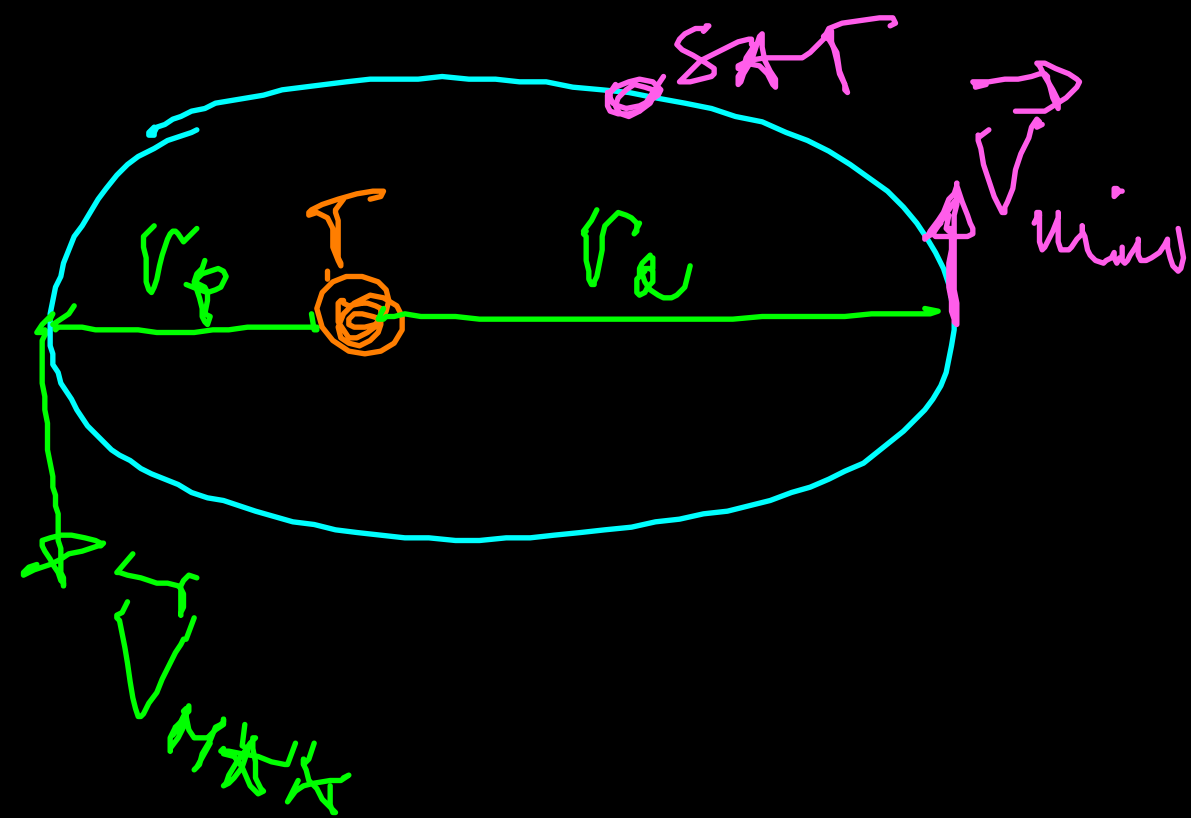


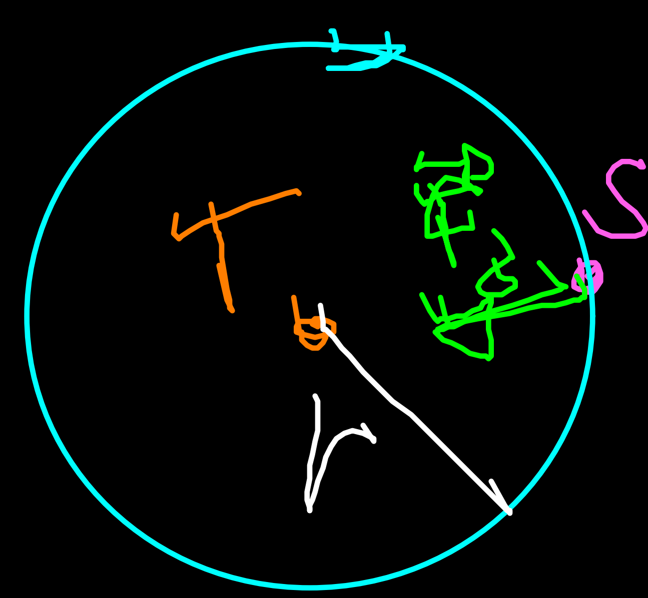

$\vec{F}_G$ è conservativa $\Rightarrow$ E è costante

Per il SAT.

$$E = \frac{1}{2} m v^2 - \underbrace{\frac{G M_T m_s}{r}}_{\to U(r)}$$

In pratica i SAT. si muovono su orbite circolari con $r = \text{cost}$




$$\vec{F}_G = m_s \vec{a}_c$$

