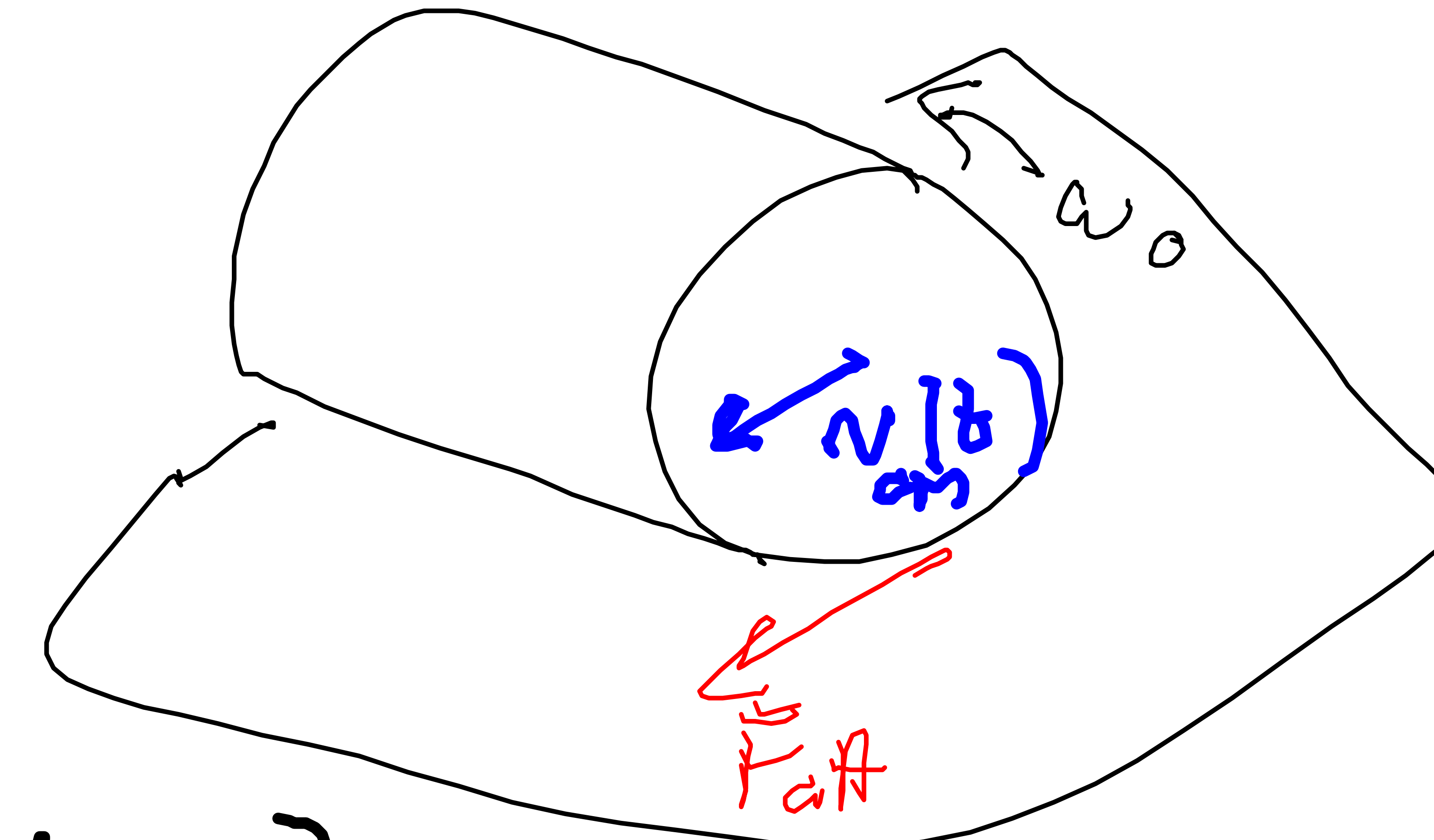
Cilindro pieno

$R = 15\ cm$, $m = 2\ kg$, $\omega_0 = 40\ rad/s$

$\mu_d = 0.25$

a) $t' = \dfrac{\omega_0 R}{3\mu g} = 0.82\ s$

b) $v(t') = ?$ c) N giri at t' d) energia dissipata = ?



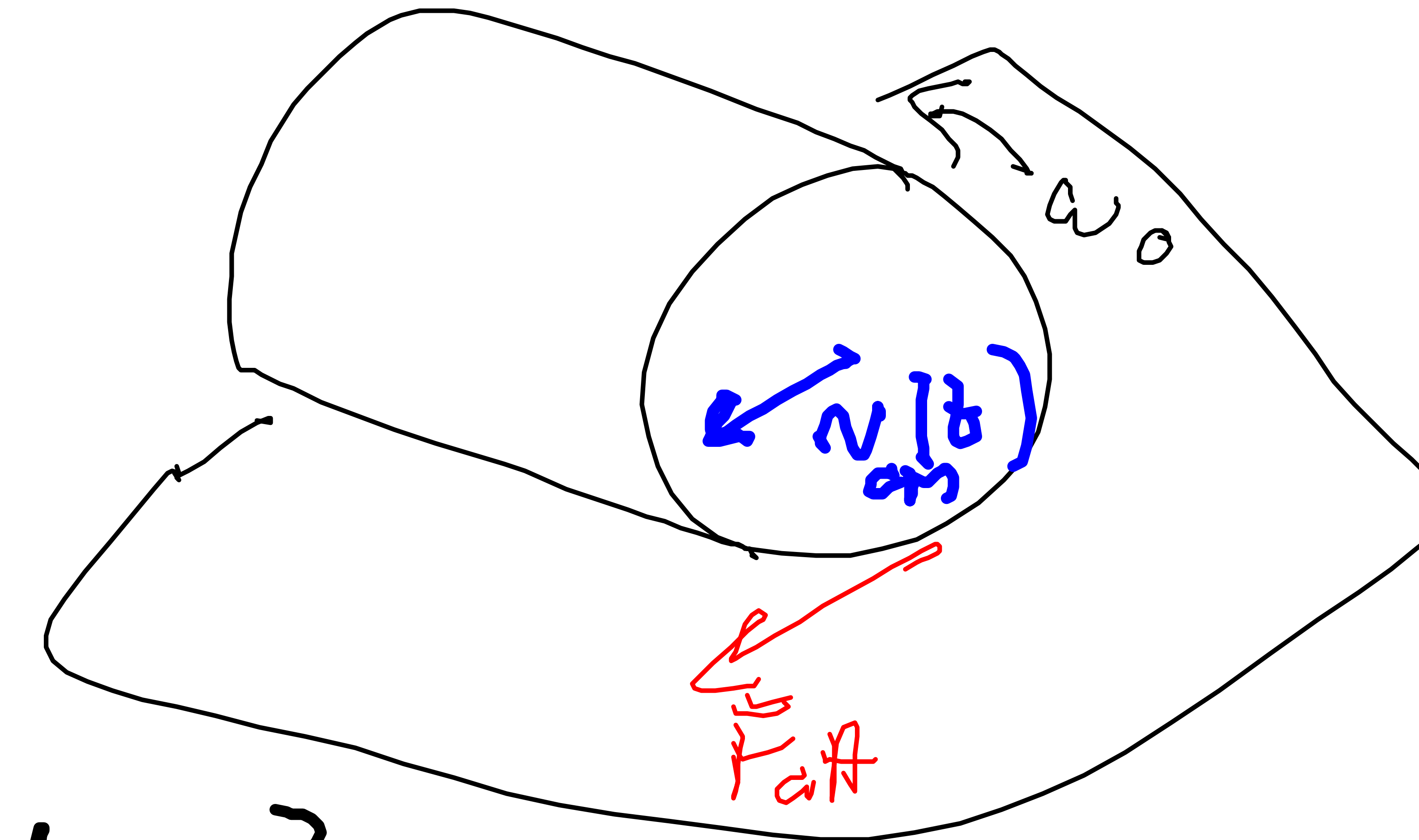
$$\alpha = \mu m g \frac{R}{I} = \frac{2\mu_d g}{R} = 32.7\ rad/s = \frac{a_{cm}}{R}$$

$$v_{cm}(t') = a_{cm}\, t' = \alpha R t' = 2.0\ m/s^2$$

$$\begin{cases} \omega(t) = \omega_0 - \alpha t \\ \vartheta(t) = \vartheta_0 + \omega_0 t - \frac{1}{2}\alpha t^2 = 21.8\ rad \end{cases}$$

$$N = \frac{\vartheta(t')}{2\pi} = 3.47\ giri$$

Cilindre plene



$R = 15\,cm,\ M = 2\,kg,\ \omega_0 = 40\ rad/s$

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a) $t' = \dfrac{\omega_0 R}{3\mu g} = 0.82\,s$

b) $v(t') = ?$ c) N giri at t' d) energia dissipata $= ?$

$$K_i = K_i^{rot} = \frac{1}{2} I\omega_0^2 = \frac{1}{2}\frac{1}{2}MR^2\omega_0^2 = 18\,J$$

$$K_f = K_f^{rot} + K_f^{tr} = \frac{1}{2} I\omega^2(t') + \frac{1}{2} M v_{cm}^2(t')$$

$$= \frac{1}{2}\frac{1}{2} M \underbrace{R^2\omega^2(t')}_{v_{cm}^2(t')} + \frac{1}{2} M v_{cm}^2 = \frac{3}{4} M v_{cm}^2(t') = 6\,J$$

$$\boxed{E^{diss} = K_i - K_f = 12\,J}$$

TRASL

$\bar{x}(t)$ $\quad \bar{v}(t)$ $\quad \bar{a}(t)$ $\quad m$; $\quad \bar{F}(t)$ $\qquad \bar{p} = m\bar{v}$

$$\sum \bar{F} = m\bar{a} \qquad \bar{F} = \frac{d\bar{p}}{dt}$$

$$\updownarrow \qquad\qquad \updownarrow$$

ROT

$\vartheta(t)$ $\quad \bar{\omega}(t)$ $\quad \bar{\alpha}(t)$ $\quad I$ $\quad \bar{\tau}(t)$ $\qquad \bar{L} = I\bar{\omega}$

$$\sum \bar{\tau} = I\bar{\alpha} \qquad \bar{\tau} = \frac{d\bar{L}}{dt}$$

$$\bar{L} = \bar{r} \times \bar{p}$$

$$\bar{\tau} = \bar{r} \times \bar{F}$$

Giroscopio

oggetto a simmetria cilindrica

$I_z \neq I_x = I_y$ rotazione su z

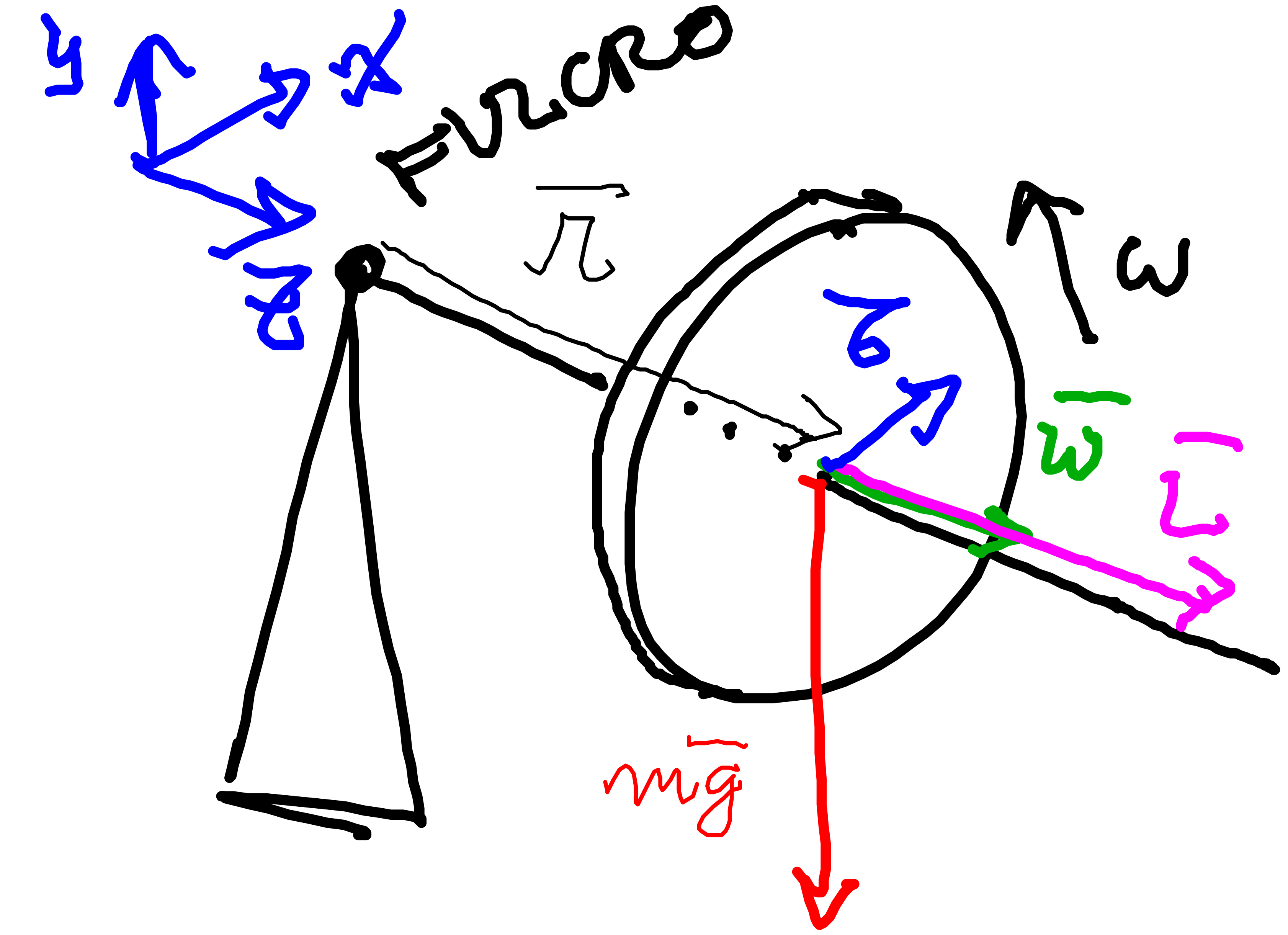
se penso alle sole forze, l'oggetto dovrebbe cadere

