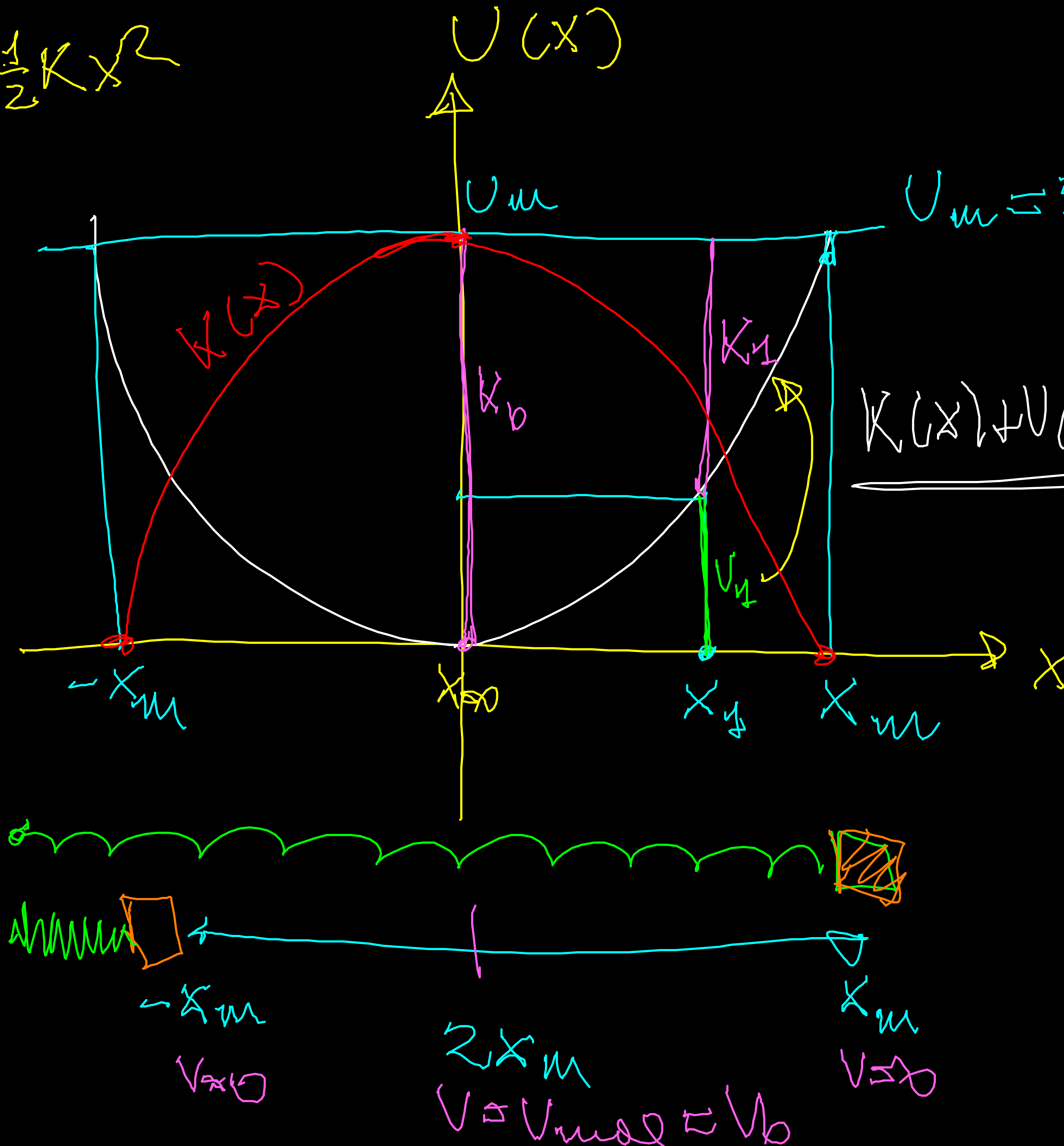
$U = \frac{1}{2} K X^2$



$U_m = E_m$

$E = U + K$

$E_m = \frac{1}{2} K X_m^2 \qquad X_m$

$E_m = \frac{1}{2} K X_1^2 + \frac{1}{2} m V_1^2 \qquad X_1$

$K(x) + U(x) = E_m$