$$m\, a_c = m \frac{v^2}{r} = F_G = \frac{G M_T m}{r^2}$$

$$K_S = \frac{1}{2} m v^2 = \frac{1}{2} \frac{G M_T m}{r} = \frac{1}{2} |U(r)|$$

$$E_S = \underbrace{\frac{1}{2} \frac{G M_T m}{r}}_{K_S} - \frac{G M_T m}{r} = -\frac{G M_T m}{2r} < 0$$

CORI

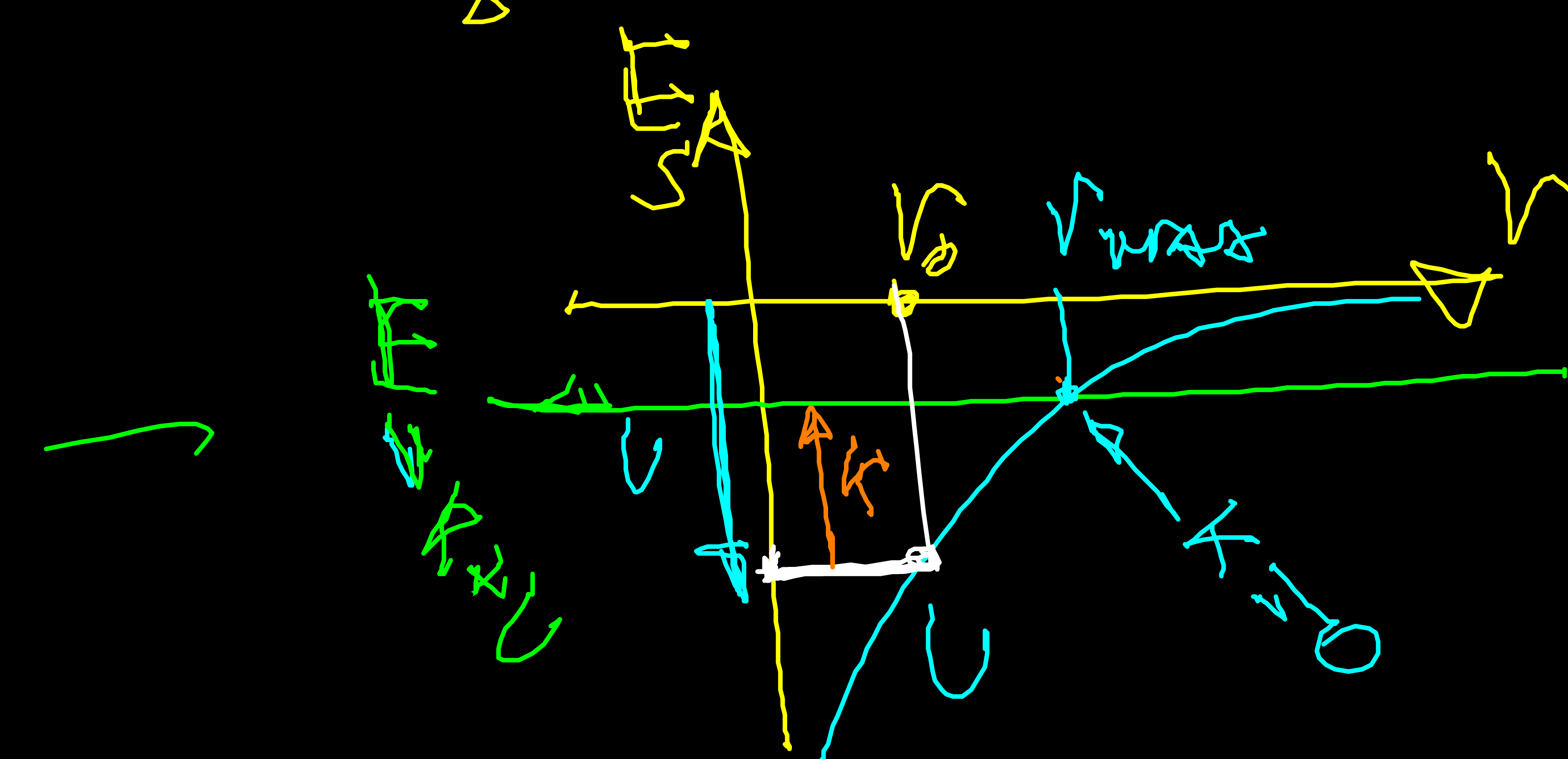
SAT. "LIBERO" HA $U(r) \to 0$ per $r \to \infty$