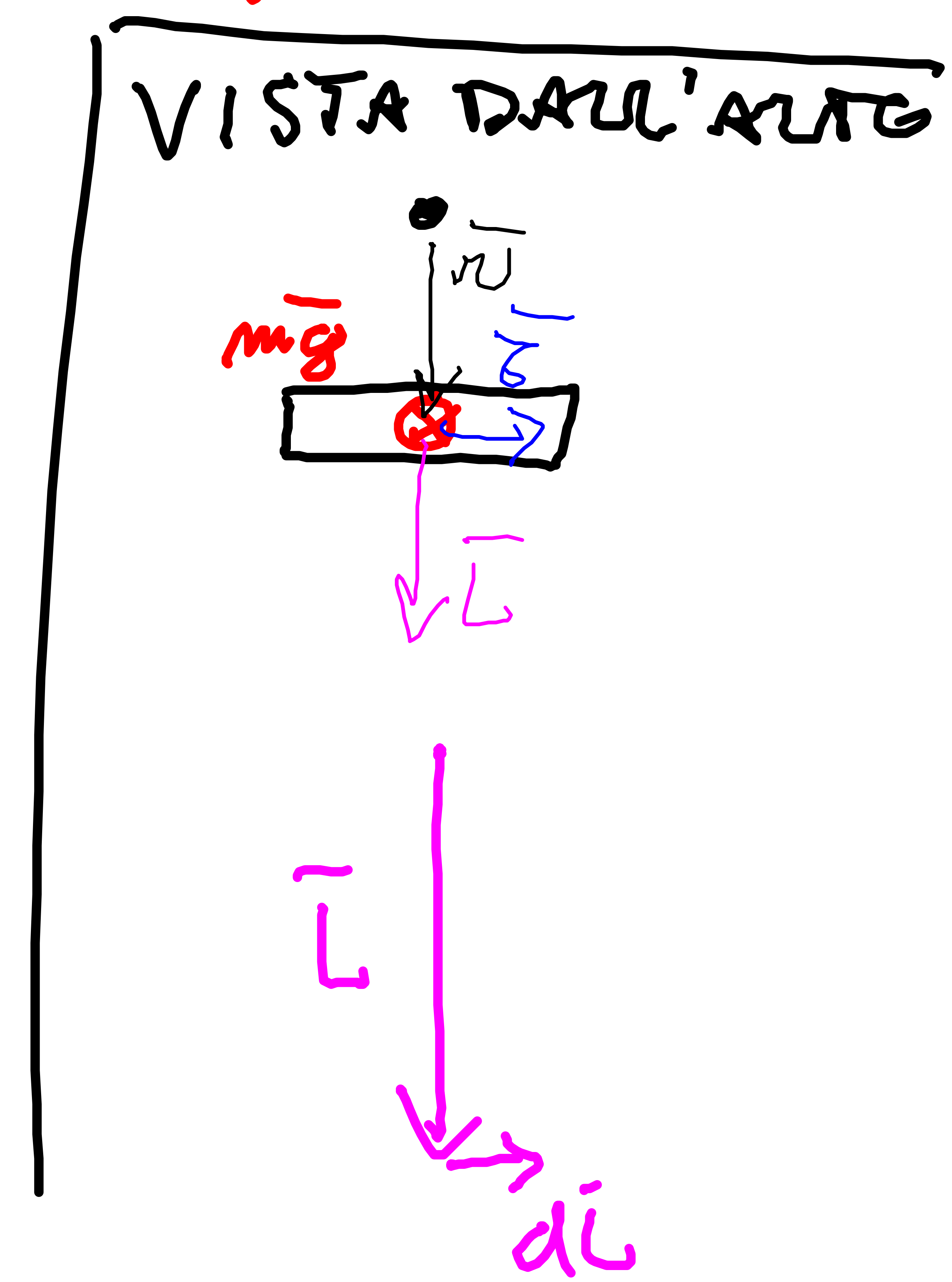
se cadesse si creerebbe un $\bar{\tau} = \bar{r} \times (m\bar{g})$

se ho già un $\bar{L}$ preesistente $\bar{\tau} = \frac{d\bar{L}}{dt}$

in un lasso di tempo dt $d\bar{L} = \bar{\tau}\,dt$ è PERPENDICOLARE al $\bar{L}$

