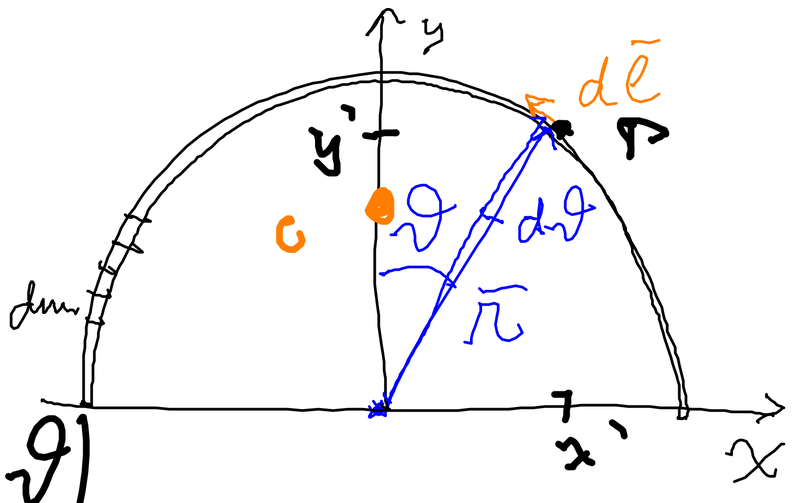
Mezzo anello

Trovare il centro di massa di un mezzo anello di raggio r e massa m



$$m : \pi r = dm : d\ell$$

$$dm = d\ell \frac{m}{\pi r} \lambda$$

$$P = (x', y') = (r\sin\vartheta ; r\cos\vartheta)$$

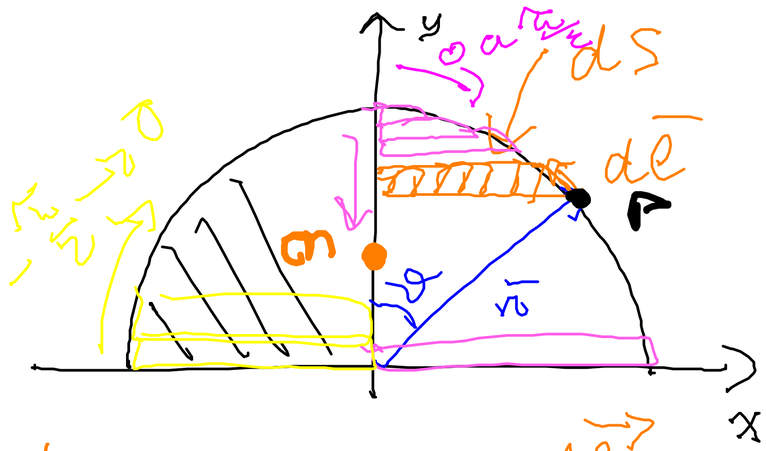
$$d\ell = r\, d\vartheta$$

$$x_{CM} = \frac{1}{m}\int_{TUTTO} x'\, dm = \frac{1}{m}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\sin\vartheta \frac{m}{\pi r} r\, d\vartheta = \frac{r}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\vartheta\, d\vartheta = 0$$

$$y_{CM} = \frac{1}{m}\int_{TUTTO} y'\, dm = \frac{1}{m}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\cos\vartheta \frac{m}{\pi r} r\, d\vartheta = \frac{r}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\vartheta\, d\vartheta = \frac{r}{\pi}\left[\sin\vartheta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2r}{\pi}$$

