$$\Delta U \equiv U_f - U_i \equiv -\int_{x_i}^{x_f} F_x(x)\, dx$$

Con questa def

$U(x)$ definita a meno di una cost. arbitraria nel caso solo forze cons.

$$K_f - K_i = W_{tot} = -(U_f - U_i)$$

conservazione en. mecc. E

$$E = K + U$$

$$K_f + U_f = K_i + U_i \qquad E_f = E_i$$

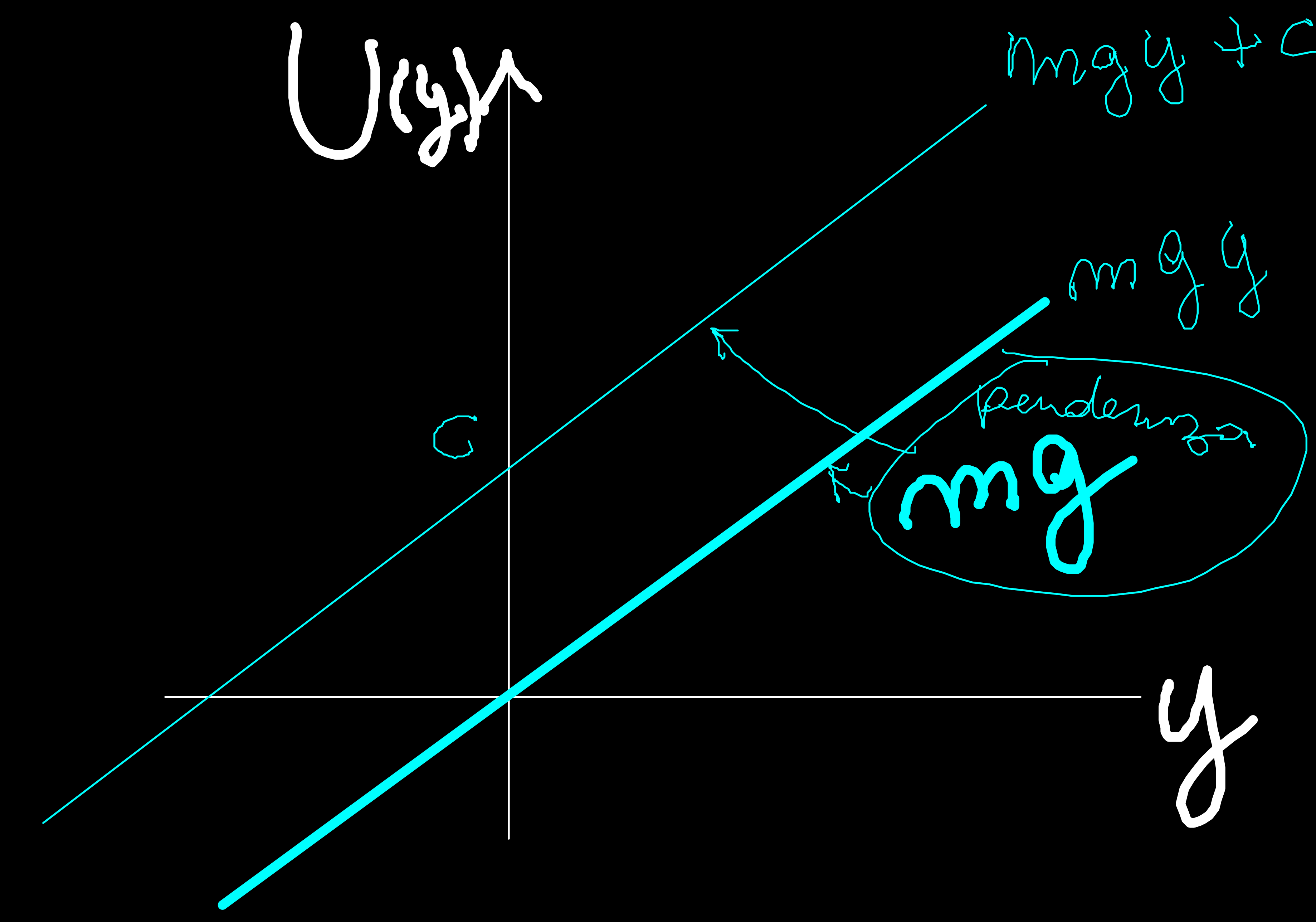
1) $$U_f - U_i = mgy_f - mgy_i$$

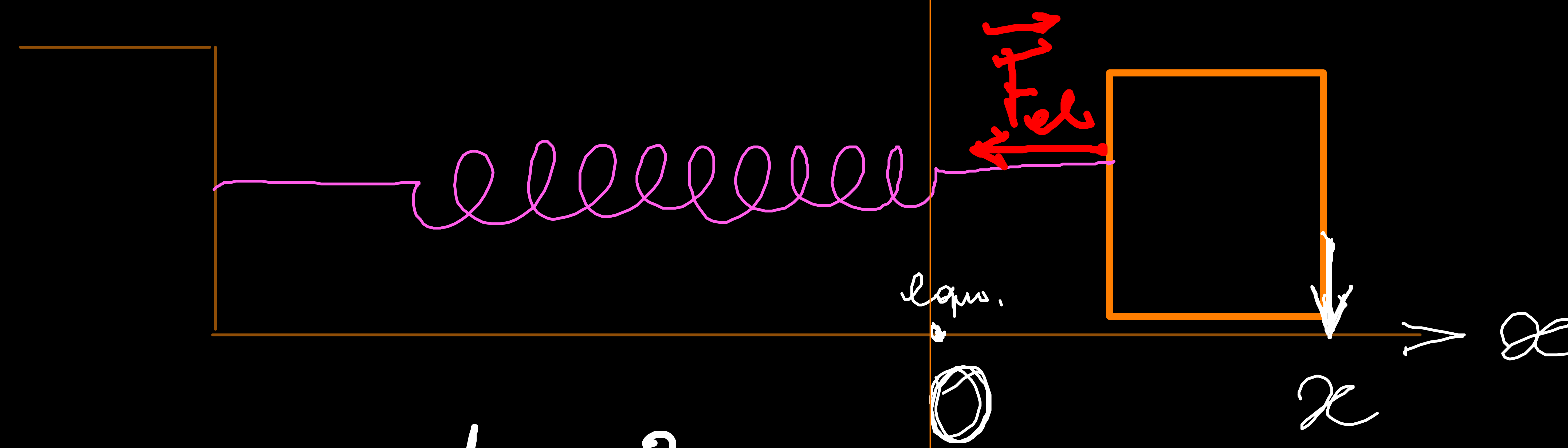
$$U(y) = mgy + C$$ ← cost arb.