$E_m = \frac{1}{2} m V_0^2 \qquad X = 0$

$\frac{1}{2} m V_0^2 = \frac{1}{2} K X_m^2$

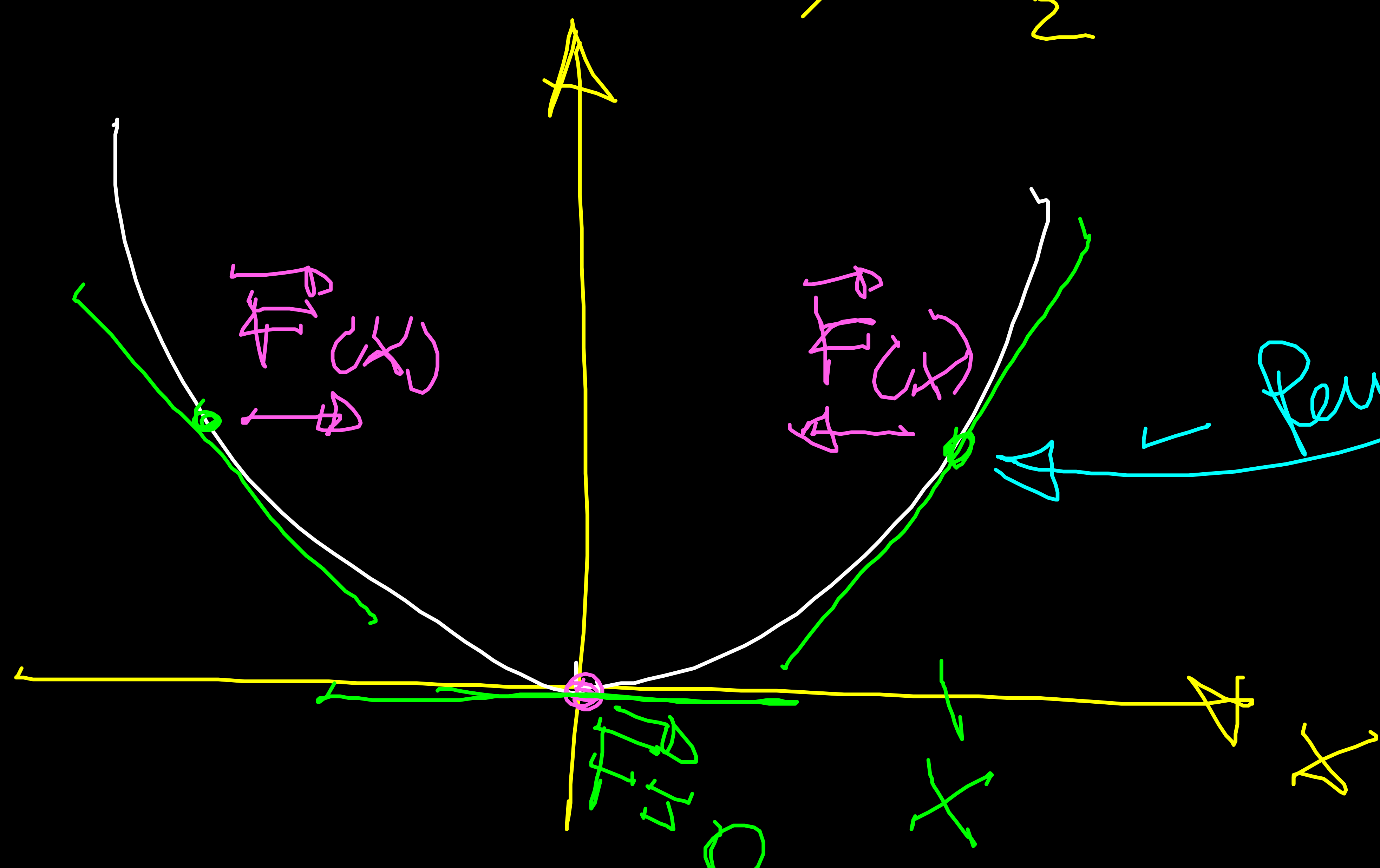
$V_0 = \sqrt{\frac{K}{m}} \cdot X$

$$U(x) - \underbrace{U(0)}_{=0} = -\int_0^x F(x)\,dx \Rightarrow U(x) = -\int_0^x F(x)\,dx$$