Mezzo disco

Trovare il centro di massa di un mezzo disco di raggio R e massa M

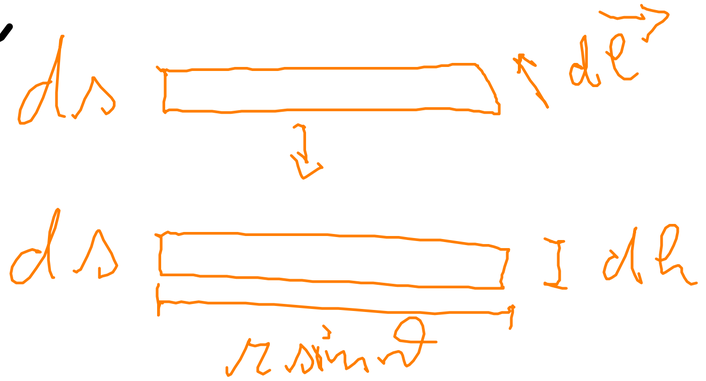


$$M : \frac{\pi R^2}{2} = dm : ds$$

$$dm = ds \left(\frac{2M}{\pi R^2}\right) \sigma$$

$$ds = r^2 \sin^2\vartheta \, d\vartheta$$

$$dl = r\, d\vartheta \qquad dh = r\sin\vartheta\, d\vartheta \qquad \vec{r} = (r\sin\vartheta;\ r\cos\vartheta)$$



$$dh = dl \sin\vartheta$$

$$x_{CM} = 0 \quad \text{per simmetria}$$

$$y_{CM} = \frac{1}{M}\int_{\text{mezzo disco}} y'\, dm = \frac{1}{M}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\cos\vartheta \, r^2 \sin^2\vartheta \, \frac{2M}{\pi R^2}\, d\vartheta$$

$$= \frac{2R}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\vartheta\cos\vartheta\, d\vartheta = \frac{2R}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\vartheta\, d(\sin\vartheta) = \frac{2R}{\pi}\left[\frac{\sin^3\vartheta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{3}\frac{R}{\pi}$$

Mezza sfera

Trovare il centro di massa di un mezza sfera di raggio R e massa M

$$m : \frac{2}{3}\pi R^3 = dm : dV$$

$$dm = dV \frac{3m}{2\pi R^3}$$

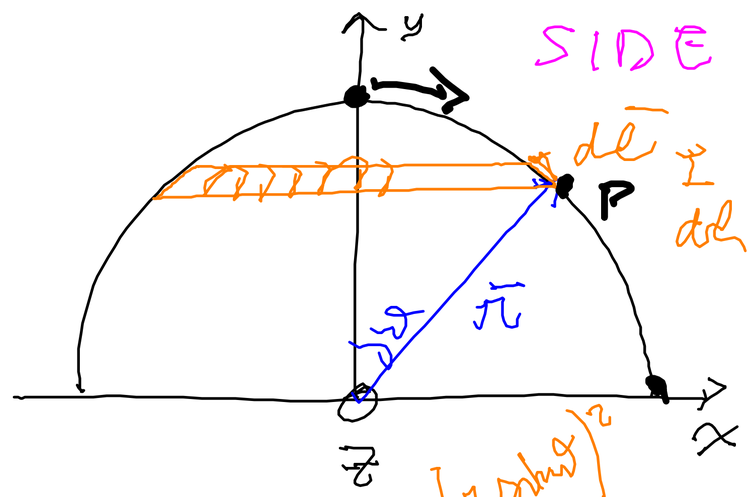
$$dV = dh\, \pi r^2 \sin^2\vartheta = \pi R^3 \sin^3\vartheta\, d\vartheta \qquad dl = r\, d\vartheta$$