$y = 0$ assumo arbitrariamente

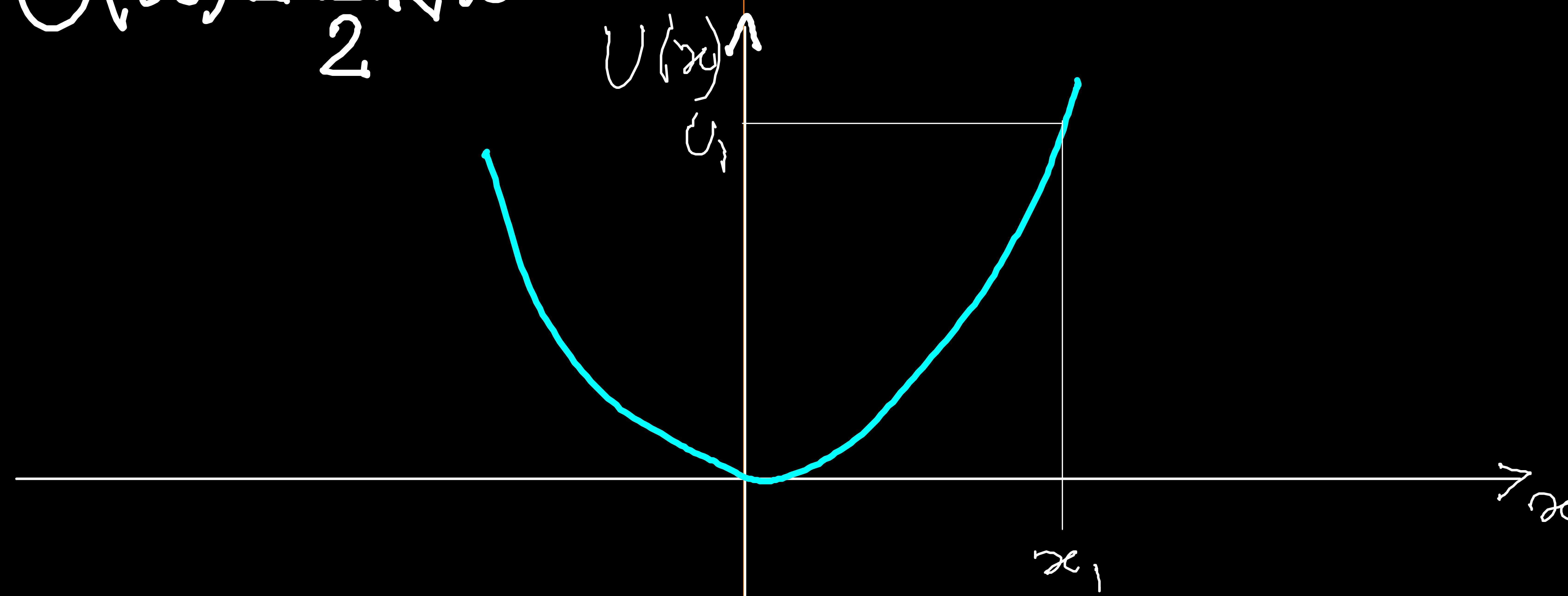
$$U(y=0) = 0 \qquad C = 0$$

$$\longrightarrow U(y) = mgy$$