$$\Downarrow$$

$$dU(x) = -F(x)\,dx$$

$$U(x) = \frac{Kx^2}{2}$$

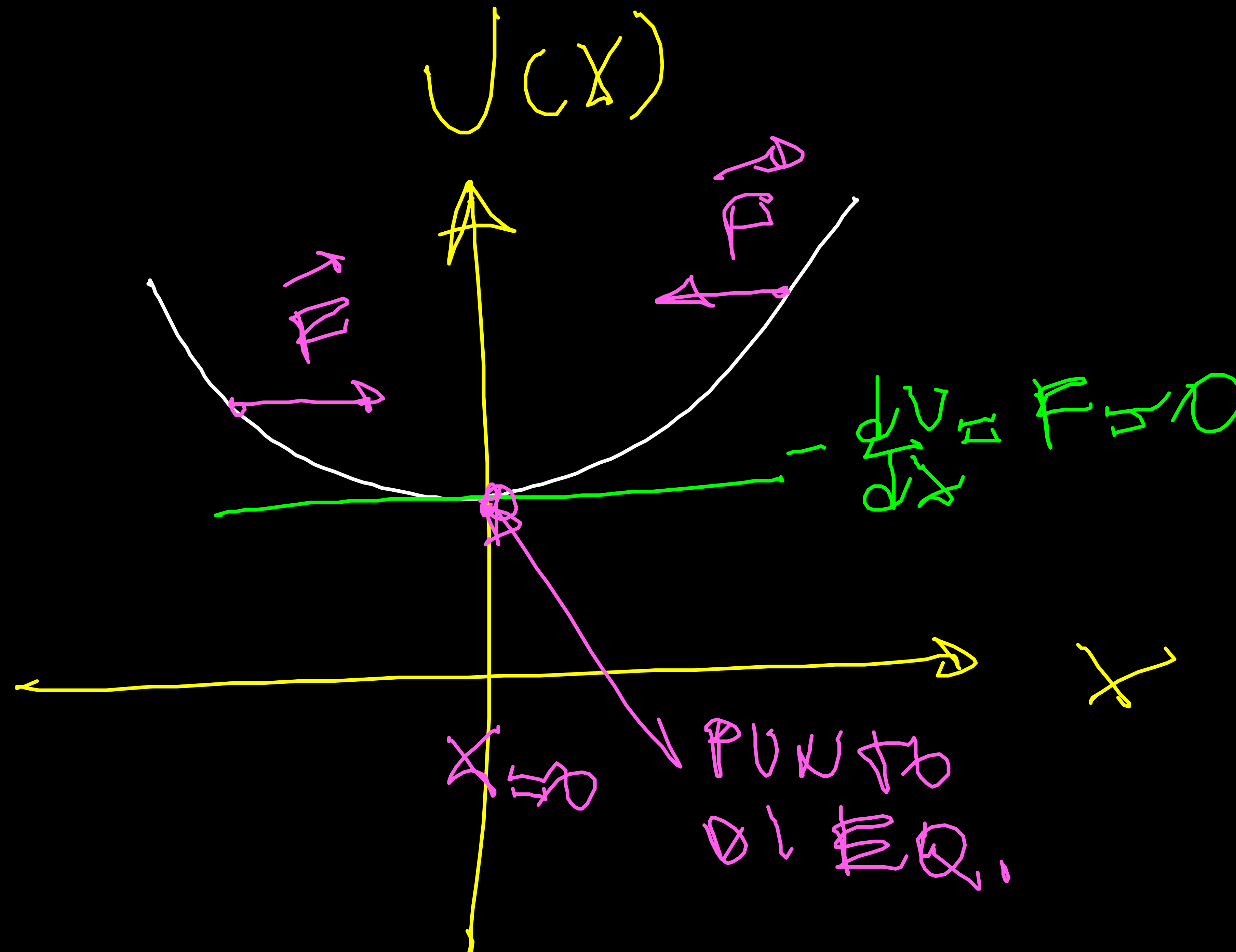


Pendenza

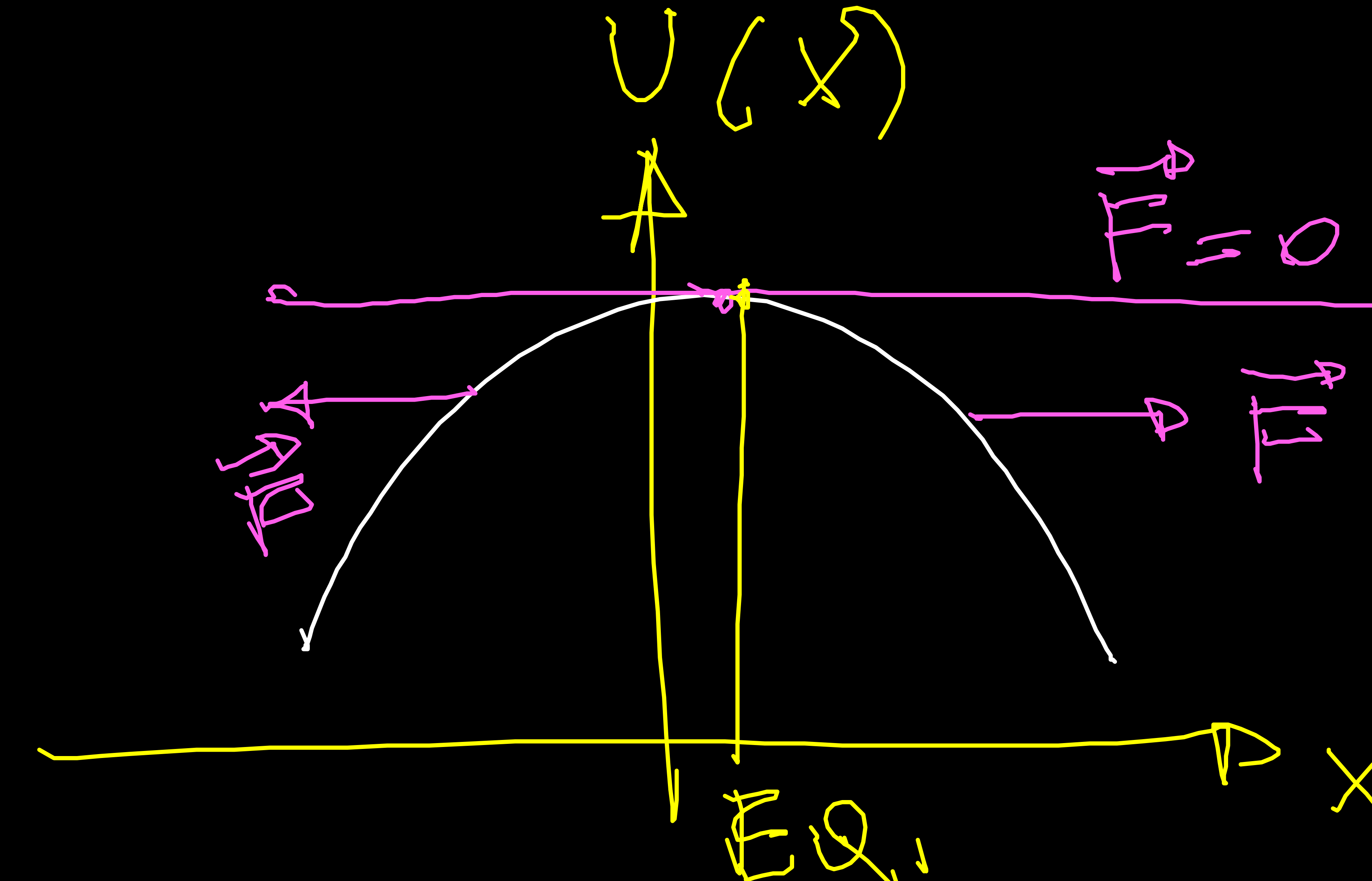
$$\boxed{F(x) = -\frac{dU}{dx}(x)}$$

$$U_{el} = \frac{Kx^2}{2} \Rightarrow -\frac{dU}{dx} = -\frac{2Kx}{2} = -Kx$$

$$U_p = mgy \Rightarrow -\frac{dU}{dy} = -mg$$



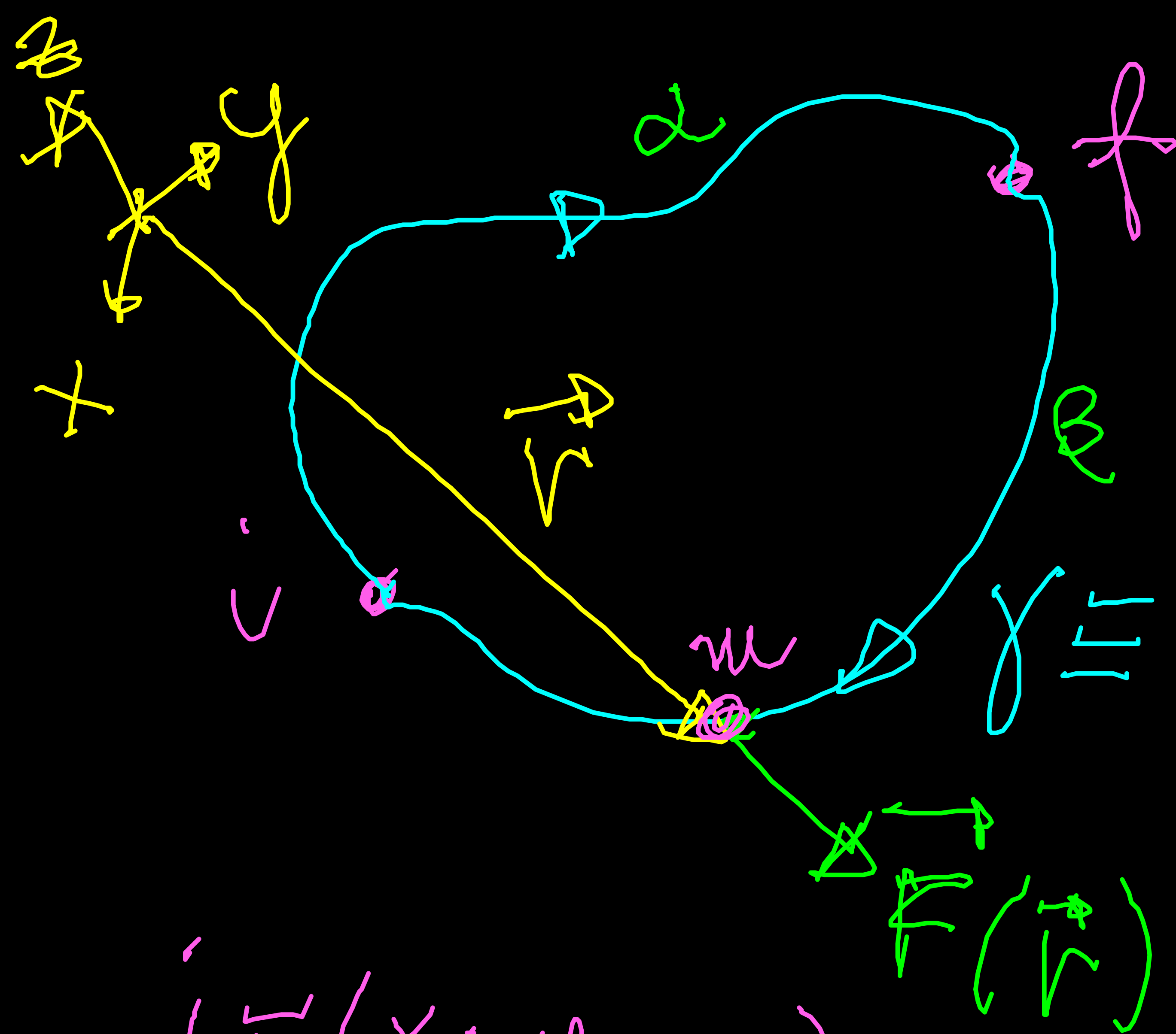