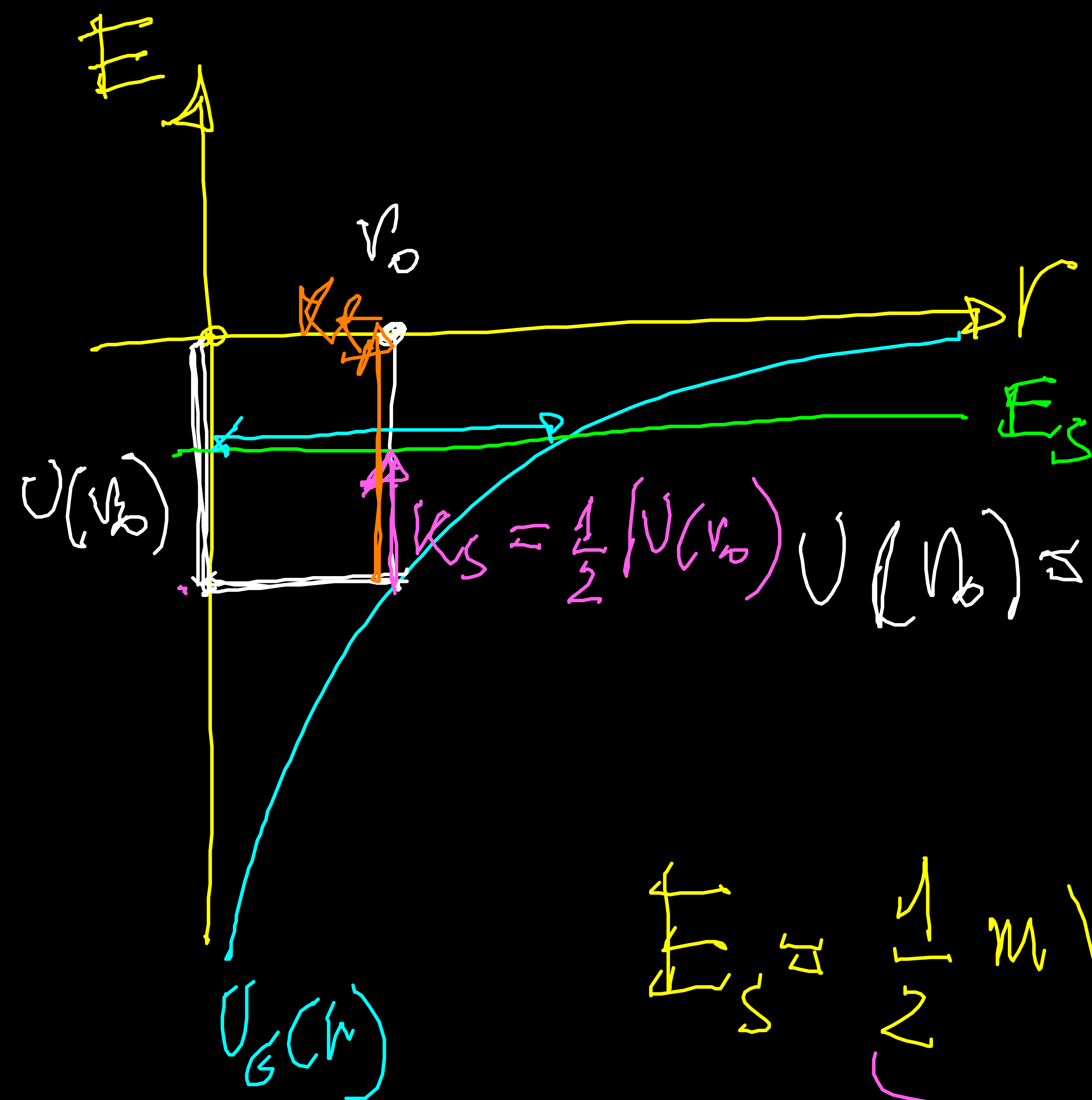
$E_{mec} \simeq \frac{1}{2} m v^2 \geq 0$ $\quad \underbrace{K \geq 0}$ e $U \simeq 0$