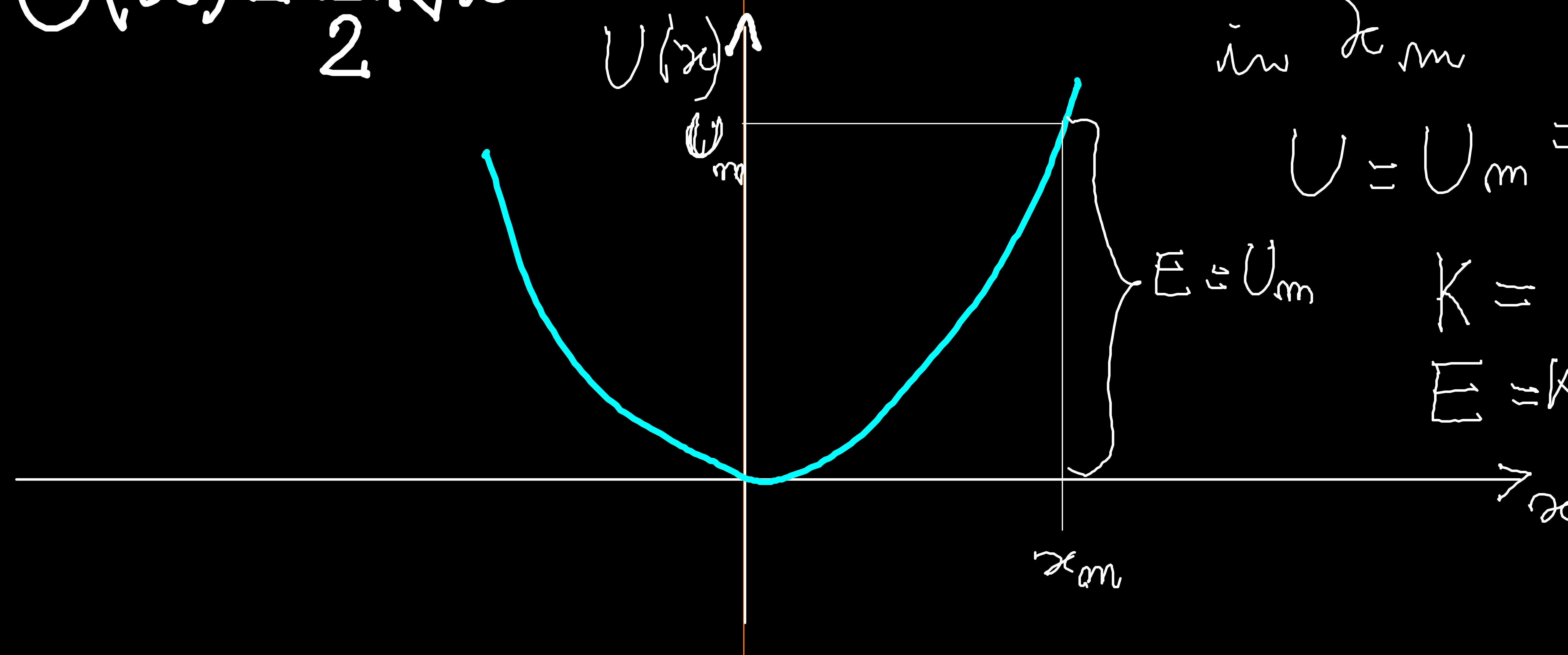




$$U(x) = \frac{1}{2} k x^2$$



$$U(x) = \frac{1}{2} k x^2$$

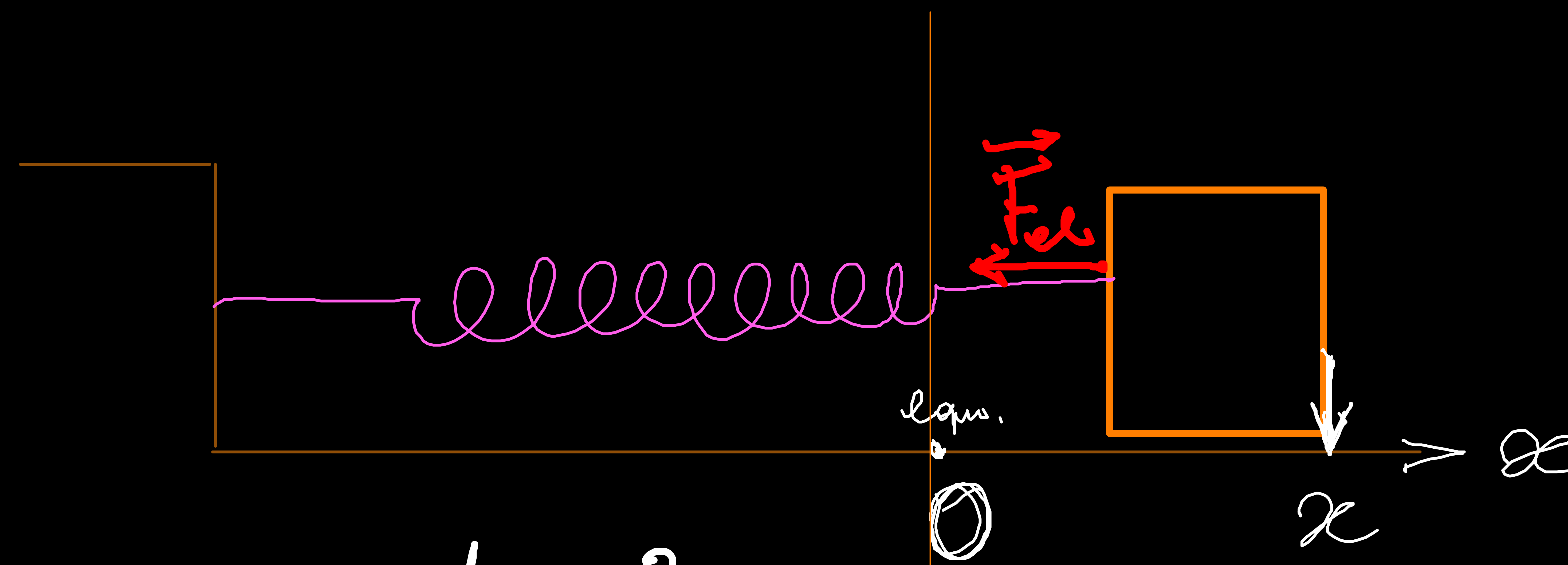


in x_m