EQ. STABILE



EQ. INSTABILE

EQ. INDIFF.

FORZE CONSERVATIVE IN 3D



$\vec{F}(\vec{r})$ È CONSERVATIVA SE

$$\oint_{\gamma\ \text{CHIUSO}} \vec{F}\cdot d\vec{r} = 0 \quad \forall \gamma$$

$\gamma \equiv$ PERCORSO CHIUSO

$i \equiv (x_i, y_i, z_i)$

$f \equiv (x_f, y_f, z_f)$

$$\underbrace{i \to f}_{\alpha} \underbrace{\to i}_{\beta} \qquad \alpha + \beta \equiv \gamma$$

$$\Rightarrow \int_{i,\alpha}^{f} \vec{F}\cdot d\vec{r} + \int_{f,\beta}^{i} \vec{F}\cdot d\vec{r} = 0$$

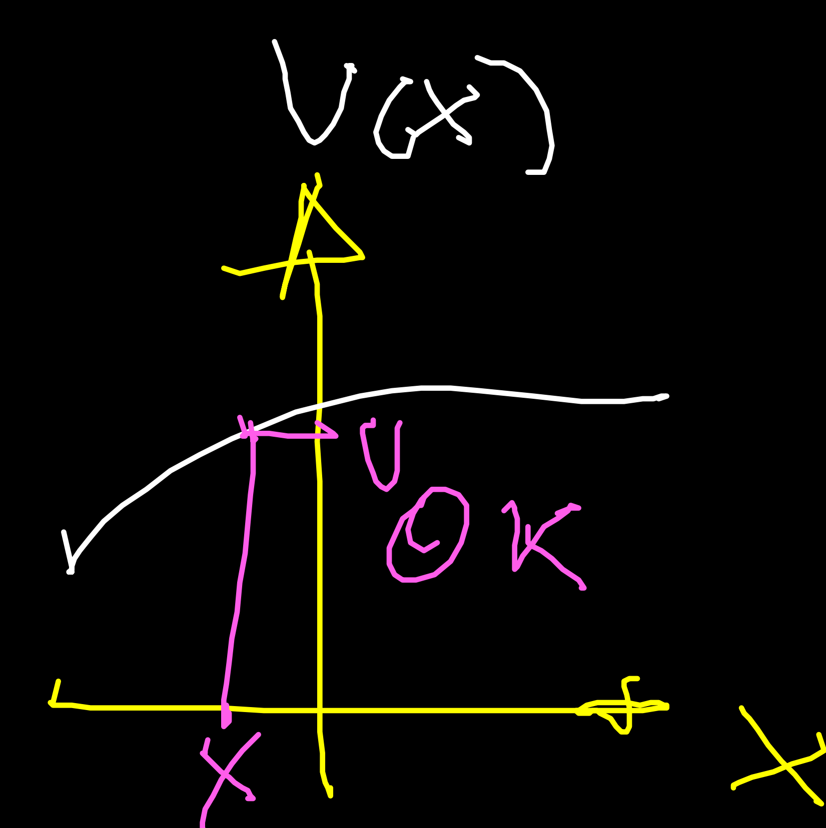
$$\int_{i,\alpha}^{f} \vec{F}\cdot d\vec{r} - \int_{i,\beta}^{f} \vec{F}\cdot d\vec{r} = 0 \Rightarrow \boxed{\int_{i,\alpha}^{f} \vec{F}\cdot d\vec{r} = \int_{i,\beta}^{f} \vec{F}\cdot d\vec{r}}$$