$\Rightarrow |\bar{L}|$ non cambia $\Rightarrow$ CONSERVAZIONE MOMENTO ANGOLARE

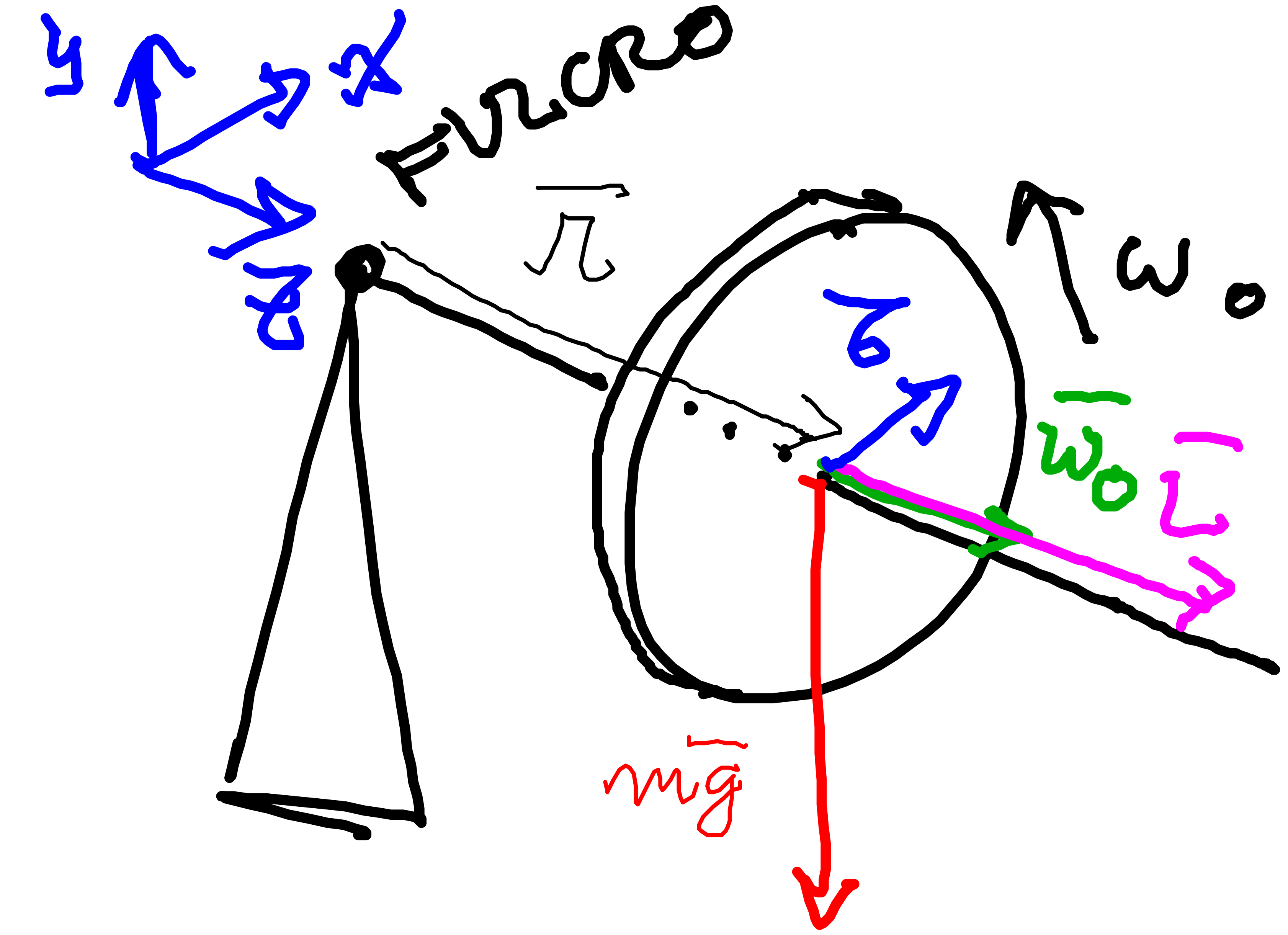


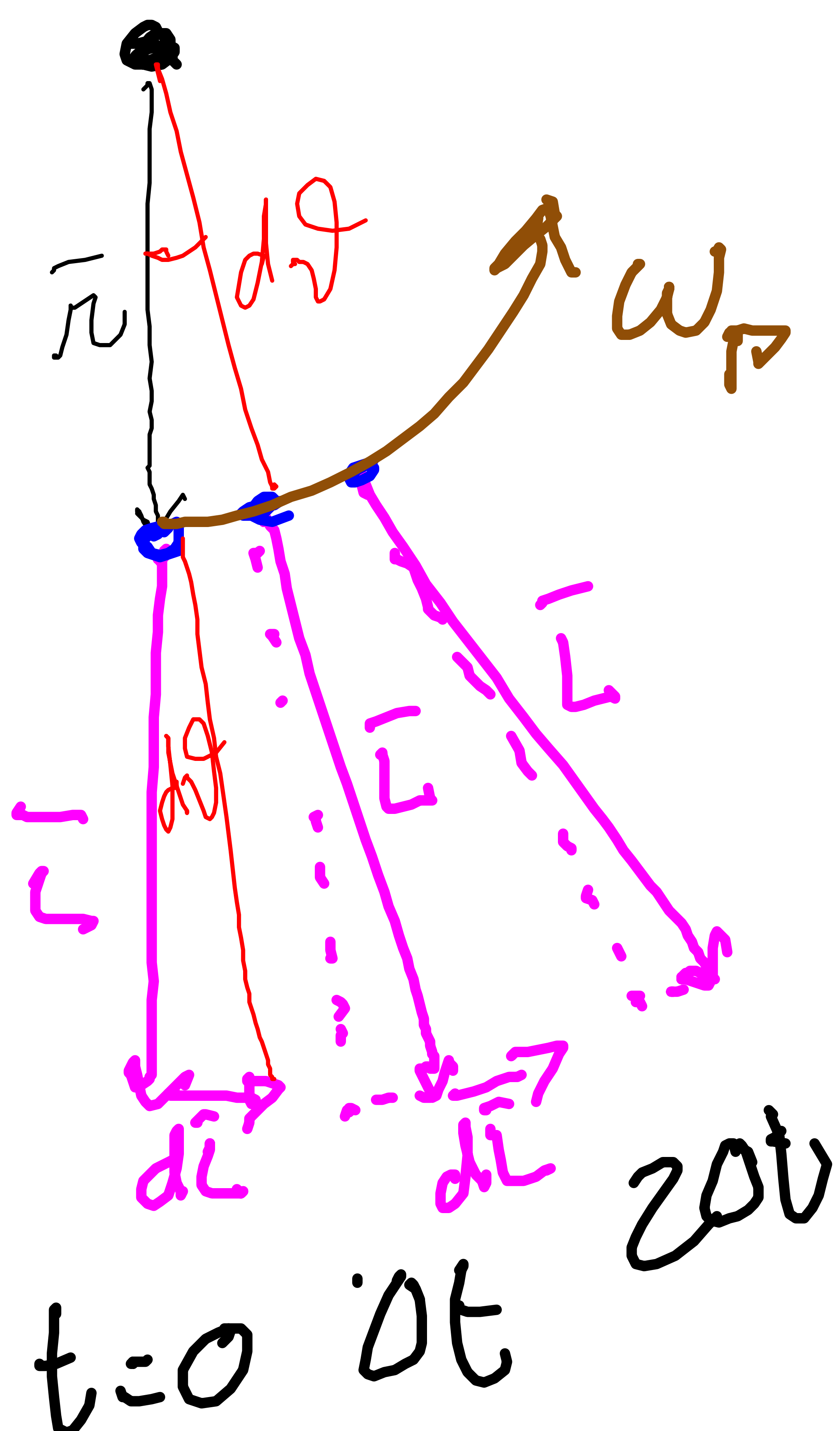


Giroscopio

oggetto a simmetria cilindrica

$I_z \neq I_x = I_y$ rotazione su z





MOTO DI PRECESSIONE

$$dL = L\,d\vartheta$$

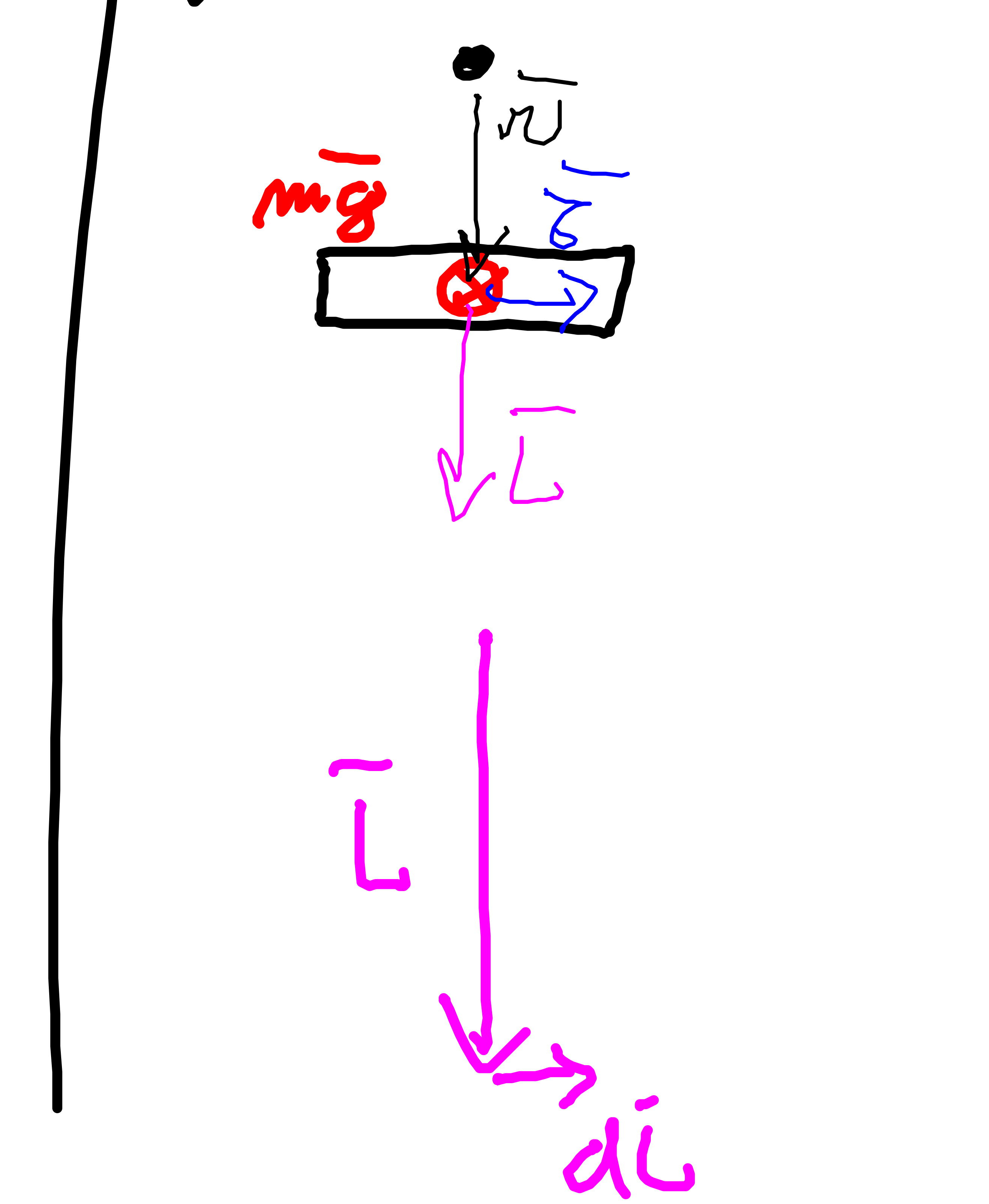
$$\bar{\tau} = \frac{d\bar{L}}{dt}$$

$$dL = \tau\,dt = rmg\,dt$$

$$rmg\,dt = L\,d\vartheta$$