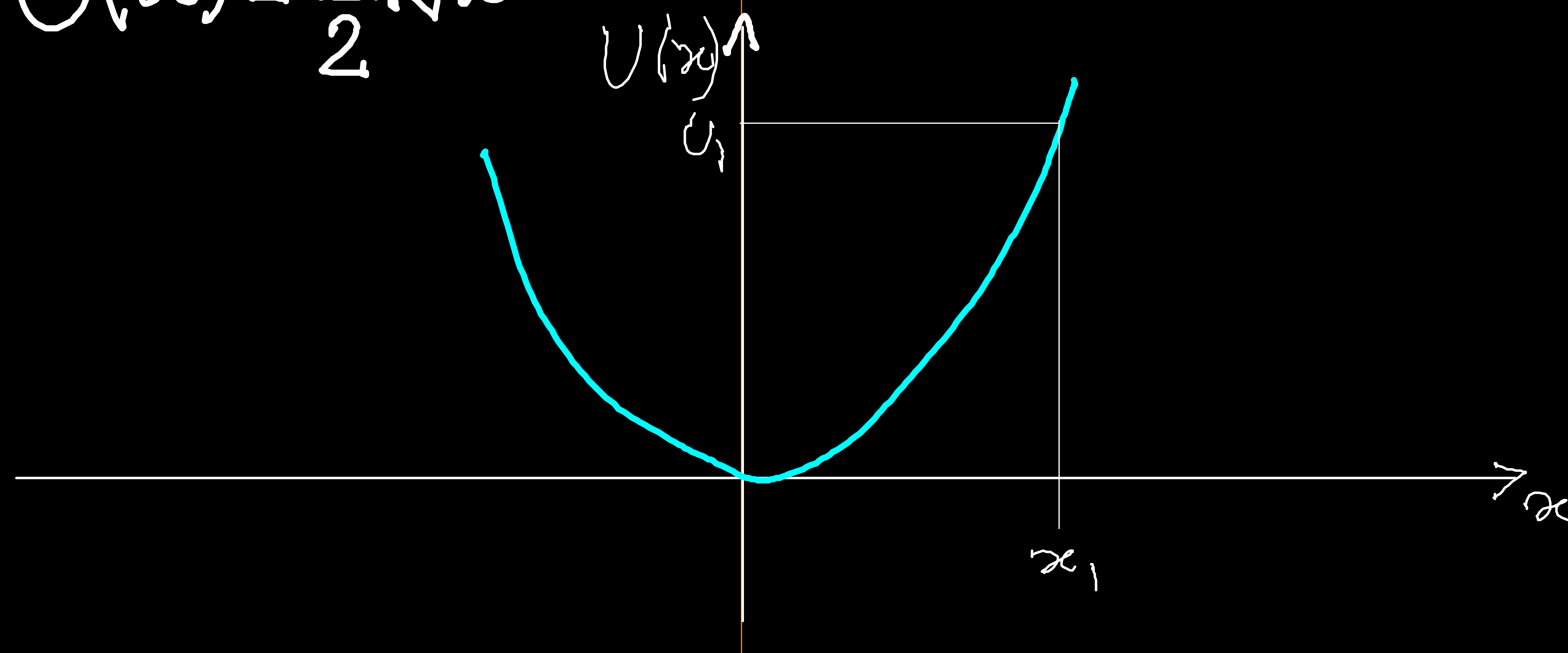
$$U = U_m = \frac{1}{2} k x_m^2$$

$$K = 0$$

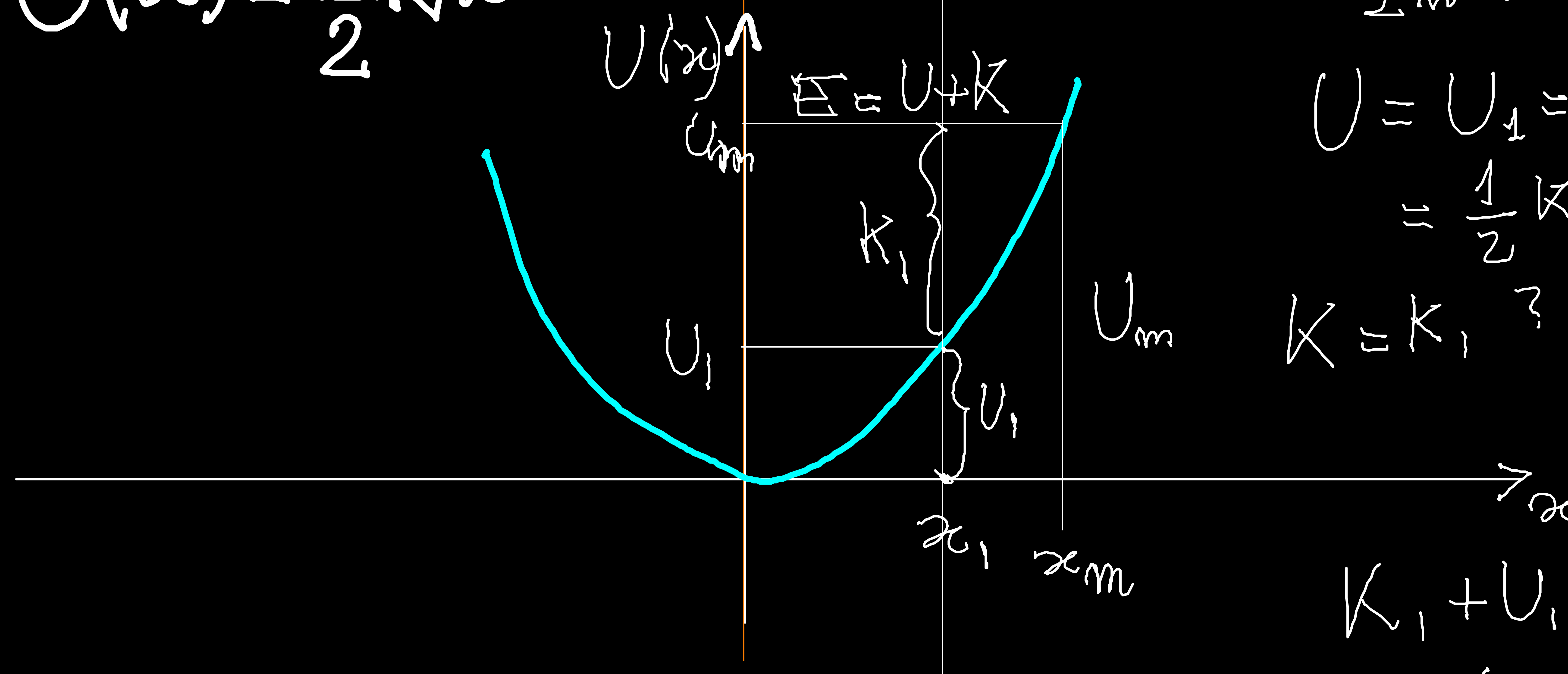
$$E = K + U = U_m$$



$$U(x) = \frac{1}{2} k x^2$$