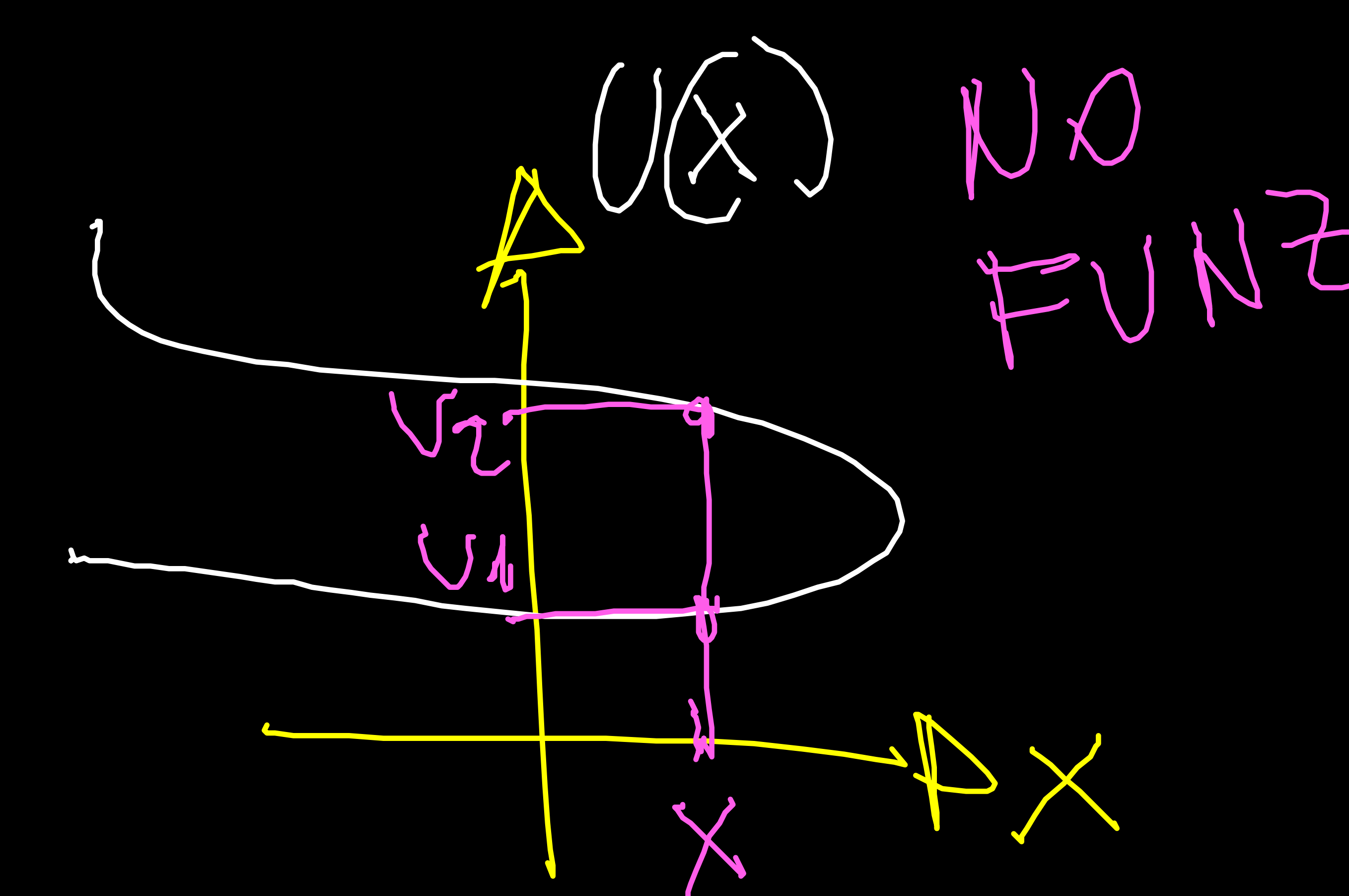
SE $\vec{F}$ È CONS.