$$\omega_p = \frac{d\vartheta}{dt} = \frac{rmg}{I\omega_0} = \text{cost}$$

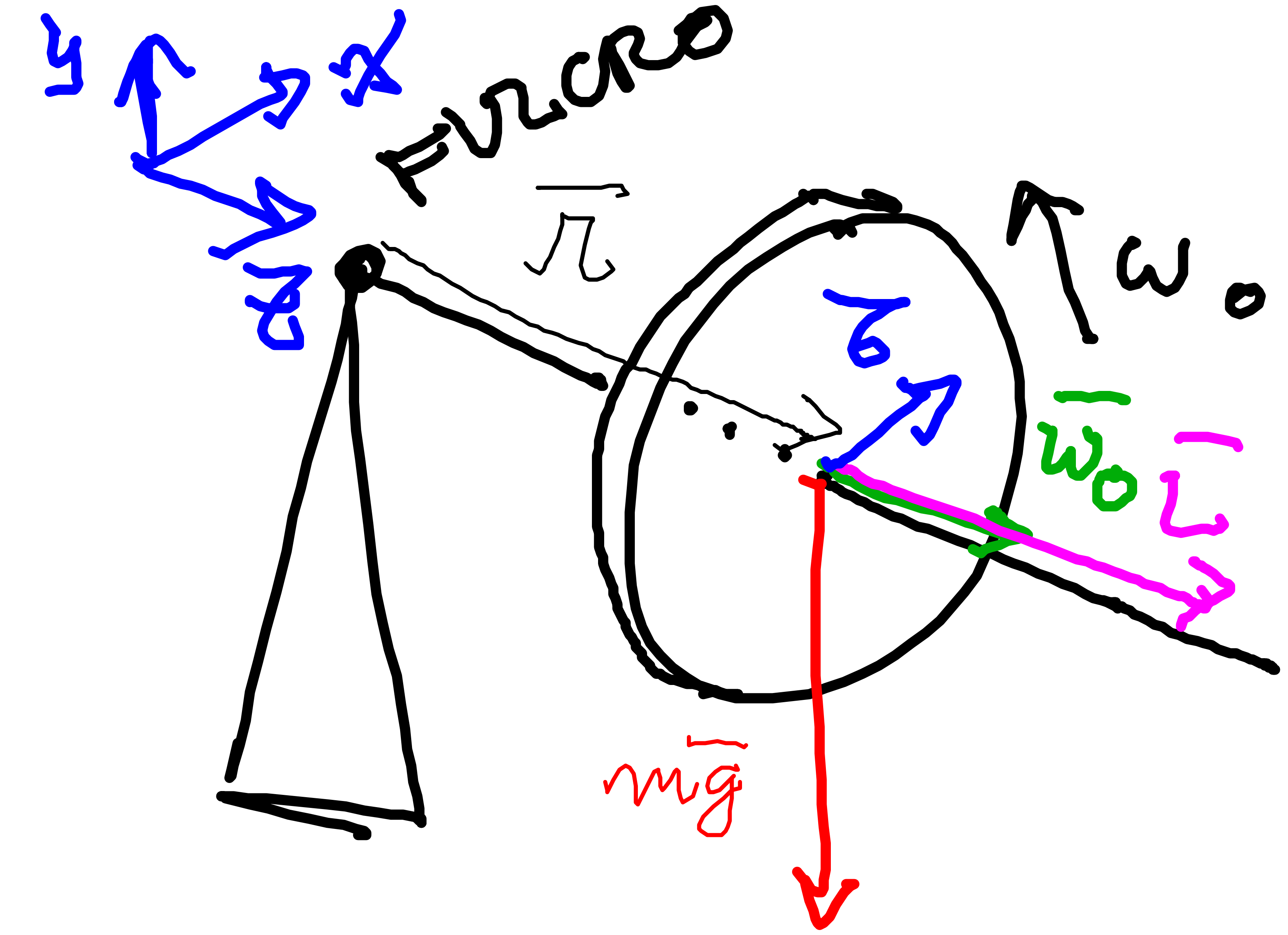
VISTA DALL'ALTO

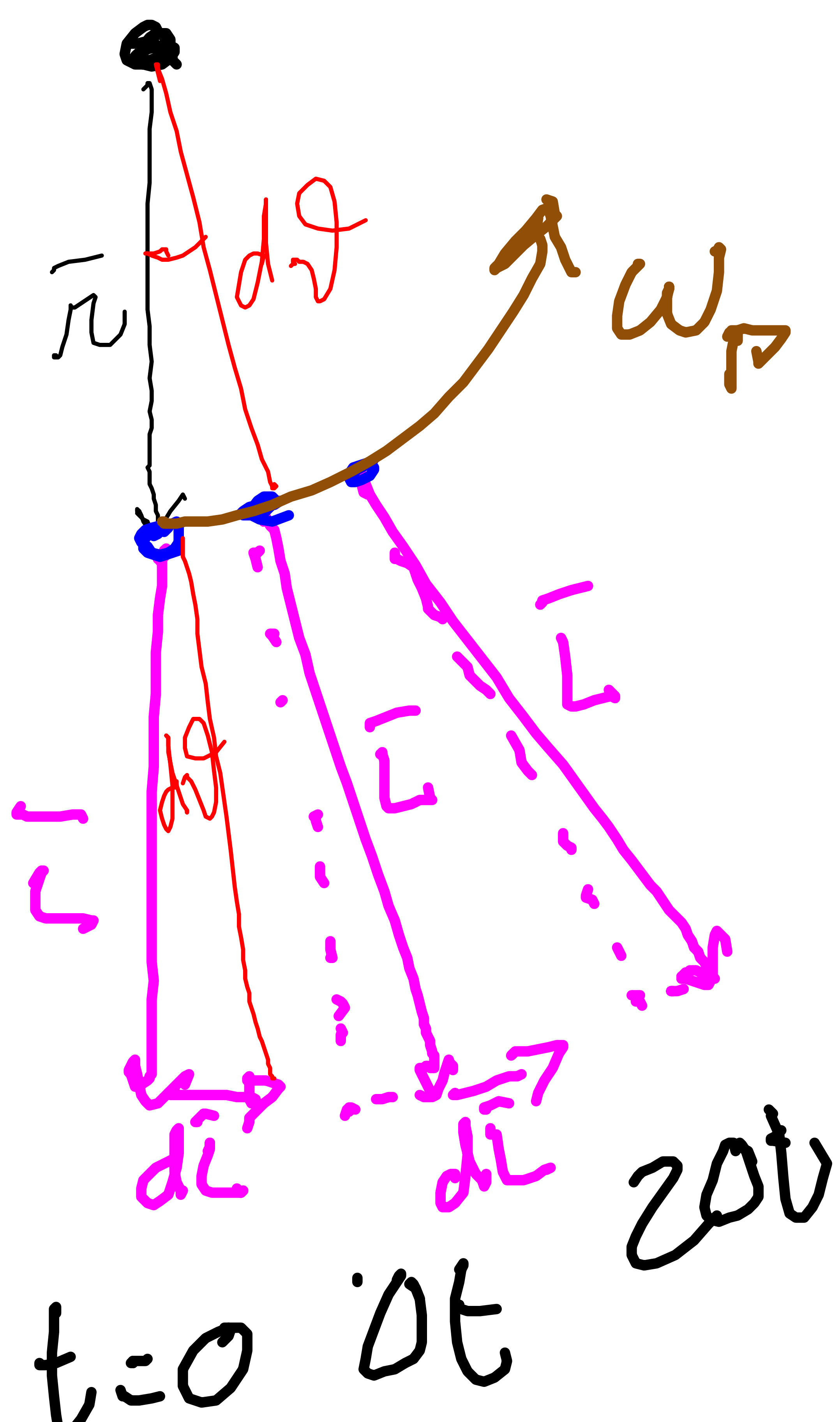


Giroscopio

oggetto a simmetria cilindrica

$I_z \neq I_x = I_y$ rotazione su z





MOTO DI PRECESSIONE

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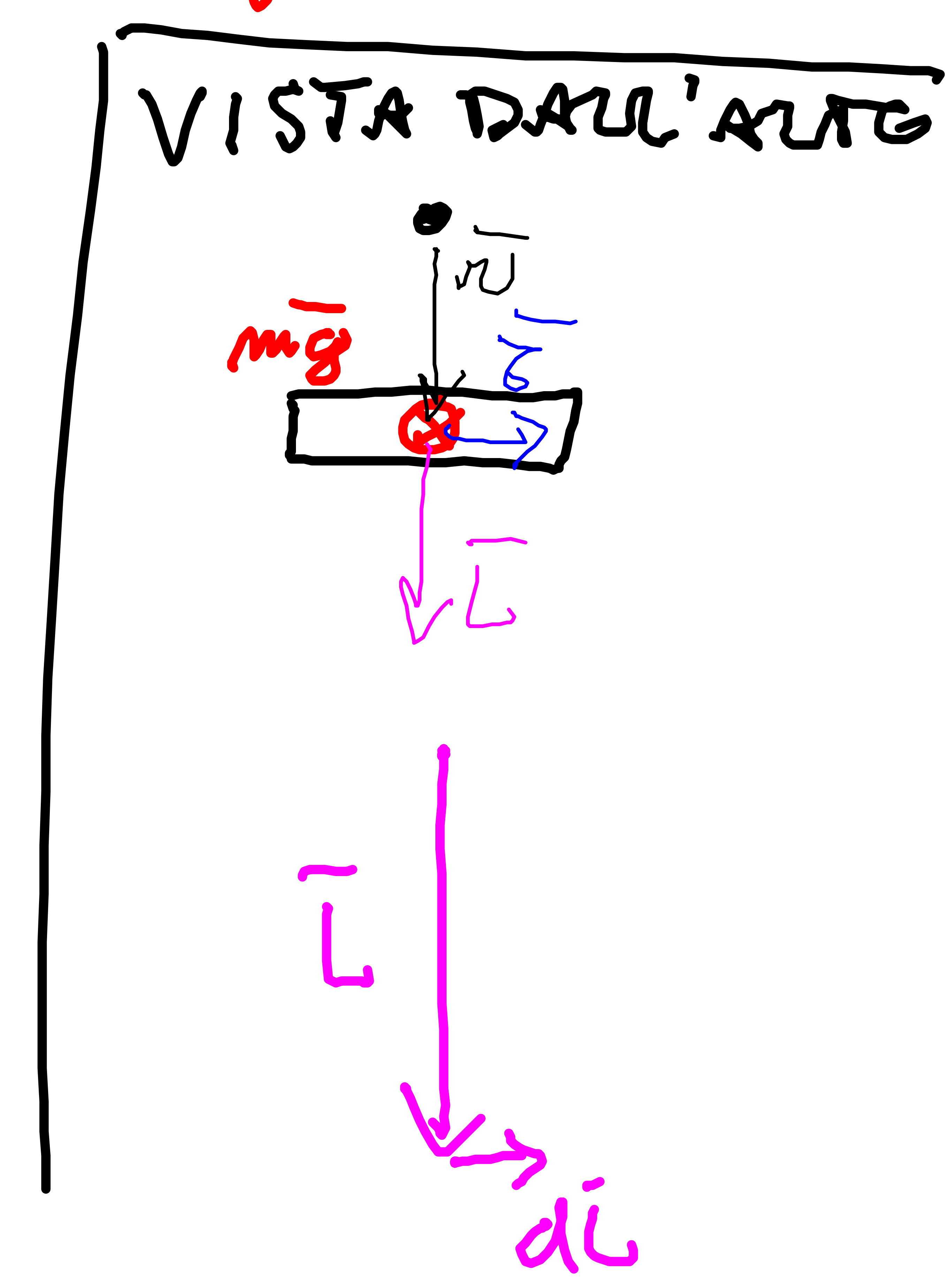
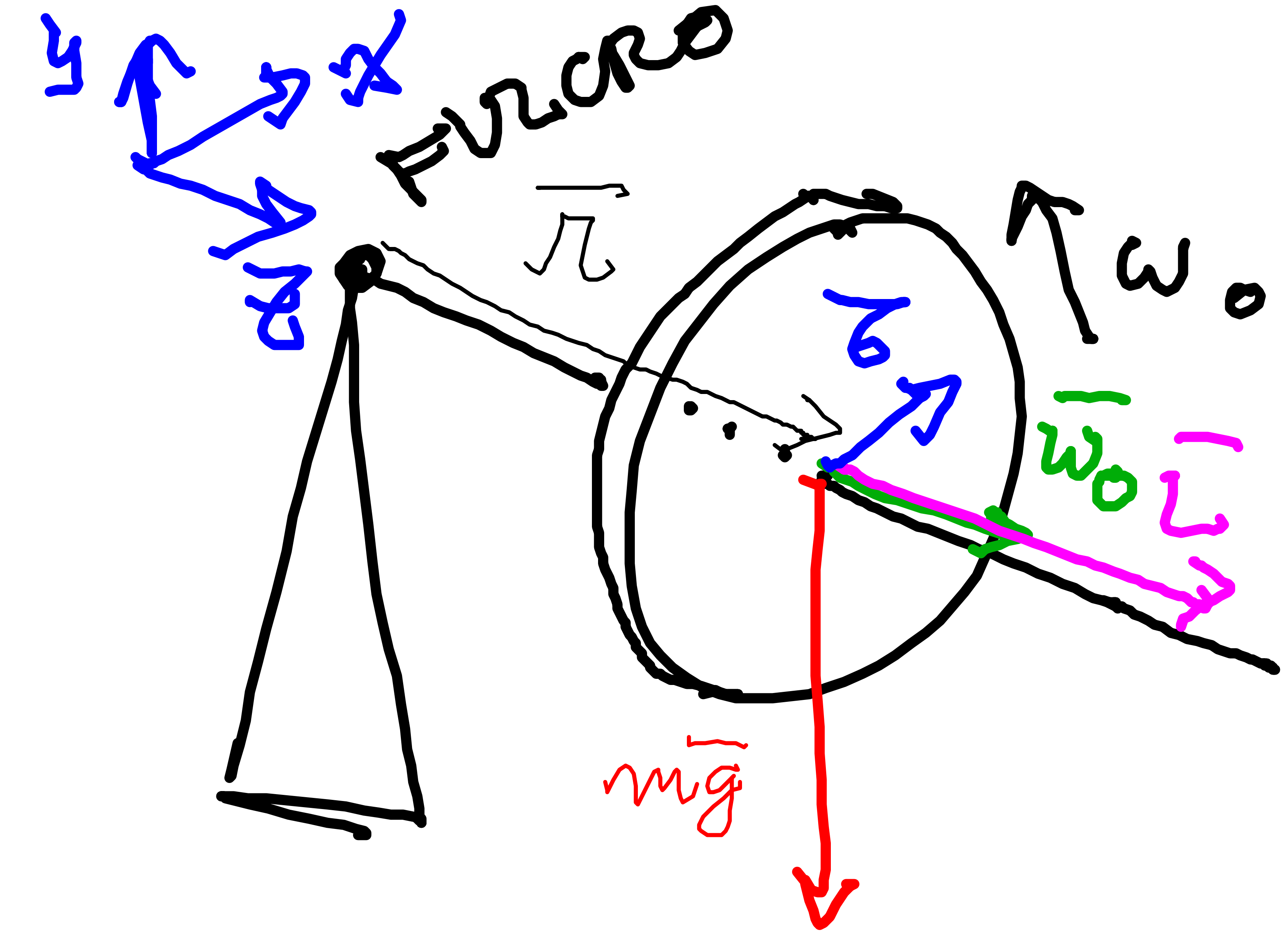