$$U(x) = \frac{1}{2} k x^2$$



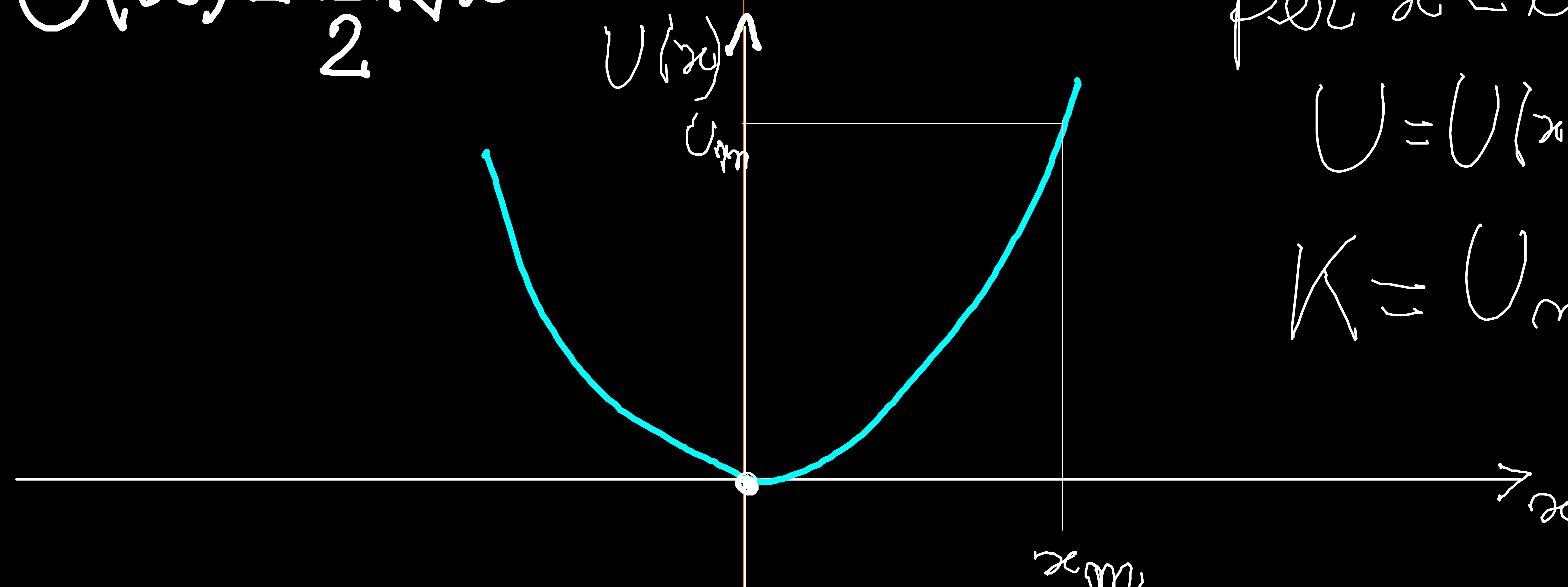
In $x = x_1$

$$U = U_1 = U(x_1)$$
$$= \frac{1}{2} k x_1^2$$

$K = K_1$?