DEF. GEN. DI ENERGIA POT. U
GENERATA DA UNA FORZA CONS. $\vec{F}$

$$-\int_i^f \vec{F} \cdot d\vec{r} = U_f - U_i$$

DIPENDE SOLO
DA i ED f



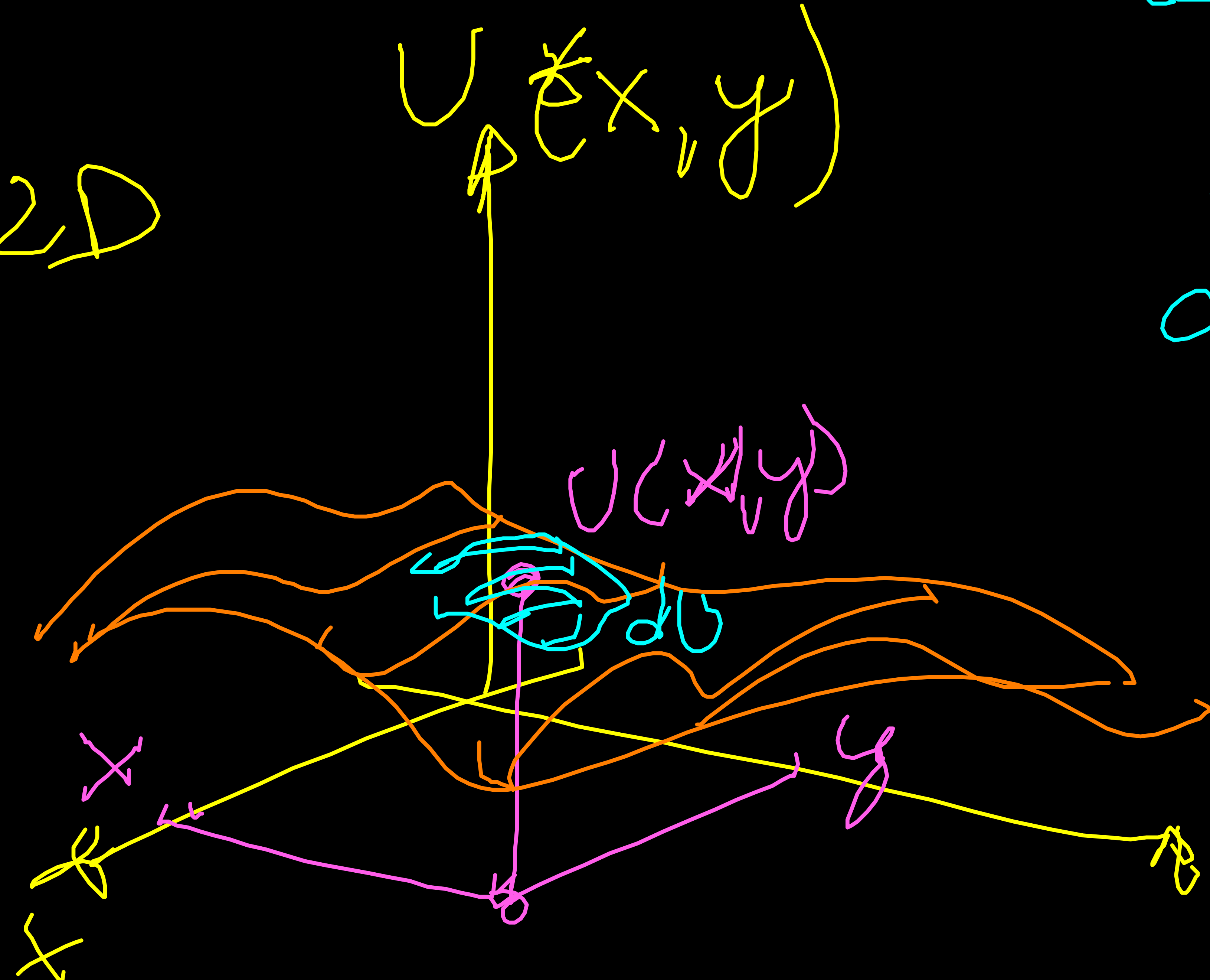


$\vec{F}$ CONSERVATIVA

$$dL = \vec{F}\cdot d\vec{r} =$$

$$\boxed{dU(x,y,z) = -\vec{F}\cdot d\vec{r} = -dL}$$

IN 2D



$$dU(x,y,z) = \underbrace{\frac{\partial U}{\partial x}}_{\text{DERIV. PARZIALI}} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz$$

$$=$$

$$-(\vec{F}\cdot d\vec{r}) = -(F_x\, dx + F_y\, dy + F_z\, dz)$$

$$\boxed{-\frac{\partial U}{\partial x} = F_x \qquad -\frac{\partial U}{\partial y} = F_y \qquad -\frac{\partial U}{\partial z} = F_z}$$

$$\frac{\partial U}{\partial x} = \lim_{\Delta x \to 0} \frac{U(x+\Delta x, y, z) - U(x,y,z)}{\Delta x}$$

$$F_x = -\frac{\partial U}{\partial x}$$

$$F_y = -\frac{\partial U}{\partial y}$$

$$F_z = -\frac{\partial U}{\partial z}$$

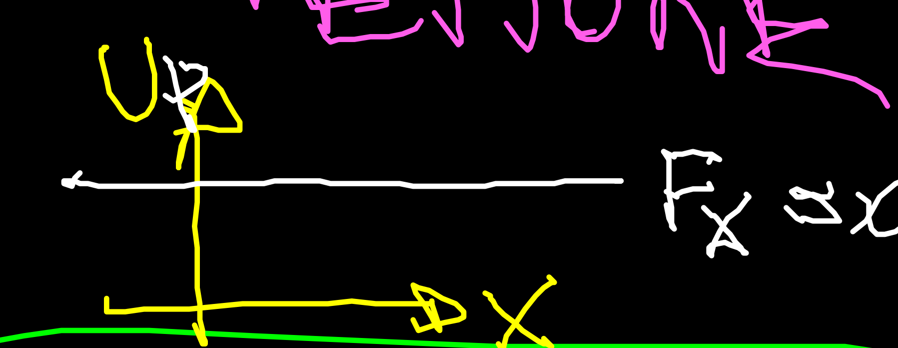
$$\vec{\nabla} \equiv \left(-\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right)$$ "NABLA"