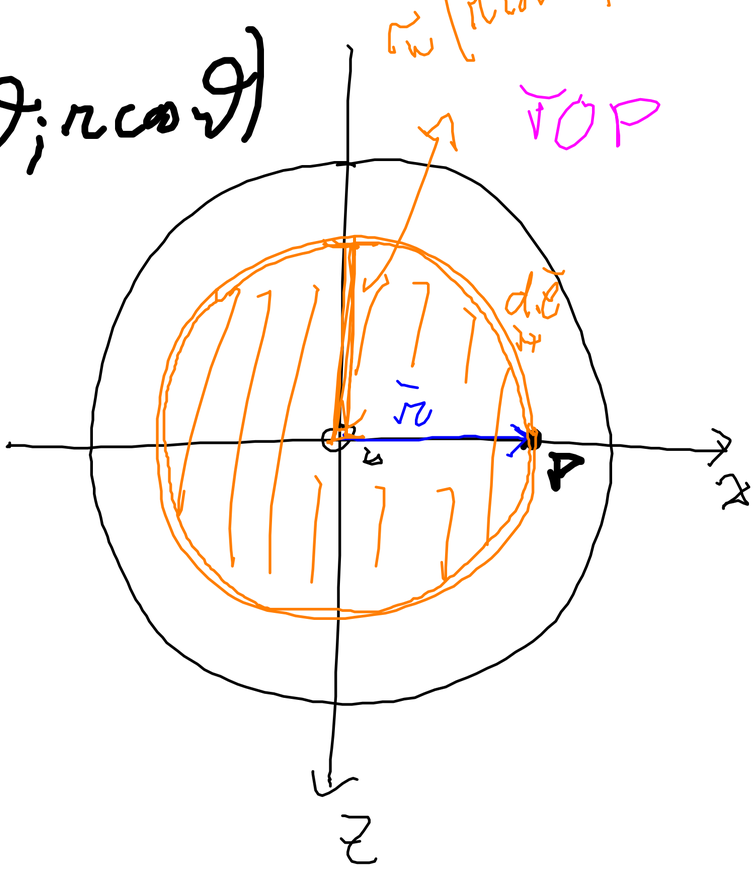
$$P = (r\sin\vartheta ; r\cos\vartheta)$$

$x_{cm} = 0$, $z_{cm} = 0$ per simmetria

$$y_{cm} = \frac{1}{m}\int_{TUTTO} y'\, dm = \frac{1}{m}\int_0^{\frac{\pi}{2}} r\cos\vartheta \, \frac{3m}{2\pi R^3}\, \pi R^3 \sin^3\vartheta\, d\vartheta$$

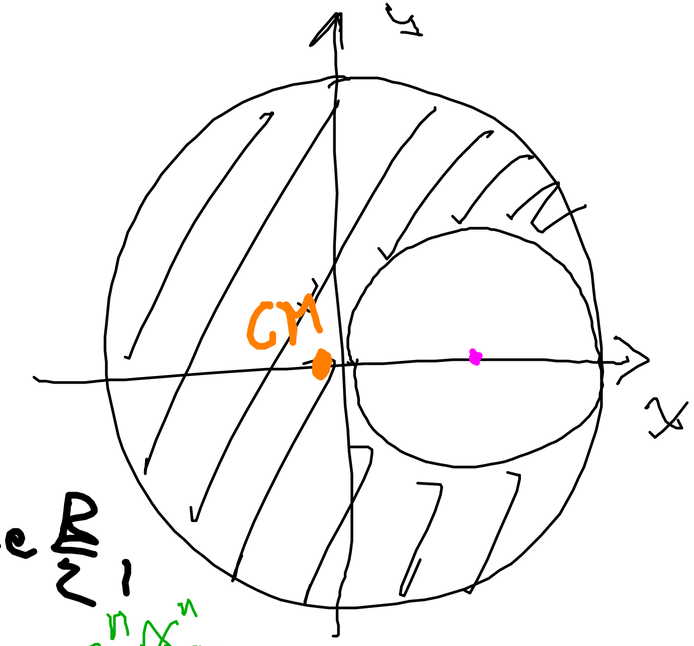
$$= \frac{3R}{2}\int_0^{\frac{\pi}{2}} \sin^3\vartheta\cos\vartheta\, d\vartheta = \frac{3R}{2}\left[\frac{\sin^4\vartheta}{4}\right]_0^{\frac{\pi}{2}} = \frac{3}{8}R$$