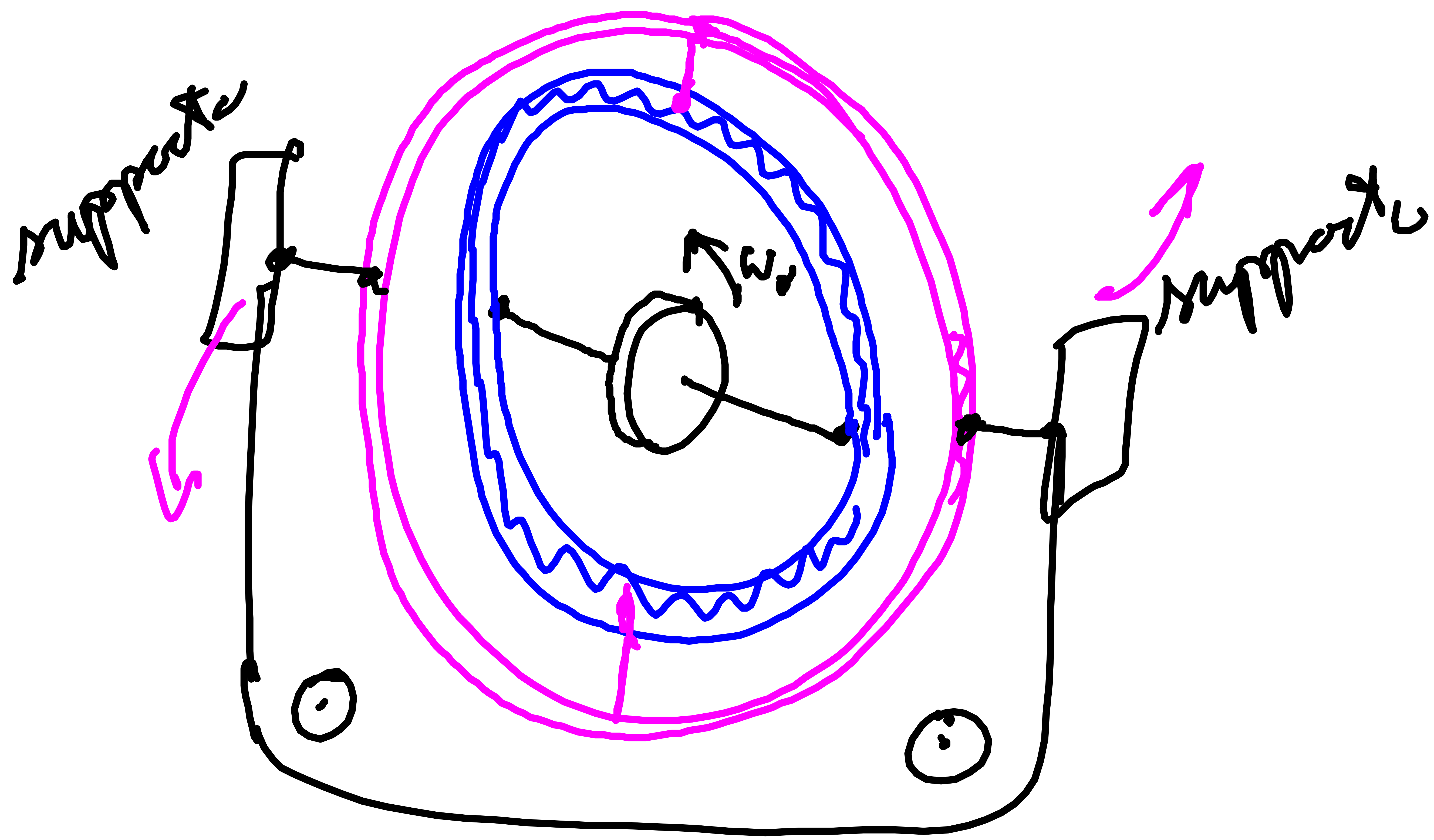
Giroscopio

oggetto a simmetria cilindrica

$I_z \neq I_x = I_y$ <u>rotazione su z</u>



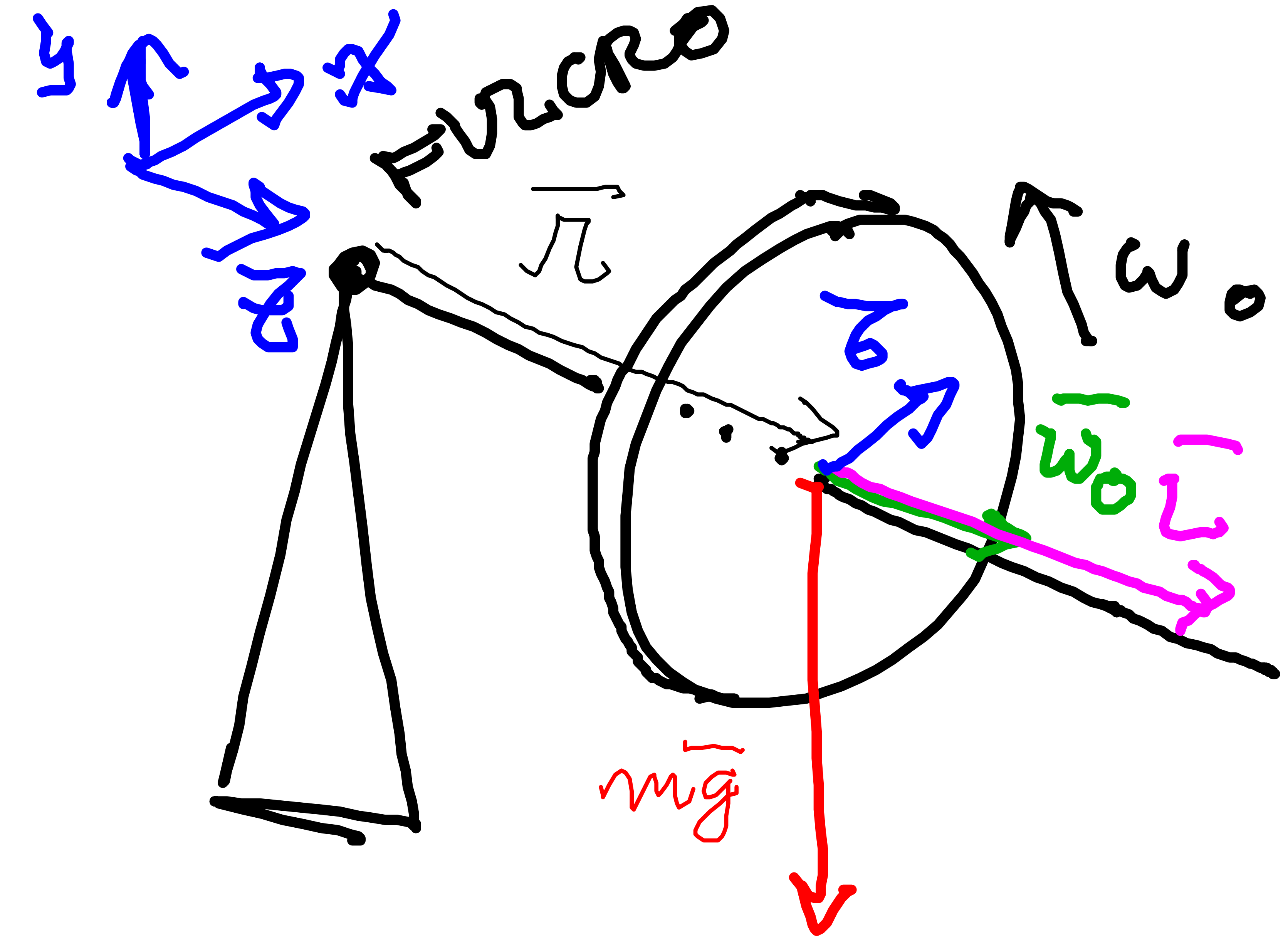
SOSPENSIONE CARDANICA 2D



Giroscopio

oggetto a simmetria cilindrica

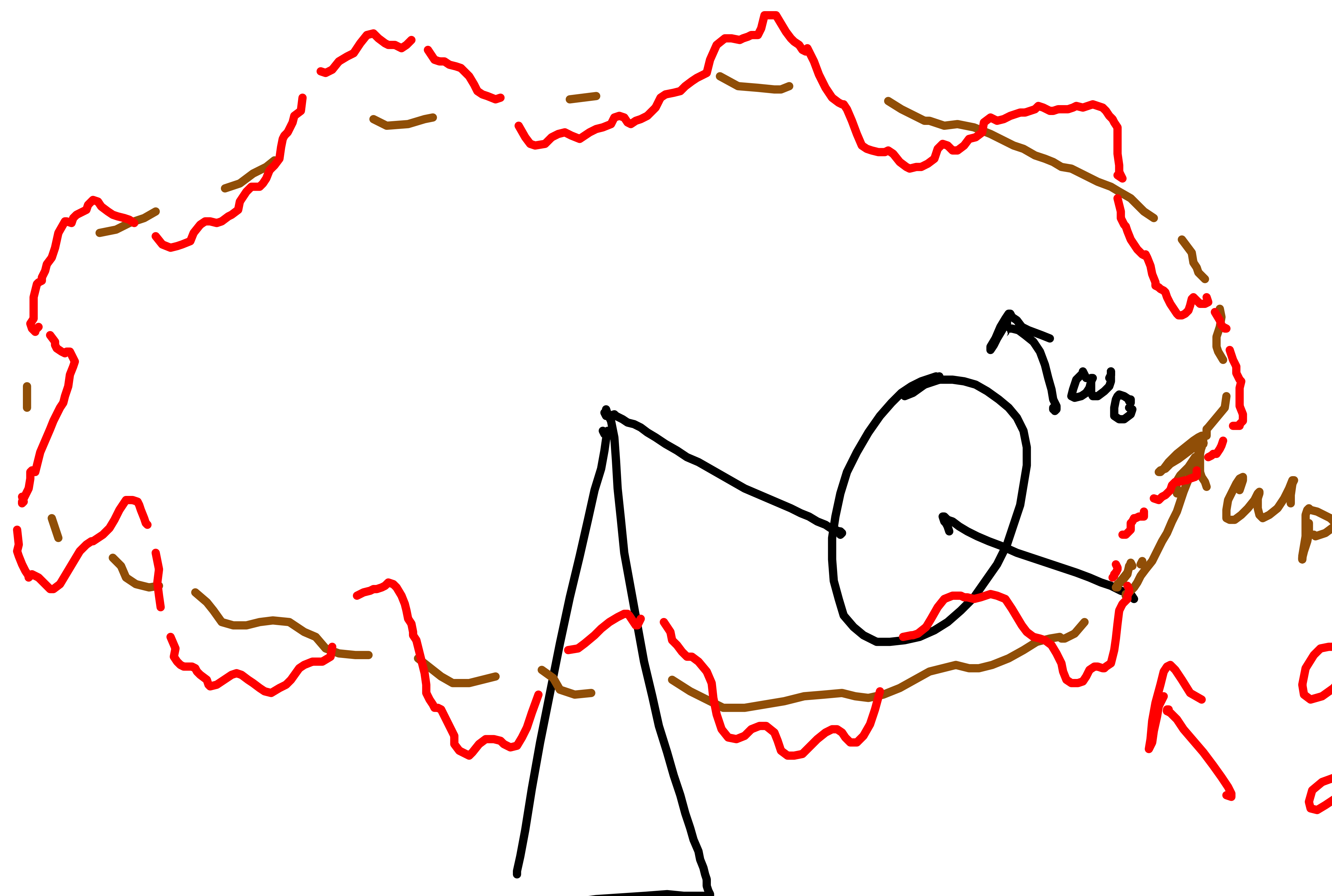
$I_z \neq I_x = I_y$ rotazione su z



NUTAZIONE

ω_p su piano xz

$\bar{\omega}_p \parallel \hat{j}$



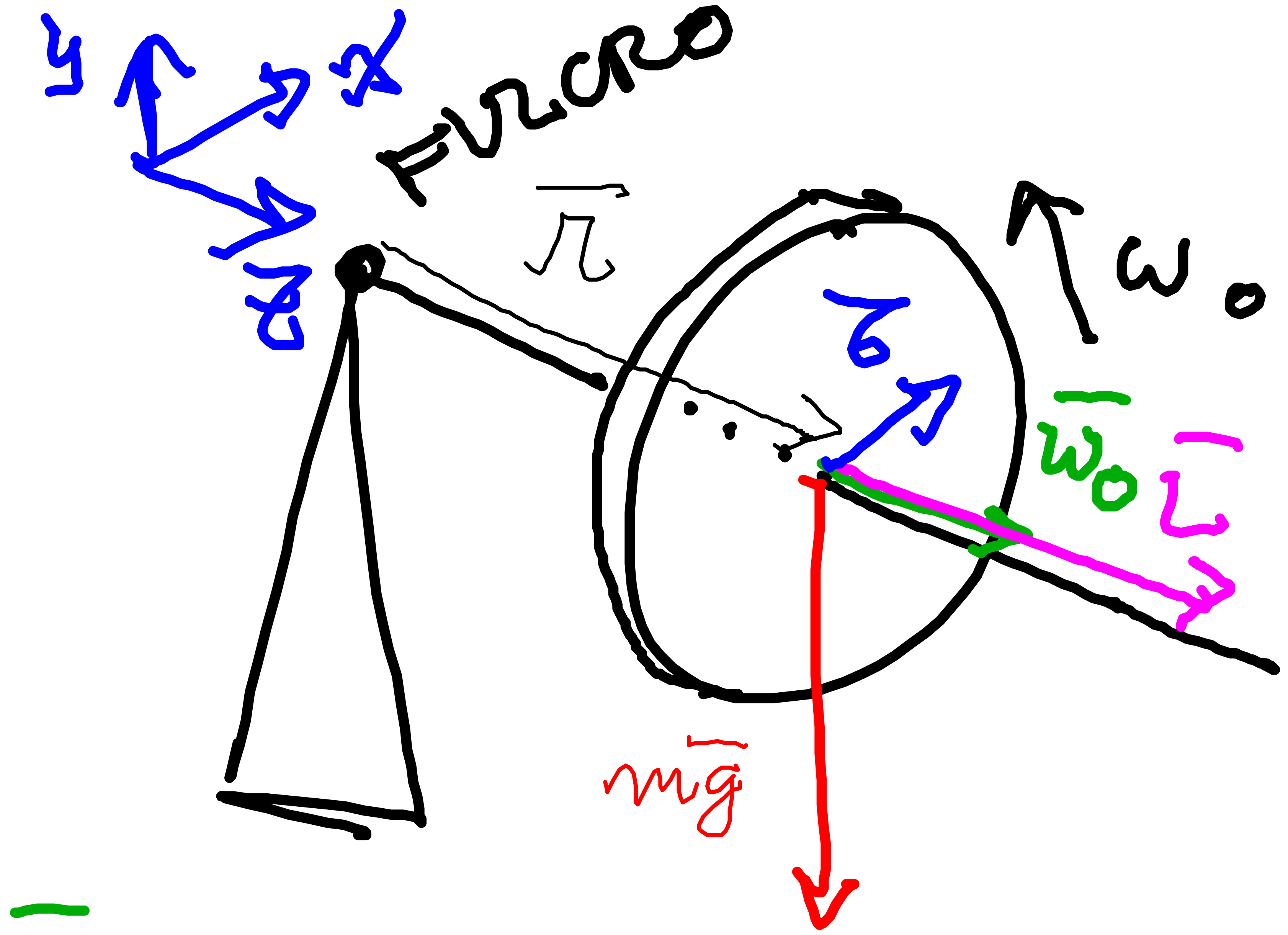
oscillazione attorno a traiettoria ω_p

NUTAZIONE

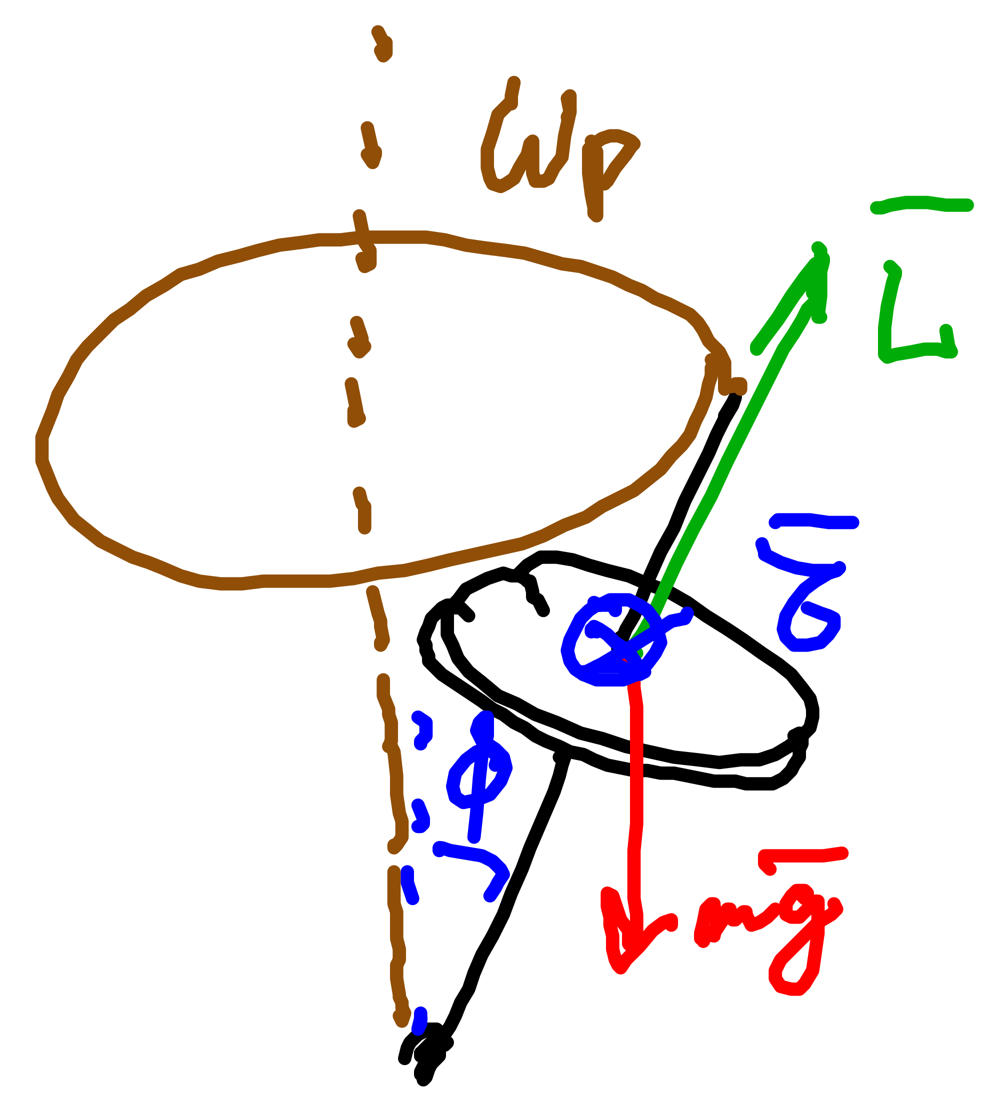
Giroscopio

oggetto a simmetria cilindrica

$I_z \neq I_x = I_y$ rotazione su z



TROTTOLA



$$\omega_p = \frac{\tau}{I\omega_0}$$

per attrito

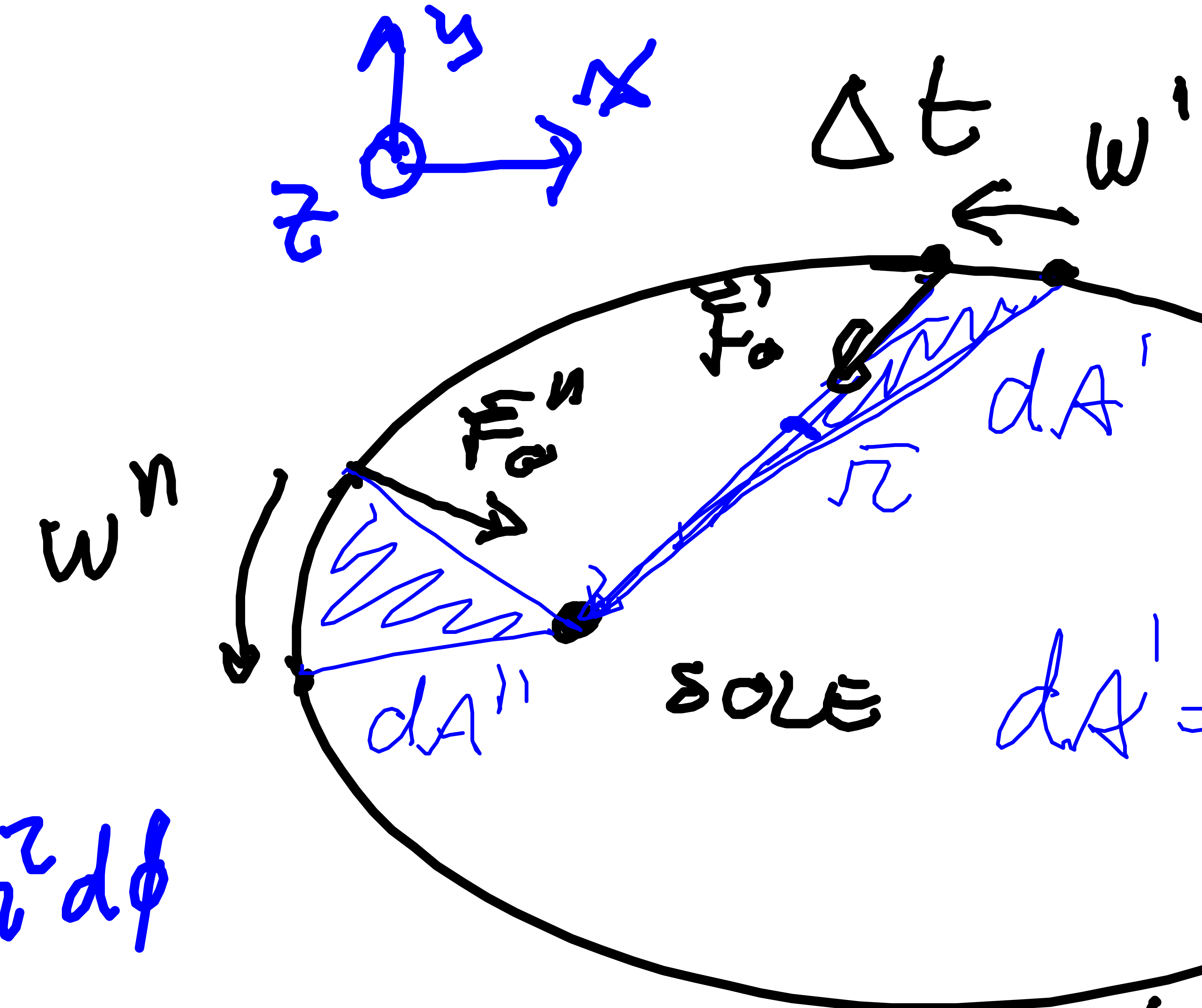
ω diminuisce

→ ω_p aumenta

→ ϕ aumenta

Dimostrazione Keplero II

