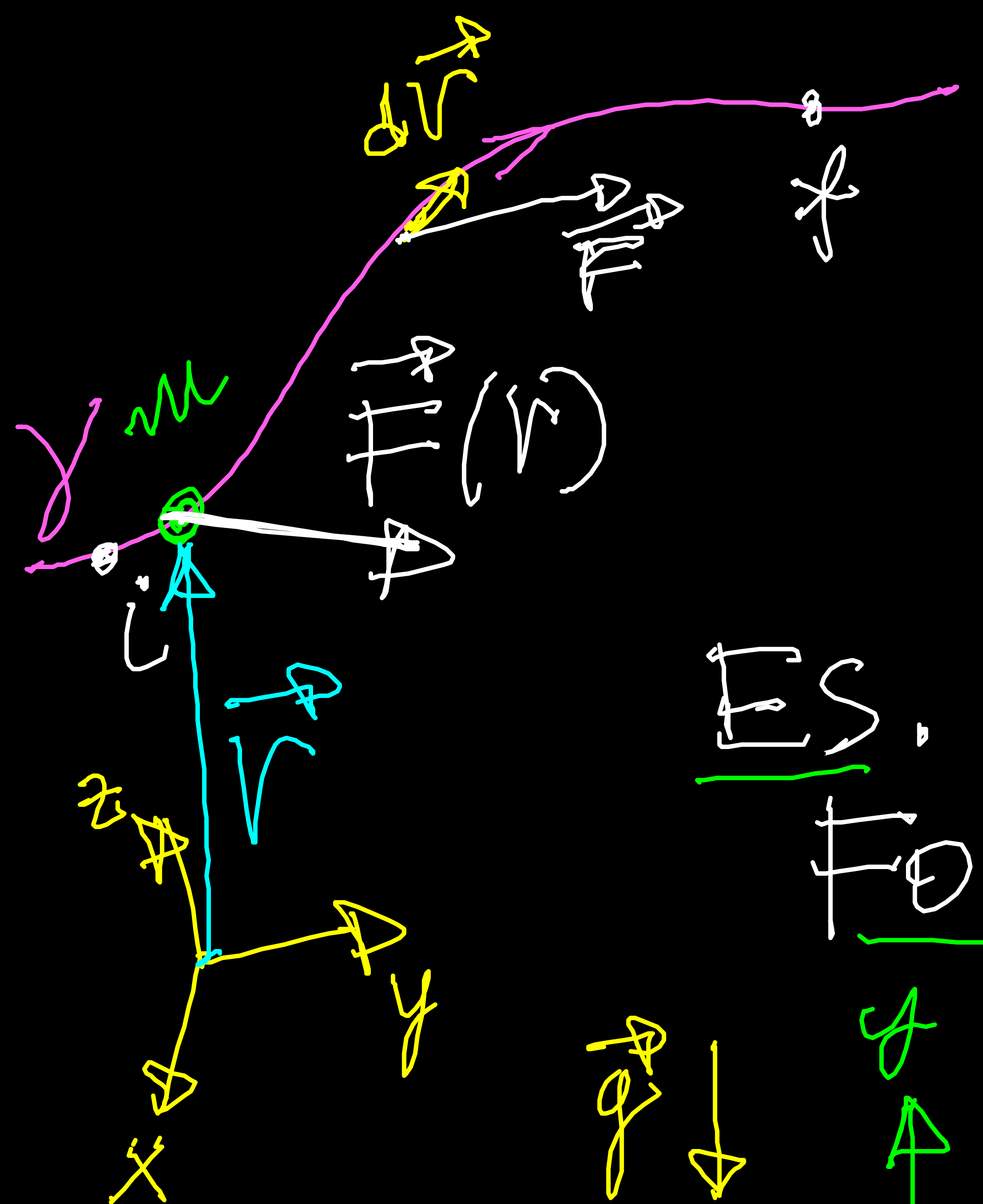