Sfera con foro

Determinare il C.M. di una sfera di raggio $R = 70\,cm$ al cui interno è stata tolta una sfera di raggio $\frac{R}{2}$ tangente alla sfera maggiore



$y_{cm} = 0$ simmetria

è come se avessi due sfere uniformi con raggio R e $\frac{R}{2}$, una con densità ρ e l'altra con densità $-\rho$

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$m \cdot x_{cm}$

$\rho \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 \cdot \frac{R}{2}$

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x' dm' + \int x'' dm''}{\rho \frac{4}{3}\pi R^3 - \rho \frac{4}{3}\pi \left(\frac{R}{2}\right)^3} = \frac{0 + \rho \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 \cdot \frac{R}{2}}{\rho \frac{4}{3}\pi R^3 \left(1 - \frac{1}{8}\right)}$$

$$= \frac{R/16}{7/8} = \frac{R}{14}$$

Problema II 13.09.18

Asta NON OMOGENEA $l = 101\,cm$ e massa posa

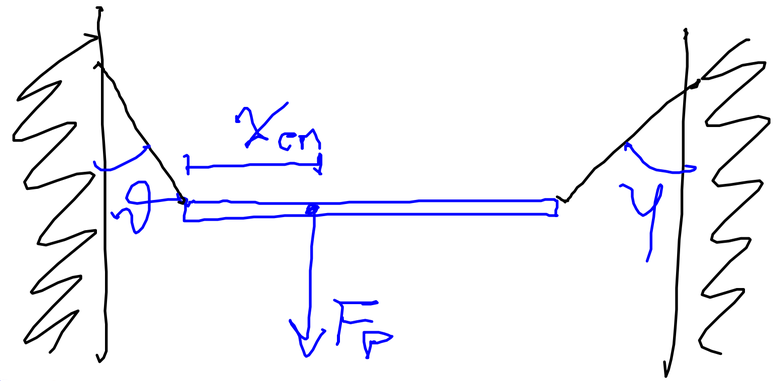
$F_P = 2.0\,N$ sospesa inquiete

$\vartheta = 30°$, $\varphi = 45°$

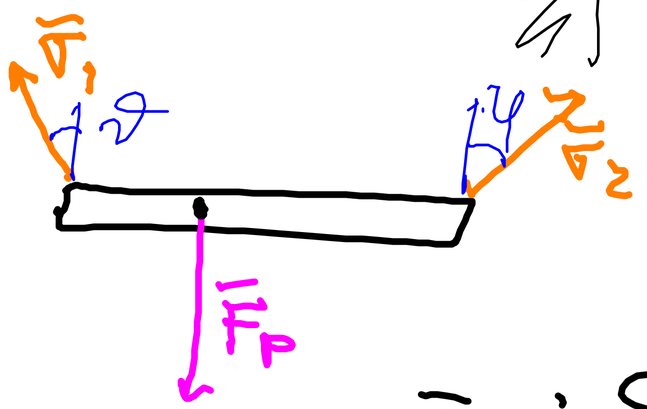
a) diagramma forze = ?

b) condizioni eq. statica e moduli forze = ?

c) determinare x_{CM} = ?



a)



b) eq. statica

$$\text{FORZE:} \begin{cases} \bar{T}_1 + \bar{T}_2 + \bar{F}_P = 0 \\ \bar{\tau}_{T_1} + \bar{\tau}_{T_2} + \bar{\tau}_{F_P} = 0 \end{cases} \quad \text{MOMENTI:}$$

su x: $-T_1 \sin\vartheta + T_2 \sin\varphi = 0$

su y: $T_1 \cos\vartheta + T_2 \cos\varphi - F_P = 0$

$$T_1 = T_2 \frac{\sin\varphi}{\sin\vartheta}$$

$$T_2 \frac{\sin\varphi}{\sin\vartheta}\cos\vartheta + T_2\cos\varphi - F_P = 0$$

$$T_2 = F_P \Big/ \left(\frac{\sin\varphi}{\tan\vartheta} + \cos\varphi\right) \simeq 1.04\,N \simeq \boxed{1.0\,N}$$