Raggio vettore pianeta spazza aree uguali in tempi uguali



FORZE CENTRALI

$\bar{F}_o \parallel \bar{r}$

$\bar{\tau}_{F_o} = 0$

$\Delta t \to dl$

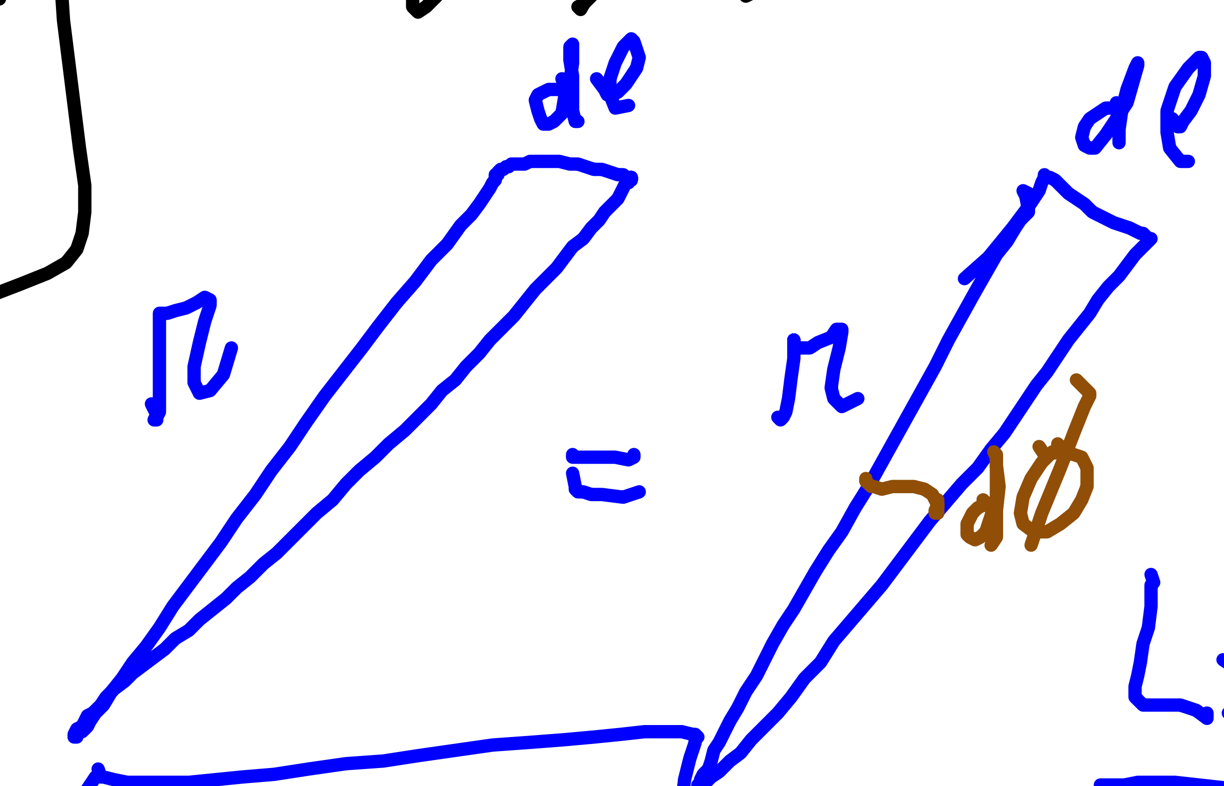
$$dA = \frac{1}{2} r\, dl = \frac{1}{2} r\, r\, d\phi = \frac{1}{2} r^2 d\phi$$

$w'' > w'$ $\quad (r')^2 (w') = (r'')^2 (w'')$

nell'orbita varia r

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} = \frac{1}{2} r^2(t)\, w(t)$$

cost $\quad \boxed{\frac{dA}{dt} = cost}$



il pianeta ha momento d'inerzia $\quad I(t) = M r^2(t)$

posso calcolare $\quad L_z(t) = I(t)\, w(t) = M r^2(t)\, w(t)$

$$\boxed{\bar{\tau} = \frac{d\bar{L}}{dt}} = 0$$

$\frac{L_z \text{ è cost}}{(r^2 w) \text{ è COSTANTE}}$

Dimostrazione Keplero II

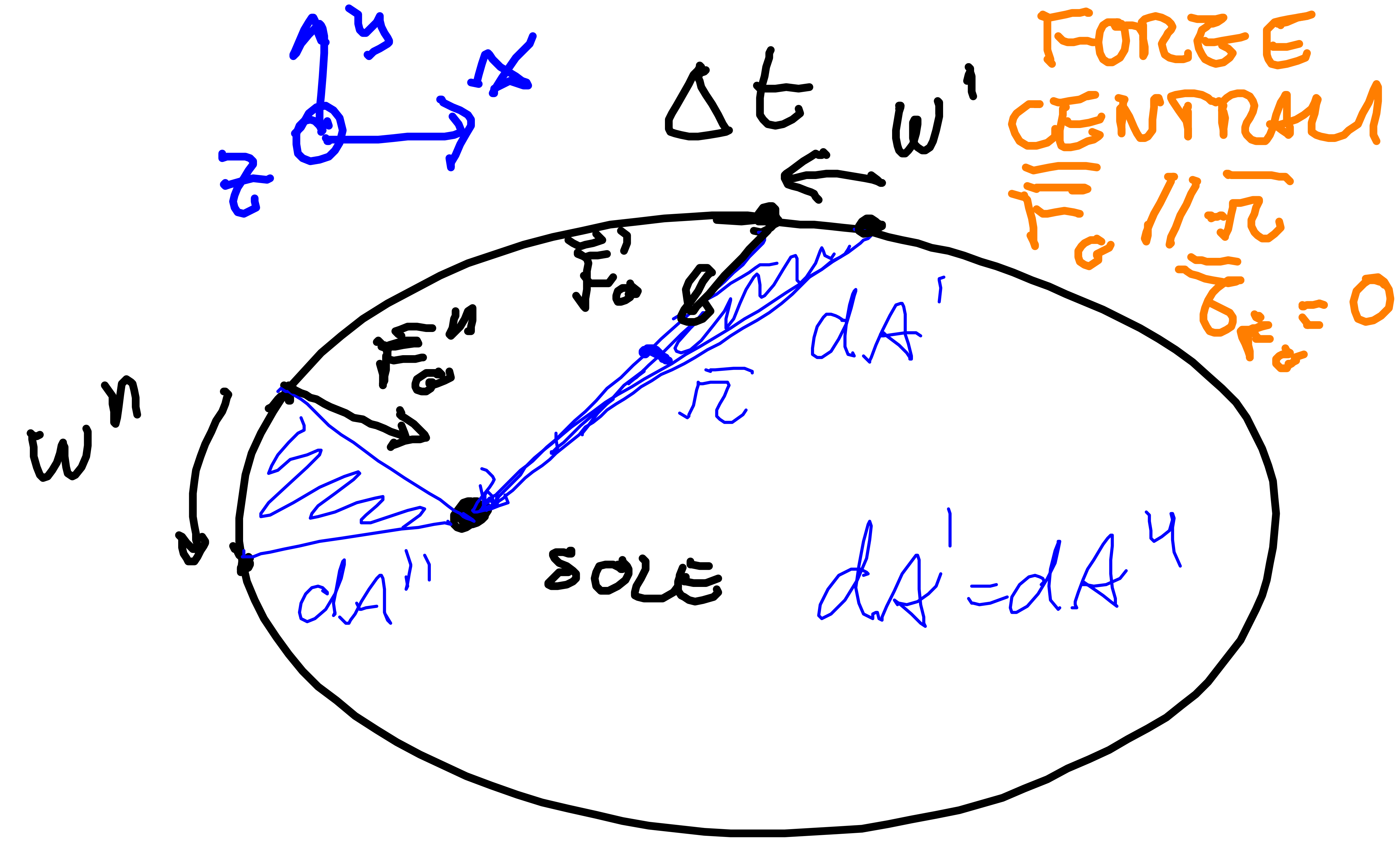
Raggio vettore pianeta spazza aree uguali in tempi uguali