$$K_1 + U_1 = E = U_m$$
$$K_1 = U_m - U_1$$

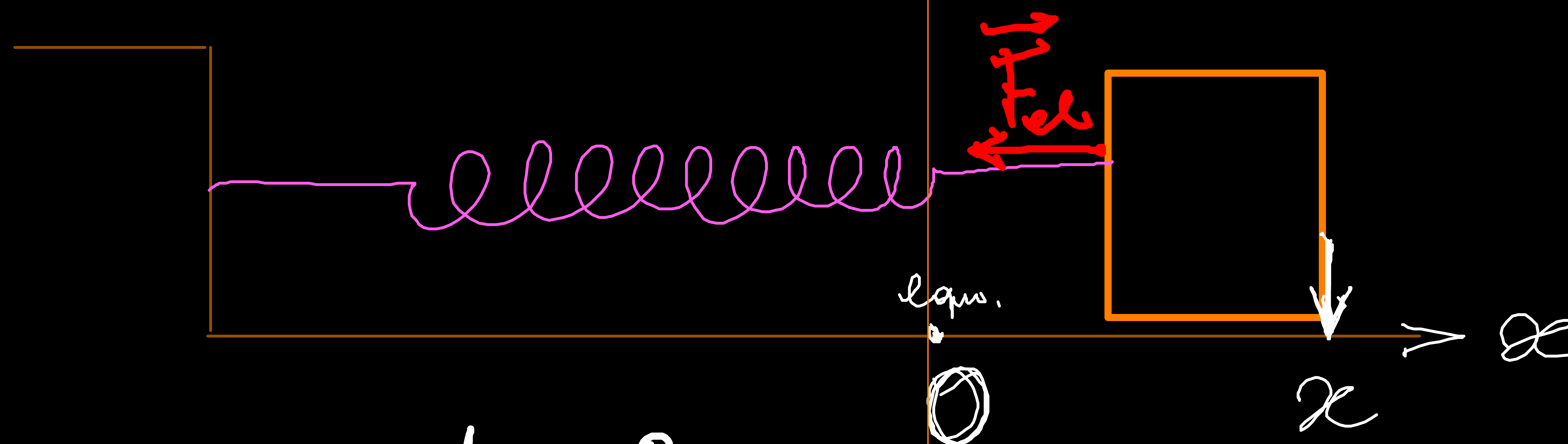
$$U(x) = \frac{1}{2} k x^2$$



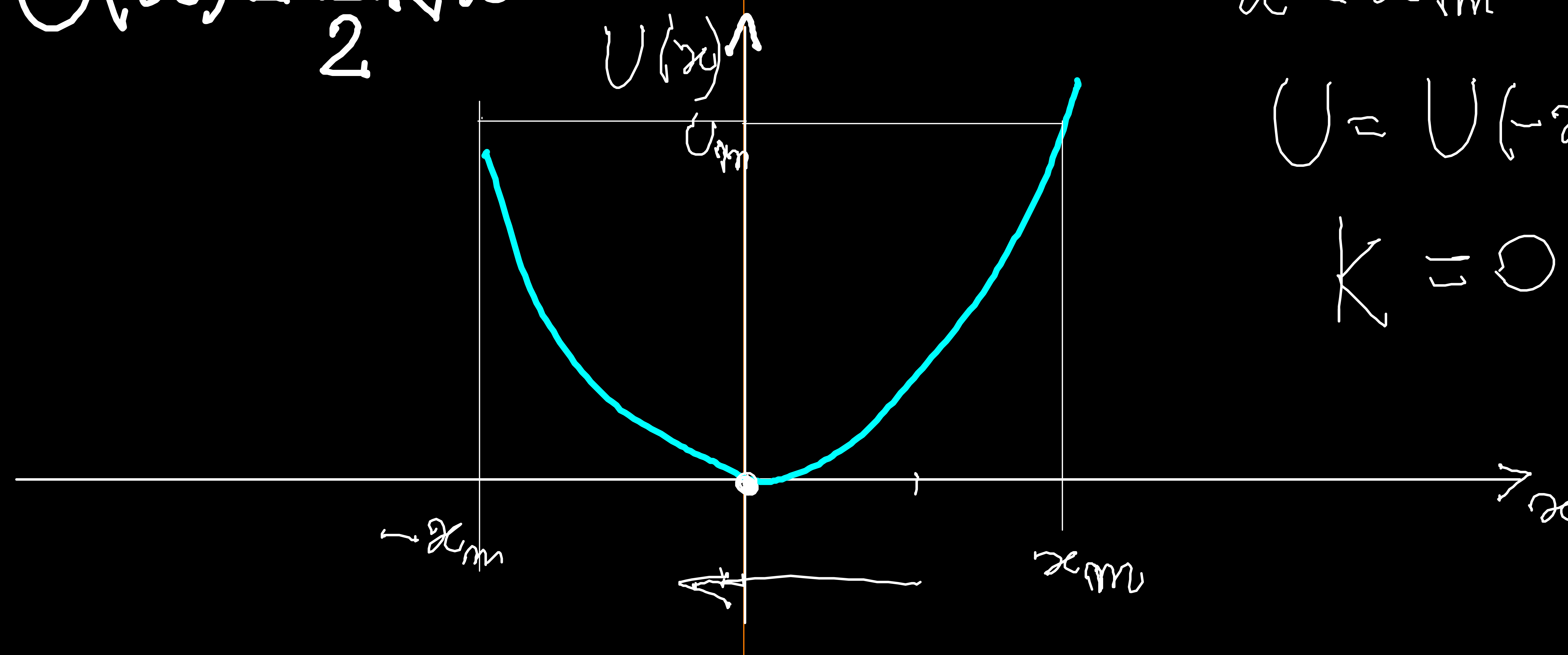
per $x = 0$

$$U = U(x=0) = 0$$

$$K = U_m$$



$$U(x) = \frac{1}{2} k x^2$$



$$x = -x_m$$

$$U = U(-x_m) = U_m$$