SE $E_s < 0$, per avere $K$ almeno $= 0$

deve essere "fornito" di energia

$\Rightarrow$ Se $E_s < 0 \Rightarrow$ IL SAT. È <u>LEGATO</u>






in ORBITA

$K_S = \frac{1}{2}|U(r_0)|$ $\quad U(r_0) = -\frac{G M_T m_S}{r_0}$

# VELOCITÀ DI FUGA

↳ Per sfuggire alla gravità devo portare

$$E_S \xrightarrow[\text{almeno}]{} 0$$

$$E_S = \underbrace{\frac{1}{2} m v^2}_{K_S} + U(r_0) = -\frac{G M_T m_S}{2 r_0}$$

$$E = \boxed{0 = -\frac{G M_T m_S}{r_0} + \frac{1}{2} m_S v_f^2} \Rightarrow v_f = \sqrt{\frac{2 G M_T}{r_0}}$$

$$a_c = \frac{v^2}{r}$$

$$v \neq v_f$$

$$V_f = \sqrt{\frac{2GM_T}{r_0}}$$

$r_0 = r_T = 6.4 \times 10^6\ m$

$M_T = 6 \times 10^{24}\ Kg$

$G = 6.67 \times 10^{-11}\ \frac{N}{Kg^2 m^2}$

$$V_f = \sqrt{\frac{2 \cdot (\cancel{6.67} \times 10^{-11})(6 \times 10^{24})}{\cancel{6.4} \times 10^6}} \simeq \sqrt{12 \cdot 10^7}\ \frac{m}{s}$$

$$= \sqrt{120 \times 10^6}\ \frac{m}{s} = 11 \times 10^3\ \frac{m}{s} = 11\ \frac{Km}{s}$$