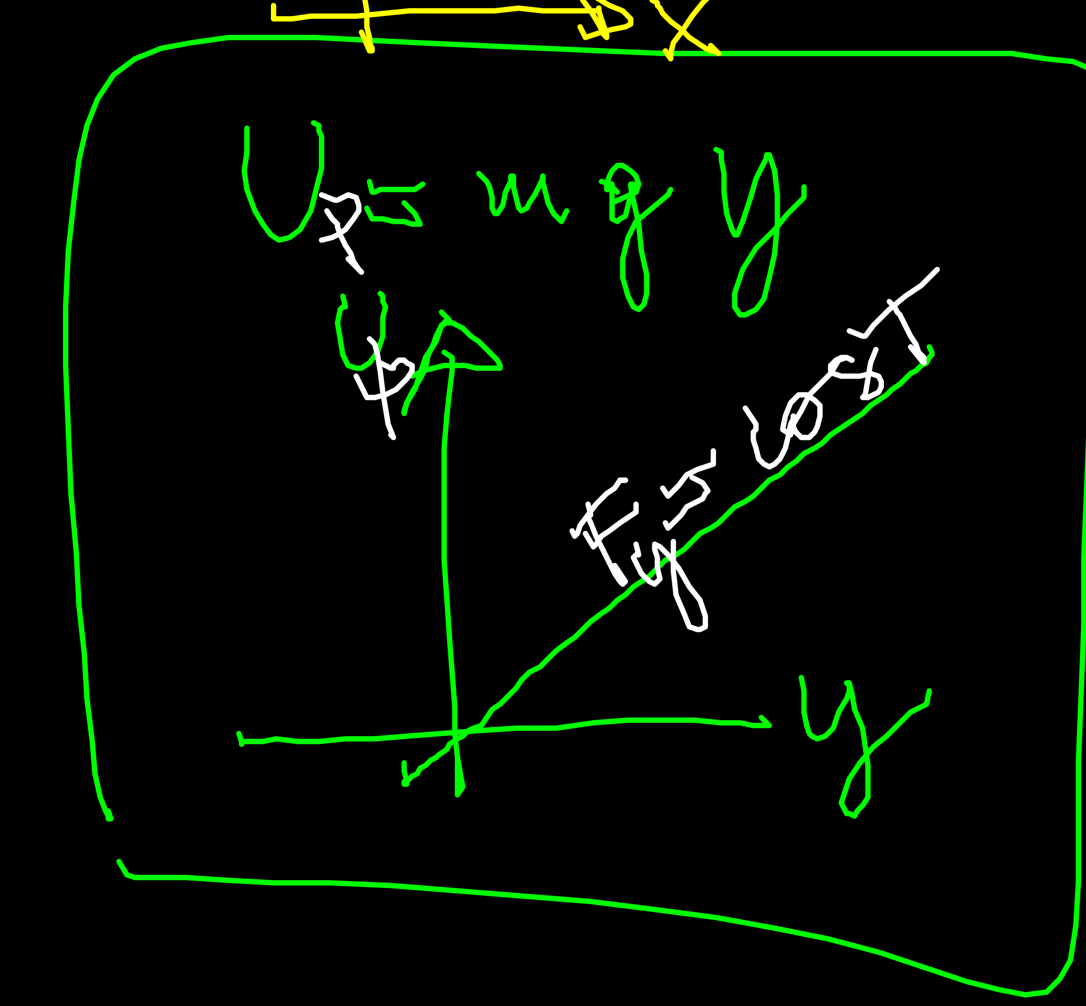
$$-\vec{\nabla} U \equiv \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right) = \underbrace{(F_x, F_y, F_z)}_{\text{VETTORE}}$$

OP. "VETTORE" su SCALARE

GRADIENTE

$$\vec{F} = -\vec{\nabla} U = -\text{grad}\, U$$





$$U_f - U_i = -\int_i^f \vec{F}_{cons} \cdot d\vec{r}$$

$$K_f - K_i = \mathcal{L}_{tot} = \mathcal{L}_{F_{c1}} + \mathcal{L}_{F_{c2}} + \dots +$$

$$K_f - K_i = -(U_f - U_i)_1 - (U_f - U_i)_2 + \dots +$$

$$= -(U_{f1} + U_{f2} + \dots) + (U_{i1} + U_{i2} + \dots)$$

$$K_f + \underbrace{(U_{f1} + U_{f2} + \dots)}_{U_{T,f}} = K_i + \underbrace{(U_{i1} + U_{i2} + \dots)}_{U_{Ti}}$$

CONS. EN. MECC.