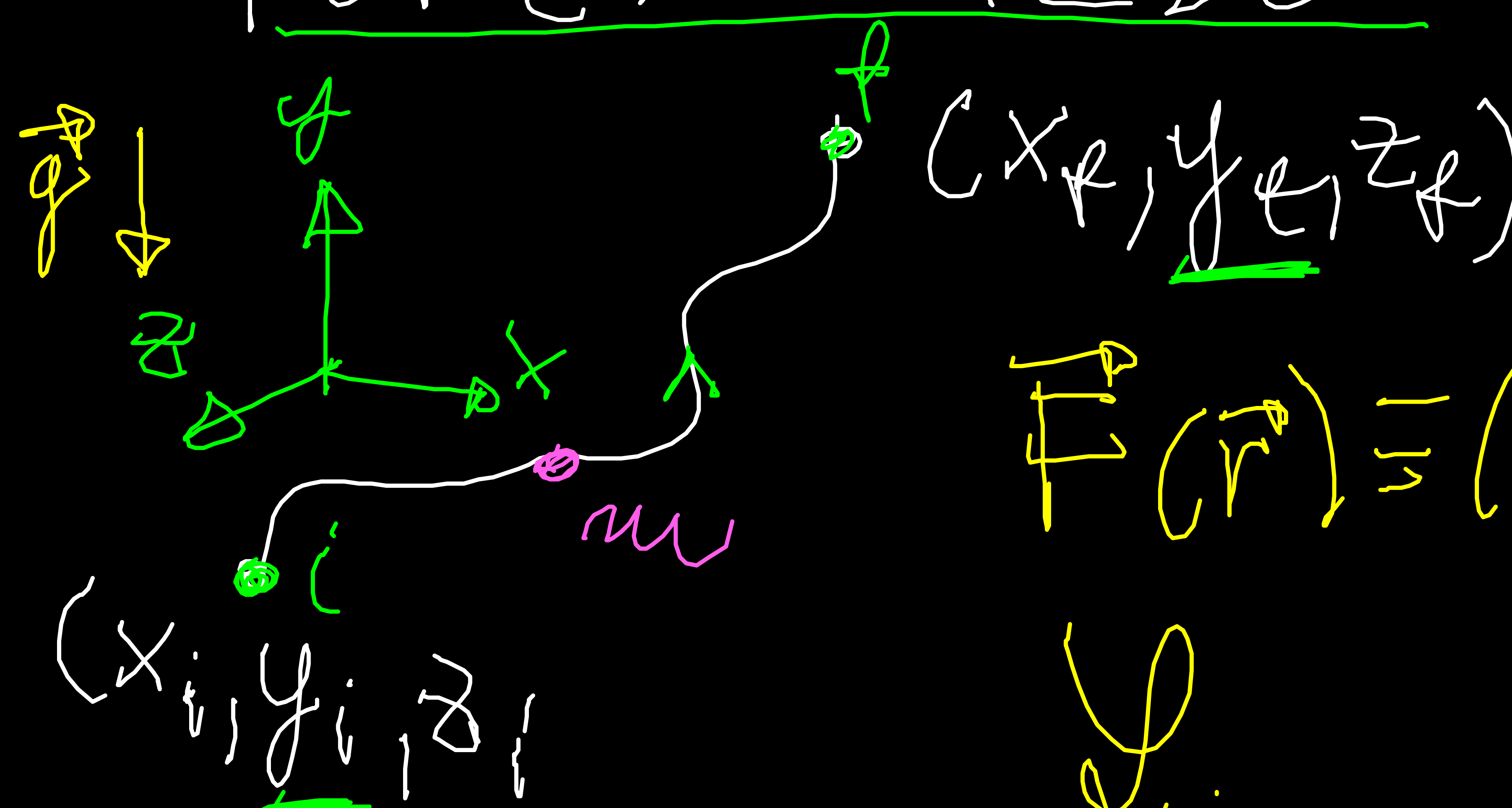
LAVORO DI UNA FORZA