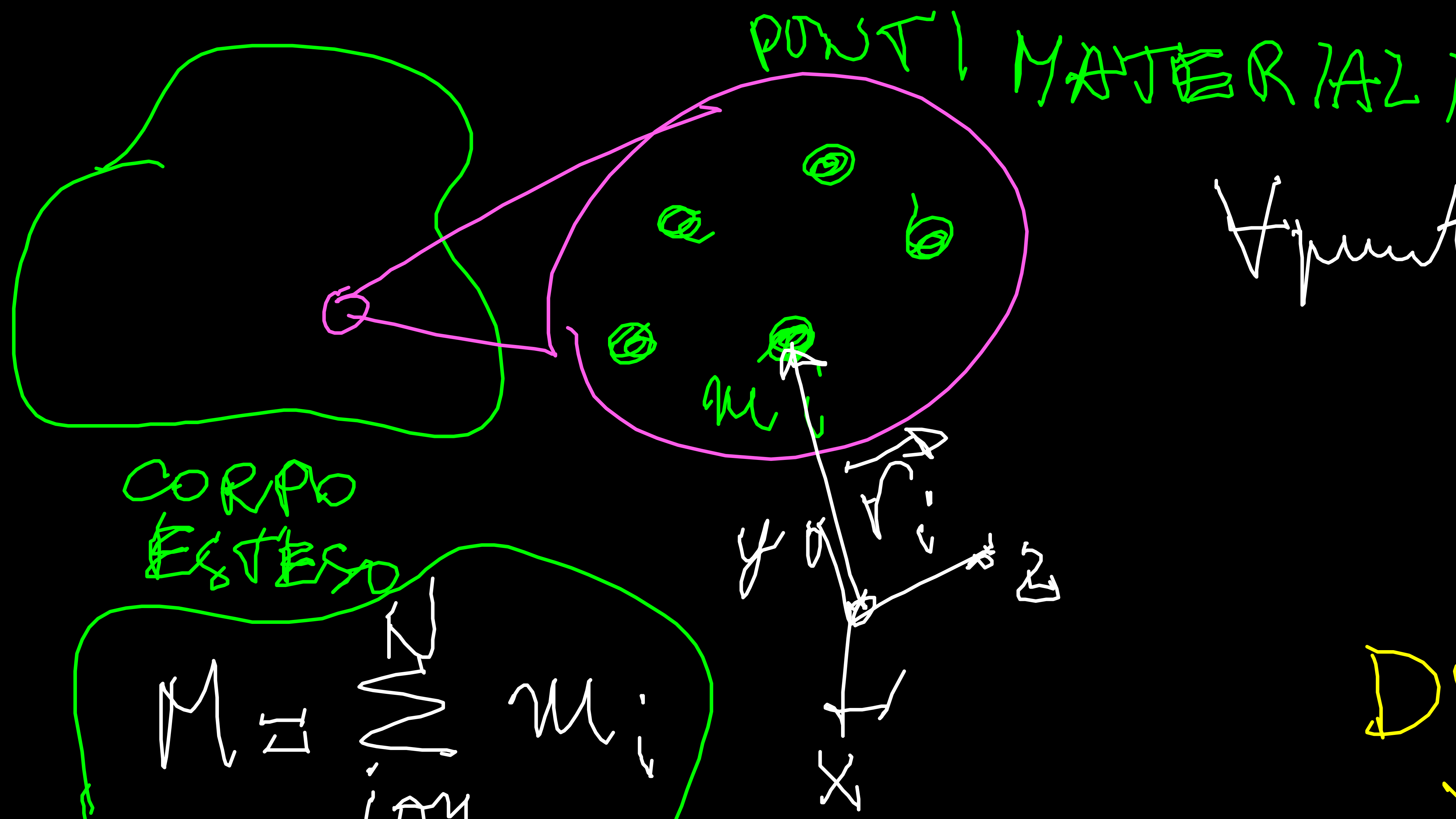
$M_\odot$

$C \simeq 3 \times 10^8\ \frac{m}{s}$

$$\frac{M_{BH}}{r_{BH}} = \frac{C^2}{2G} = \frac{2}{3} \times 10^{27}\ \frac{Kg}{m}$$

$$\frac{M_T}{r_T} = \frac{6 \times 10^{24}\ Kg}{6.4 \times 10^6\ m} \simeq 10^{18}\ \frac{Kg}{m}$$

# MOTO DEI SISTEMI (... DI PUNTI MATERIALI)




$$M = \sum_{i=1}^{N} m_i$$

$N$ = num. tot. di punti mat.

$\simeq N_A \simeq 6{,}023 \times 10^{23}$

$\forall$ punto "i" $\left[ \begin{array}{l} \vec{r}_i(t) \\ \vec{v}_{0i} \end{array} \right. \Rightarrow 6 \times N_A$ !! variabili !

IMPOSSIBILE

DEVO

"TROVARE DELLE "REGOLARITÀ" E PROPRIETÀ "MEDIE" PER RIDURRE LE VARIABILI

# CENTRO DI MASSA