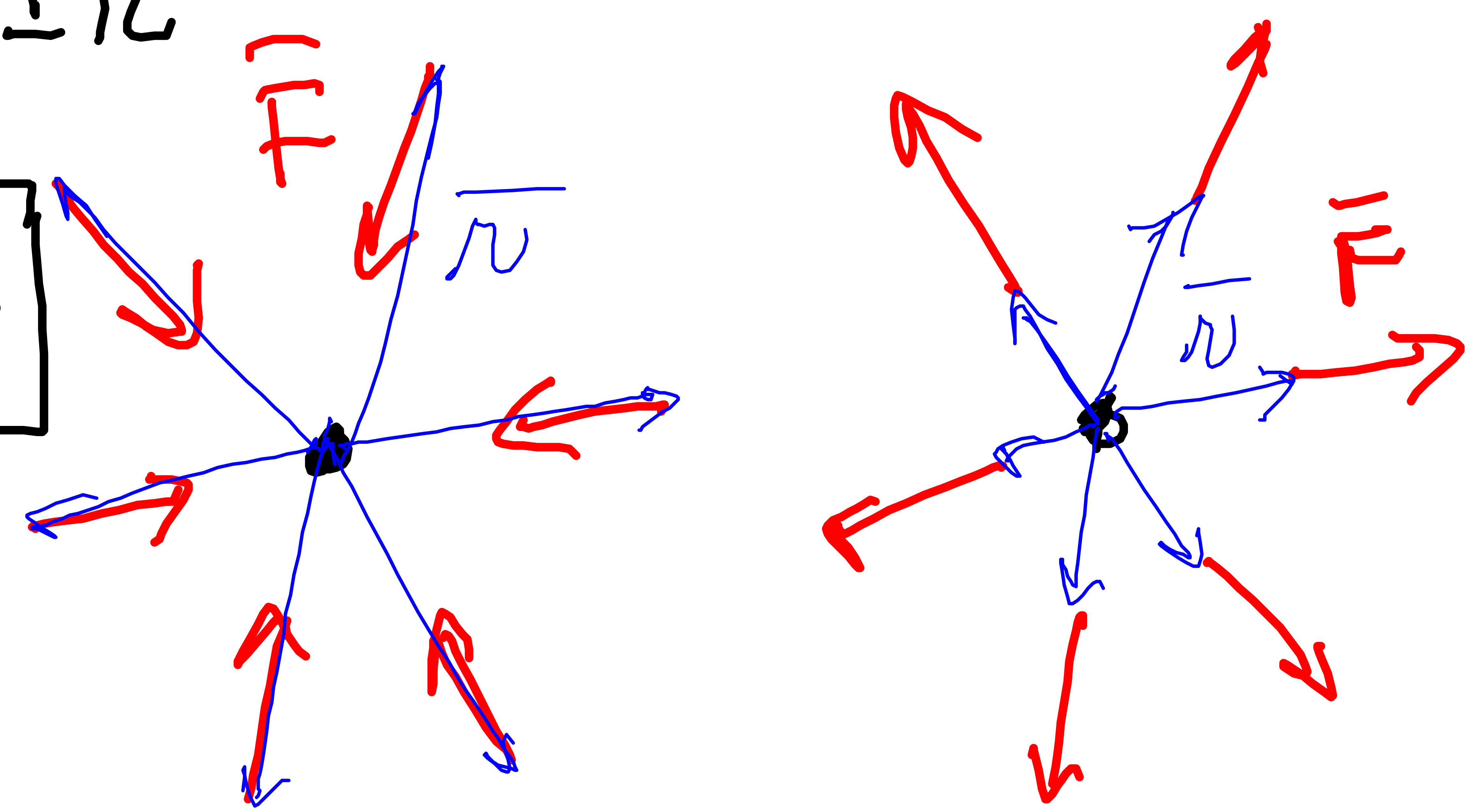
dL

FORZE CENTRALI

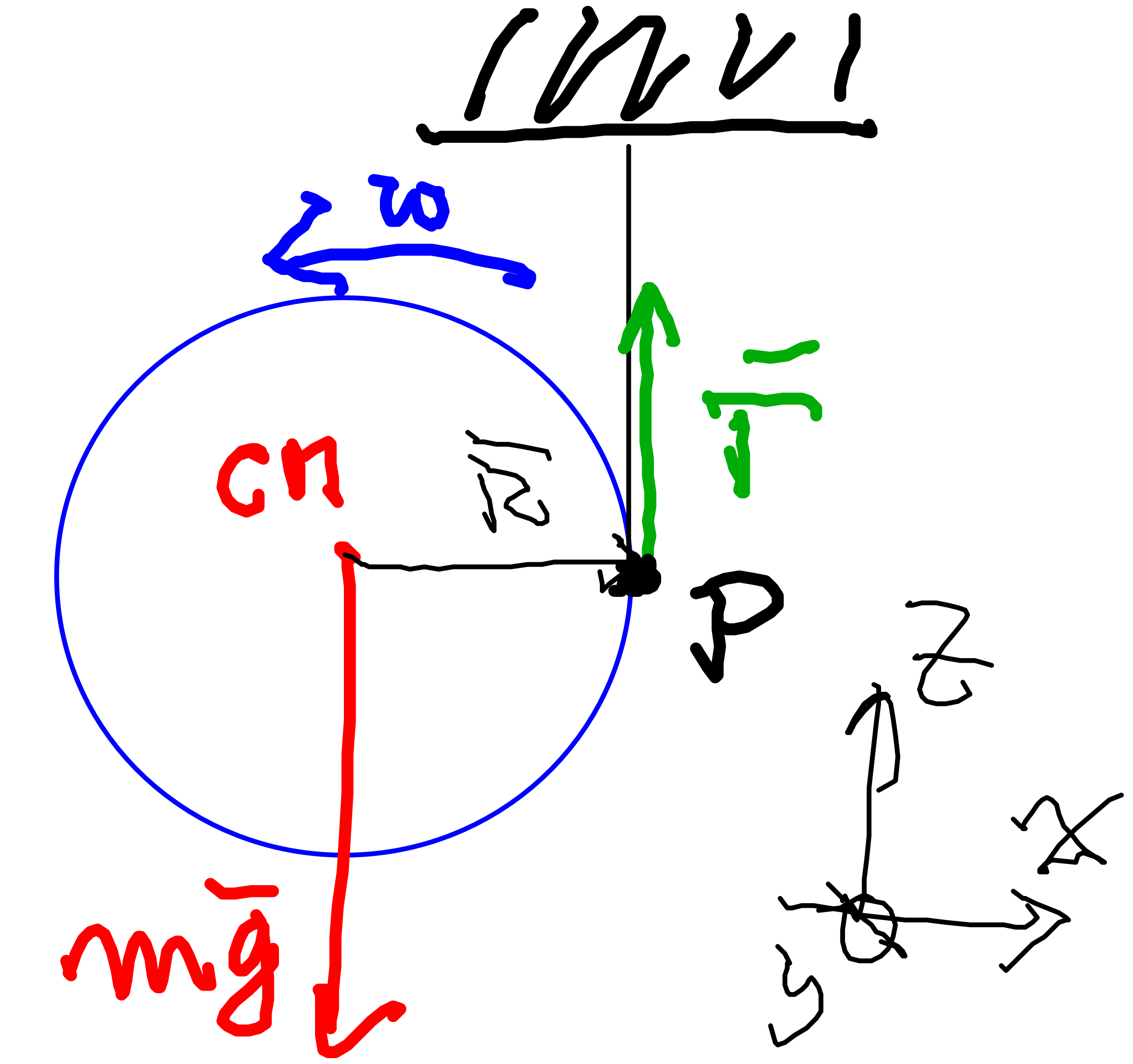
$\bar{F}_c \,//\, \pm\bar{r}$

$$\boxed{\bar{\tau}_{F_c} = \bar{r} \times \bar{F}_c = 0}$$





Problema II 4.7.19



YOYO = cilindro sospeso da una fune di massa trascurabile

$m = 0.17\ \mathrm{kg}$, $R = 7\ \mathrm{cm}$, $\ell = 73\ \mathrm{cm}$ (fune)

$v(0) = 0$ Yoyo cade

a) diagramma corpo b) v dopo aver svolto il filo

c) $\alpha(t) = ?$

$$\Sigma \bar{F} = -mg\hat{k} + T\hat{k} = m\bar{a} = ma(-\hat{k})$$

$$\Sigma \bar{\tau} = I\bar{\alpha}$$ nel punto P $$\bar{r} \times (m\bar{g}) = I\bar{\alpha} \qquad \alpha = \frac{mgR}{\frac{1}{2}mR^2}$$ ← NO!

asse rotazione non passa per P → Th. Huygens-Steiner

$$I_p = I_{cm} + I_{cm-P} = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

$$\boxed{\alpha = \frac{mgR}{\frac{3}{2}mR^2} = \frac{2}{3}\frac{g}{R}}$$

Problema II 4.7.19

YOYO = cilindro sospeso da una fune di massa trascurabile

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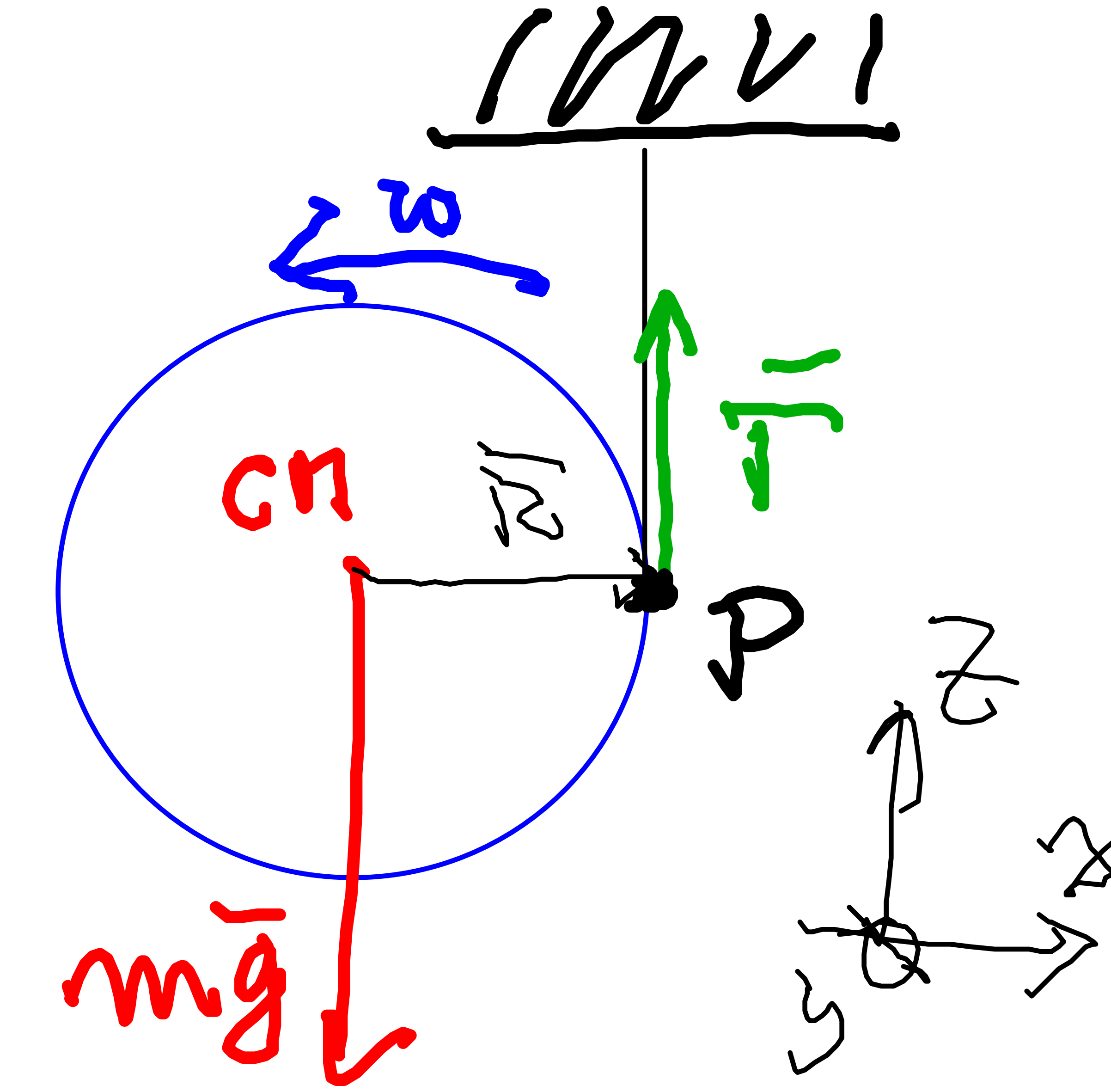

rotolamento $|\bar{a}| = |\bar{\alpha}| \cdot R = \frac{2}{3} g$

$$\begin{cases} \ell = \frac{1}{2} a t^2 \\ v = at \end{cases} \qquad \begin{cases} t = \sqrt{\frac{2\ell}{a}} = \sqrt{\frac{3\ell}{g}} \\ v = at = \frac{2}{3} g \sqrt{\frac{3\ell}{g}} = \end{cases}$$

$$\sqrt{\frac{4}{9} g^2 \frac{3\ell}{g}} = \sqrt{\frac{4}{3} g \ell} = 3.1\ \mathrm{m/s}$$

$$\boxed{\bar{v} = -\hat{z}(3.1)\ \mathrm{m/s}}$$

$$\alpha = \frac{mgR}{\frac{3}{2} m R^2} = \frac{2}{3} \frac{g}{R}$$

Problema II 4.7.19

YOYO = cilindro sospeso da una fune di massa trascurabile

$m = 0.17$ Kg, $R = 7$ cm, $\ell = 73$ cm (fune)