$$\mathcal{L}_{(i\to f,\gamma)} = W = \int_{i,\gamma}^{f} \vec{F}(\vec{r})\cdot d\vec{r}$$

ES.

FORZA PESO

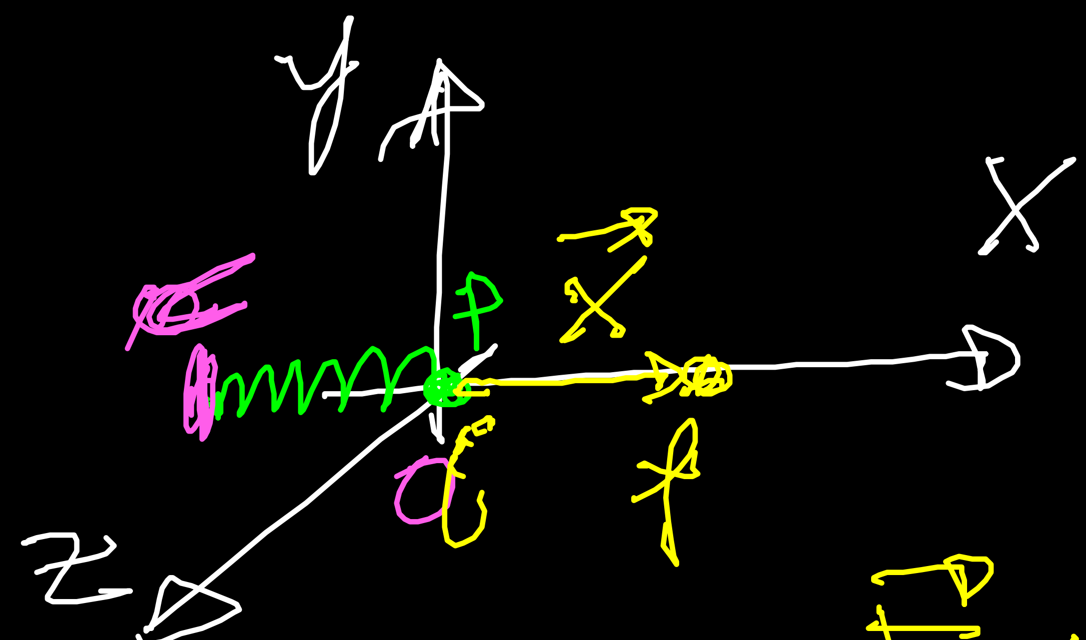


$$\vec{F}(\vec{r}) = (0, -mg, 0)$$

$$\mathcal{L}_{i\to f,\gamma} = -mg(y_f - y_i)$$

$$= -mgh$$ INDIP. DA γ

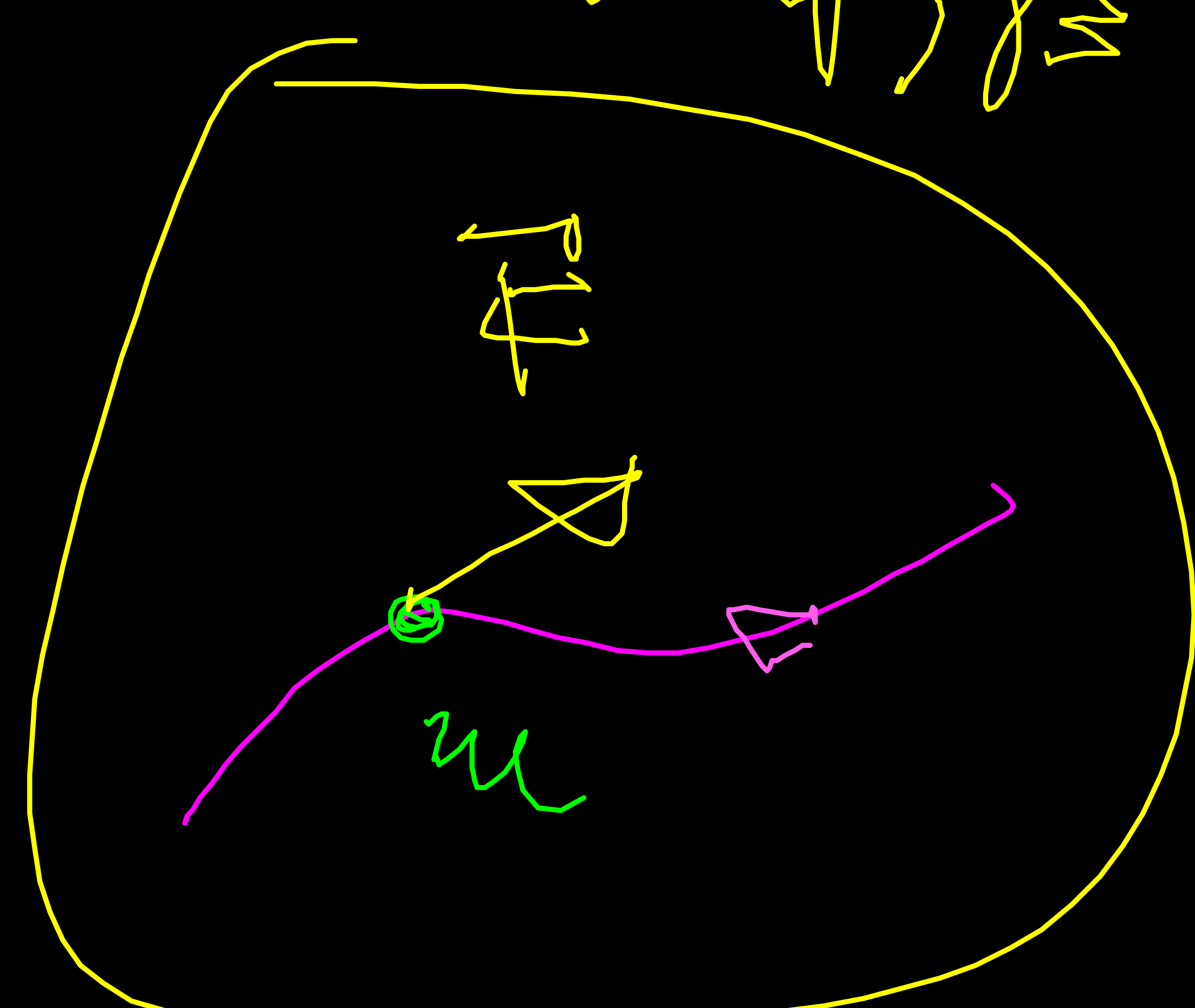
ES. FORZA ELASTICA



$$\vec{F}_{el}(\vec{r}) \equiv (-Kx, 0, 0)$$