Problema II 13.09.18

Asta NON OMOGENEA $l = 1.01\,m$ e massa posta

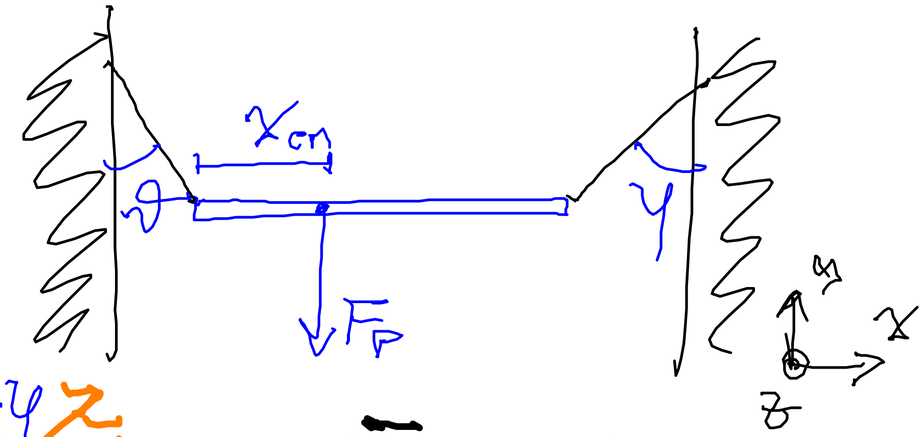
$F_P = 2.0\,N$ sospesa inquinate

$\vartheta = 30°$, $\varphi = 45°$

a) diagramma forze = ?

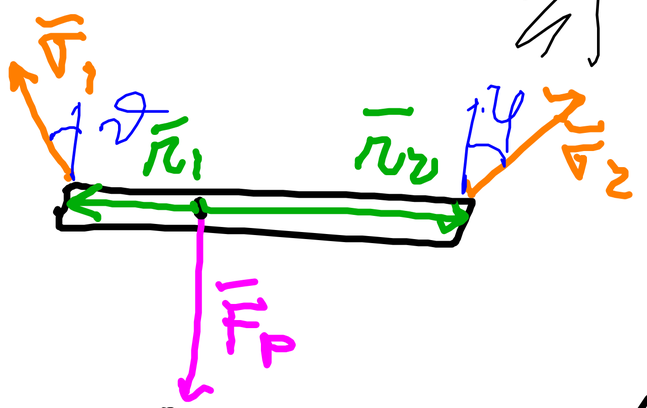
b) condizioni eq. statica e moduli forze = ?

c) determinare x_{CM} = ?



$\bar{\tau} = \bar{r} \times \bar{F}$

a)



b) eq. statica

FORZE: $\bar{T}_1 + \bar{T}_2 + \bar{F}_P = 0$

MOMENTI: $\bar{\tau}_{T_1} + \bar{\tau}_{T_2} + \bar{\tau}_{F_P} = 0$

c) su z: calcolo i momenti su CM $(\tau_{F_P} = 0)$

$\bar{\tau}_{T_1} + \bar{\tau}_{T_2} = 0$

$\bar{\tau}_{T_1} = -\hat{k}\, x_{CM}\, T_1 \cdot \sin\left(\frac{\pi}{2} - \vartheta\right)$

$\bar{\tau}_{T_2} = \hat{k}\,(l - x_{CM})\, T_2 \sin\left(\frac{\pi}{2} - \varphi\right)$

$$-x_{CM} T_1 \cos\vartheta + (l - x_{CM}) T_2 \cos\varphi = 0 \rightarrow x_{CM} = l \frac{\cos\varphi}{\frac{\sin\varphi}{\tan\vartheta} + \cos\varphi} \simeq 0.37\,m$$