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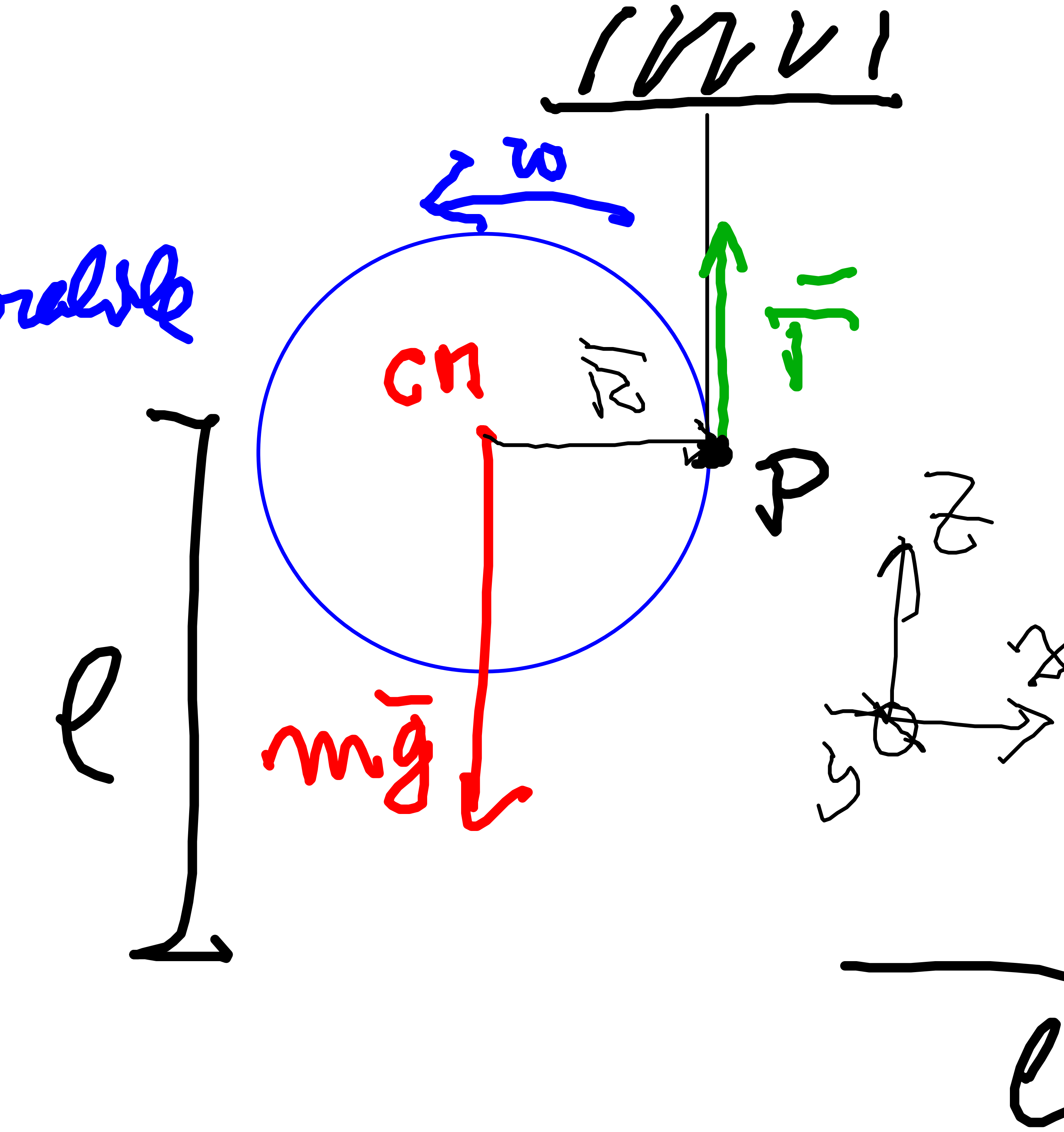


se cade

$$mg\ell = K_f^{tr} + K_f^{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v_{cm}^2 + \frac{1}{2}\frac{1}{2} m R^2 \frac{v_{cm}^2}{R^2} = \frac{3}{4} m v_{cm}^2$$

$$v_{cm} = \sqrt{\frac{4}{3}\ell g}$$

2

Problema VI 11.06.19

Uomo solidale con piattaforma girevole

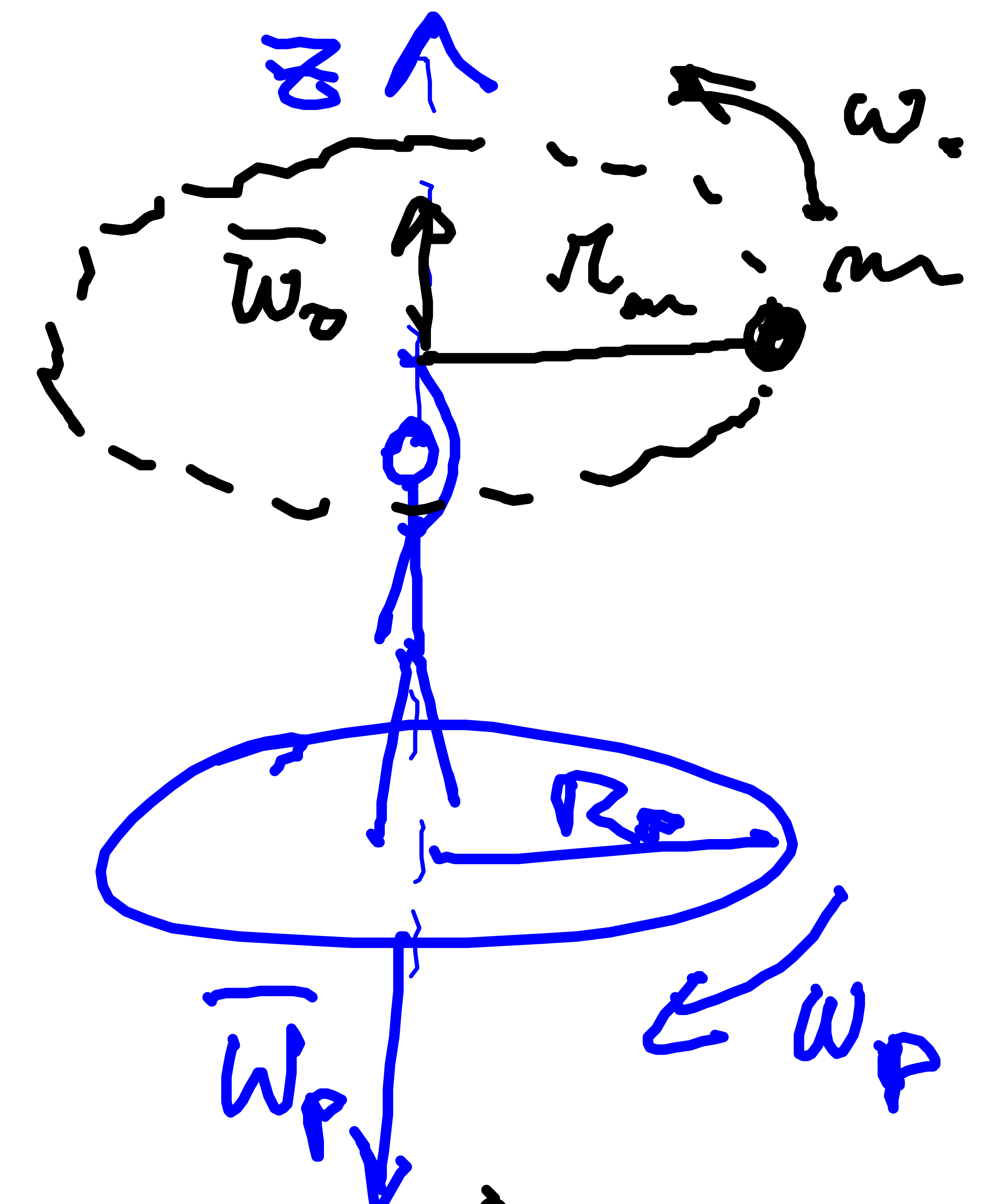
$R_p = 1.0\,m$, $M_p = 10\,kg$ in quiete

rotazione massa $m = 0.30\,kg$, $r_m = 1.0\,m$

ω_0 dall'esterno $= 21\,rad/s$

uomo $I_U = 1.1\,kg\,m^2$

a) $\omega_U = ?$ b) $W = ?$ c) $\omega_{0p} = ?$



$\vec{\omega}_p = -\hat{k}\,(1.0)\,rad/s$

L_z si conserva (non ha $\vec{\tau}$)

$$L_z^i = L_z^f \qquad 0 = L_z^{massa} + L_z^{uomo+piatt.} = I_m\omega + I_u(-\omega_p) + I_p(-\omega_p)$$

$$= m r_m^2 \omega_0 - I_u \omega_p - \frac{1}{2} M_p R_p^2 \omega_p$$

$$\omega_p\left(I_p + \frac{1}{2} M_p R_p^2\right) = m r_m^2 \omega_0$$

$$\boxed{\omega_p = \omega_0 \frac{m r_m^2}{I_p + \frac{1}{2} M_p R_p^2} = 1.0\,rad/s}$$

Problema VI 11.06.19

Uomo solidale con piattaforma girevole

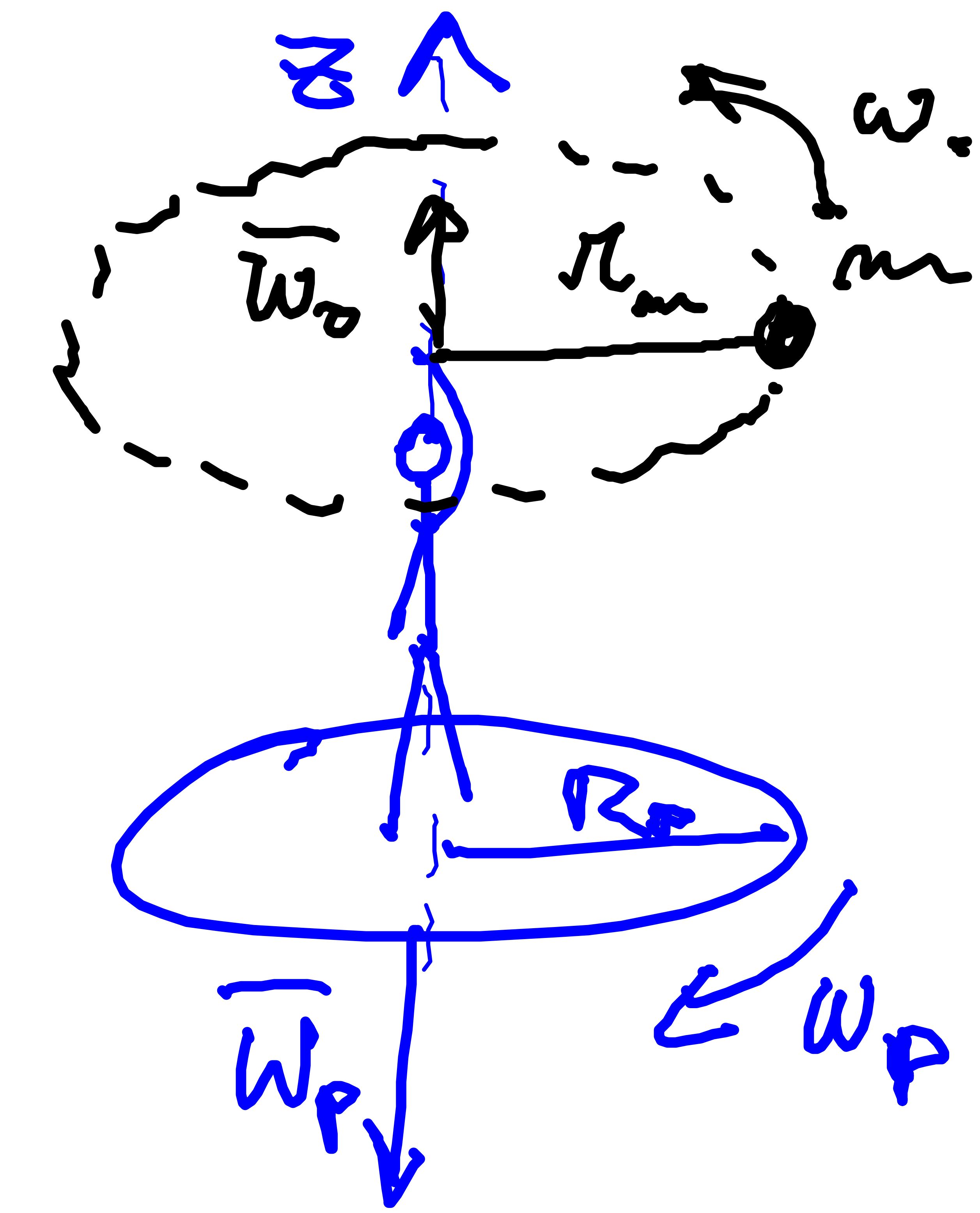
$R_p = 1.0\,m$, $M_p = 10\,Kg$ in quiete

rotazione massa $m = 0.30\,Kg$, $r_m = 1.0\,m$

ω_0 dall'esterno $= 21\,rad/s$

uomo $I_U = 1.1\,Kg\,m^2$

a) $\omega_U = ?$ b) $W = ?$ c) $\omega_{op} = ?$



$$W = \Delta K = K_f - K_i = K_f^{massa} + K_f^{uomo} + K_f^{piatt.} = \frac{1}{2}(m r_m^2)\,\omega_0^2 + \frac{1}{2} I_U \omega_p^2 + \frac{1}{2}\left(\frac{1}{2} M_p R_p^2\right)\omega_p^2 = \boxed{69\,J}$$

$$\vec{\omega}_{op} = \vec{\omega}_0 - \vec{\omega}_p = \omega_0 \hat{k} - \omega_p(-\hat{k}) = \hat{k}(\omega_0 + \omega_p) = \hat{k}\ 22\,rad/s$$