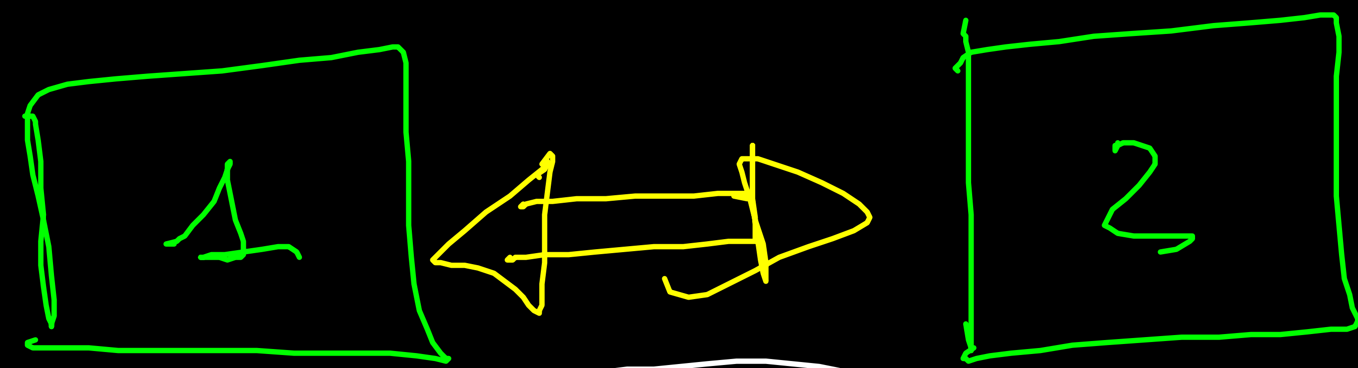
$$\vec{F}(\vec{r}) \cdot d\vec{r} = F_x dx + \cancel{F_y dy} + \cancel{F_z dz} = -Kx\,dx$$

$$\mathcal{L}_{i \to f,\, \gamma \subseteq \hat{i}} = \int_{x_i}^{x_f} F_x dx = -K \int_{x_i}^{x_f} x\,dx = -\frac{K(x_f - x_i)^2}{2}$$



Se $x_i = 0 \Rightarrow \mathcal{L} = -\frac{Kx^2}{2}$

allungamento (Δx)

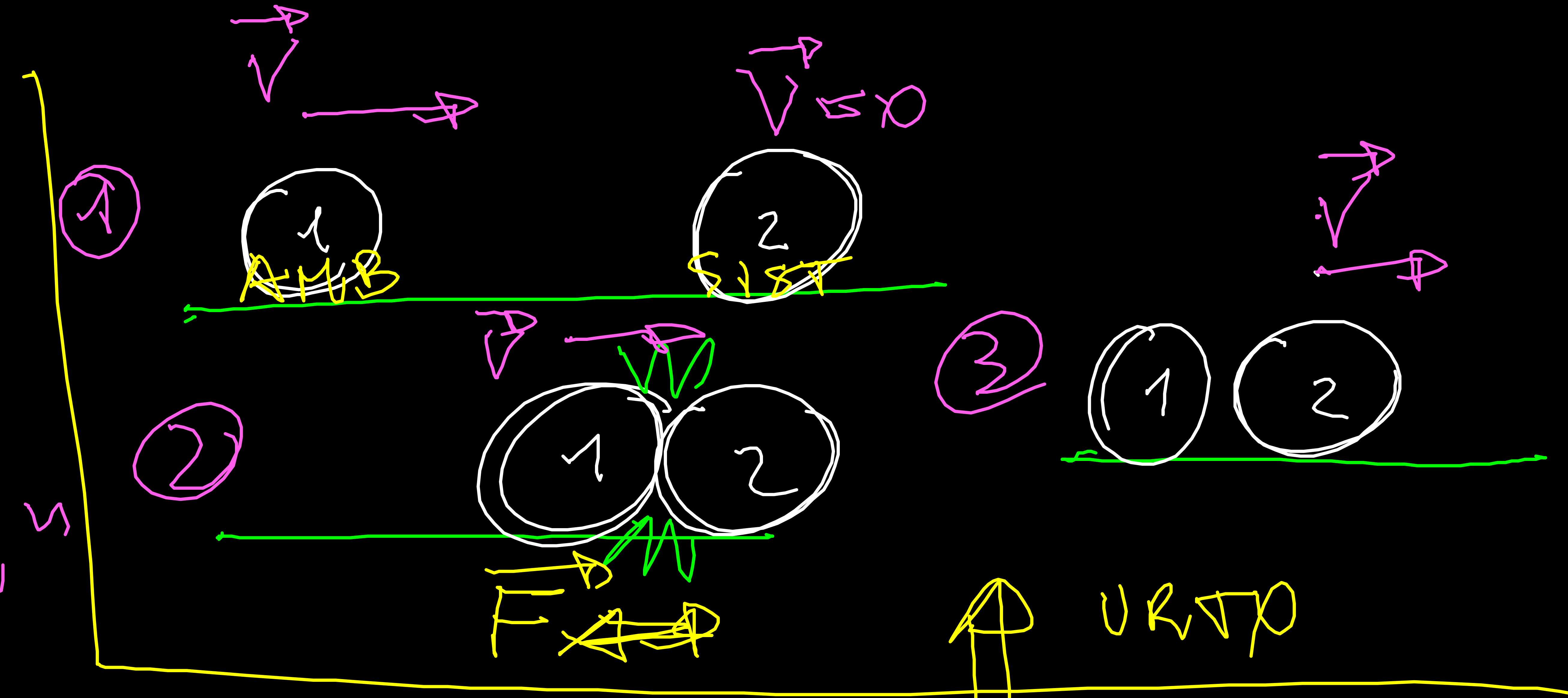


FORZA

"SISTEMA" "AMBIENTE"

f

SPOST.



IL LAVORO TRASFERISCE "ENERGIA"

TRASFORMAZIONE

LAVORO DI UNA FORZA = ENERGIA TRASFERITA

TRASFERIMENTO DI "ENERGIA"

TEOREMA LAVORO - ENERGIA $\langle \Leftrightarrow F_{ris} \rangle$
(DELL'ENERGIA CINETICA)
(DELLE FORZE VIVE)

CASO UNIDIMENSIONALE CON a_x COSTANTE

a_x costante $\Rightarrow$ MOTO UNIF. ACCELERATO

$m \longrightarrow v_x$

$x_i \qquad x_f \qquad x$

$$\boxed{v_f^2 - v_i^2 = 2a_x(x_f - x_i)}$$

$m \times$

$$\begin{cases} x_f(t) = x_i + v_i t + \frac{1}{2} a_x t^2 \\ v_f(t) = v_i + a_x t \end{cases}$$

$$m v_f^2 - m v_i^2 = 2 m a_x (x_f - x_i)$$

$$\frac{m}{2} v_f^2 - \frac{m}{2} v_i^2 = \underbrace{m a_x (x_f - x_i)}_{} \longrightarrow \text{LAVORO DI } F_x ?$$

$$\frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 = \underbrace{m a_x}_{} (x_f - x_i) = F_x (x_f - x_i)$$

$$\sum_{i=1}^{n} F_{ix} = F_x$$

∄ P.

costante

$$\frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 = F_x \Delta x = \mathcal{L}_F$$

$$K = \frac{m v^2}{2} = \text{ENERGIA CINETICA}$$

$$K = \frac{m \vec{v} \cdot \vec{v}}{2} = \frac{m |\vec{v}|^2}{2} = \frac{m v^2}{2} > 0$$

$$\mathcal{L}_F = F_x \Delta x = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = K_f - K_i$$

$$K = \frac{1}{2} m v^2$$

$$\boxed{\Delta K = \mathcal{L}_F = \mathcal{L}_{TOT}}$$

$$\Delta K = \underbrace{m a_x}\,\Delta x = \left(\sum_{i=1}^{n} F_{ix}\right)\Delta x = \sum_{i=1}^{n} F_{ix}\,\Delta x = \underbrace{\sum_{i=1}^{n} \mathcal{L}_i}_{\mathcal{L}_{TOT}}$$

$$m a_x = \sum_{i=1}^{n} F_{ix}$$

TEOREMA DELL'ENERGIA CINETICA (DIM. GEN.)

$$\vec F(\vec r) = \sum_{i=1}^{n} \vec F_i$$

$$\sum \vec F = m\vec a$$

$$d\mathcal{L} = \left(\sum \vec F\right)\cdot d\vec r = m\,\vec a \cdot d\vec r$$

$$\underbrace{\mathcal{L}_{TOT}} = \int_i^f d\mathcal{L} = \underbrace{\int_i^f \left(\sum \vec F\right)\cdot d\vec r} = \underbrace{\int_i^f m\,\vec a \cdot d\vec r}_{?}$$

$$\vec a = (a_x, a_y, a_z)$$

$$d\vec r = (dx, dy, dz)$$

$$\vec a \cdot d\vec r = a_x dx + a_y dy + a_z dz$$

?

$$\frac{a_x\,dx}{dt} = \boxed{\left(\frac{dV_x}{dt}\right)V_x}\,dt = \frac{1}{2}\frac{d}{dt}(V_x^2)\cdot dt$$

$$\frac{d}{dt}(V_x^2) = 2V_x\frac{dV_x}{dt}$$

$$a_x = \frac{dV_x}{dt}$$

$$V_x = \frac{dx}{dt} \Rightarrow dx = V_x\,dt$$

$$\frac{m}{2}\frac{d}{dt}(V_x^2)\cdot dt \qquad \frac{m}{2}\frac{d}{dt}(V_y^2)\cdot dt \qquad \frac{m}{2}\frac{d}{dt}(V_z^2)\cdot dt$$

$$\mathcal{L}_{TOT} = \int_i^f m\vec{a}\cdot d\vec{r} = \int_i^f \underline{m\,a_x dx} + m\,a_y dy + m\,a_z dz$$

$$\mathcal{L}_{TOT.} = \int_i^f m\vec{a}\cdot d\vec{r} = \int_i^f \frac{m}{2}\frac{d}{dt}(v_x^2)\cdot dt + \frac{m}{2}\frac{d}{dt}(v_y^2)\cdot dt + \frac{m}{2}\frac{d}{dt}(v_z^2)\cdot dt$$

$$= \frac{m}{2}\int_i^f \underbrace{\left[\frac{d}{dt}(v_x^2) + \frac{d}{dt}(v_y^2) + \frac{d}{dt}(v_z^2)\right]}_{\frac{d}{dt}(v_x^2 + v_y^2 + v_z^2) = \frac{d}{dt}(v^2)}\cdot dt$$

$$= \frac{m}{2}\int_i^f \frac{d}{dt}(v^2)\cdot dt = \frac{m}{2}\int_i^f d(v^2) = \frac{m}{2}\left[v_f^2 - v_i^2\right]$$

TEOREMA DELL'ENERGIA CINETICA

$$\underbrace{\mathcal{L}_{TOT}} = \mathcal{L}_{F_1} + \mathcal{L}_{F_2} + \dots + \mathcal{L}_{F_n} = \underbrace{K_f - K_i}_{\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2} = \underbrace{\Delta K}$$