$$K = 0$$

Viceversa posso usare $U(x)$

per ricavare $\vec{F} = ?$

$$\Delta U = -W = -\int F_x(x)\, dx$$

diretta

$$U(x) = \ldots + C$$

integrazione

Se invece conosco $U(x)$ posso fare operazione inversa?

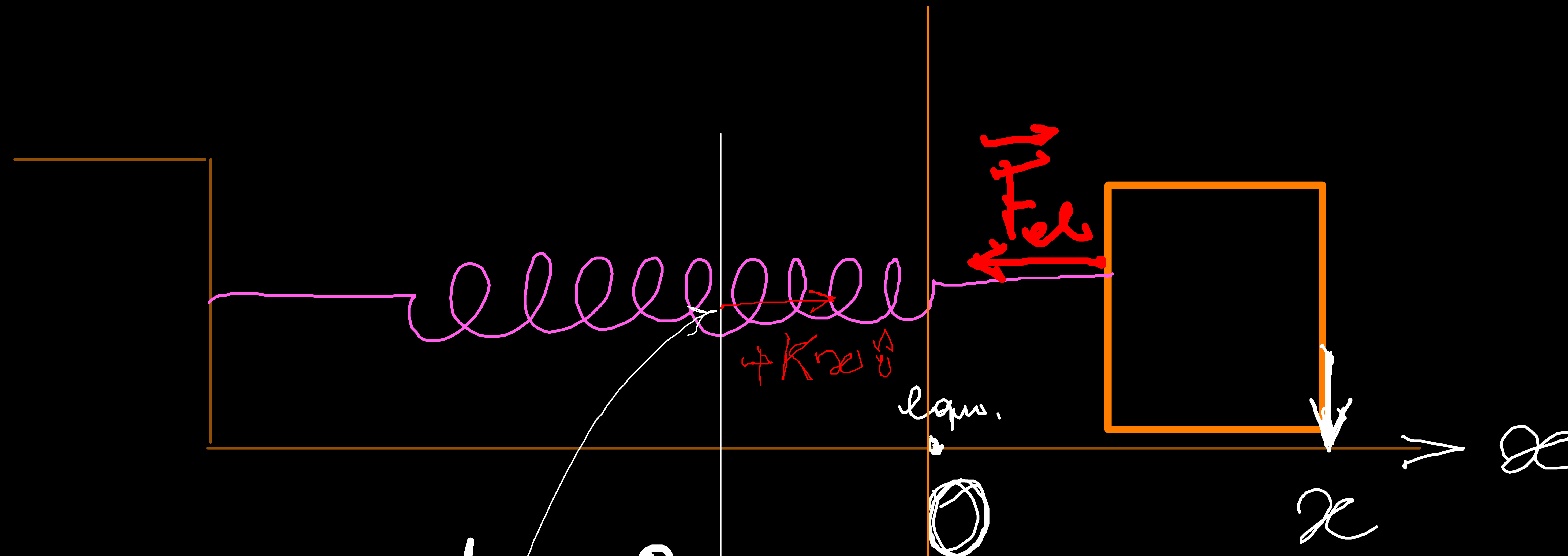
Si:

$$dU = -dW = -F_x(x)\,dx$$

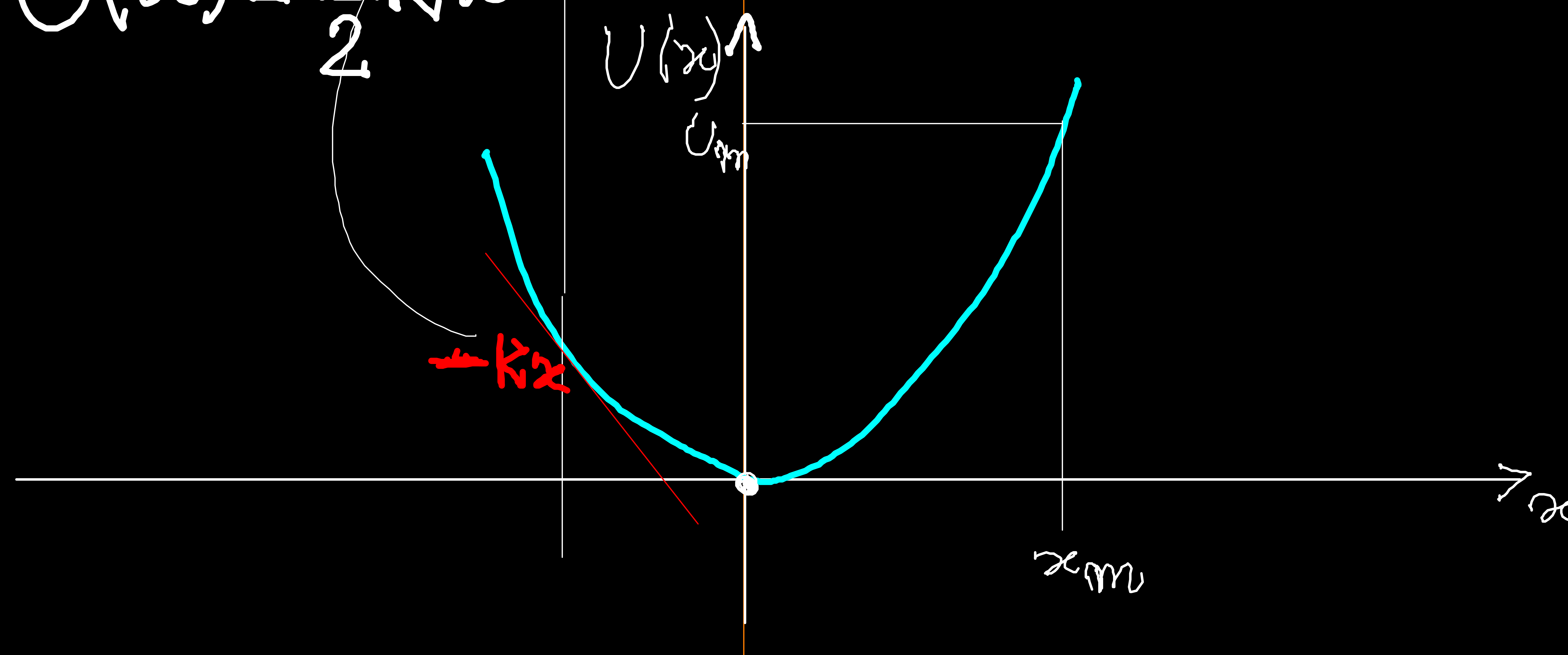
$$F_x(x) = -\frac{dU}{dx} \;\cdots\; -\frac{\partial U}{\partial x}$$

1) $\overset{\text{grav.}}{U}(y) = mgy + C \longrightarrow F_y(y) = -\frac{dU}{dy} = -mg$

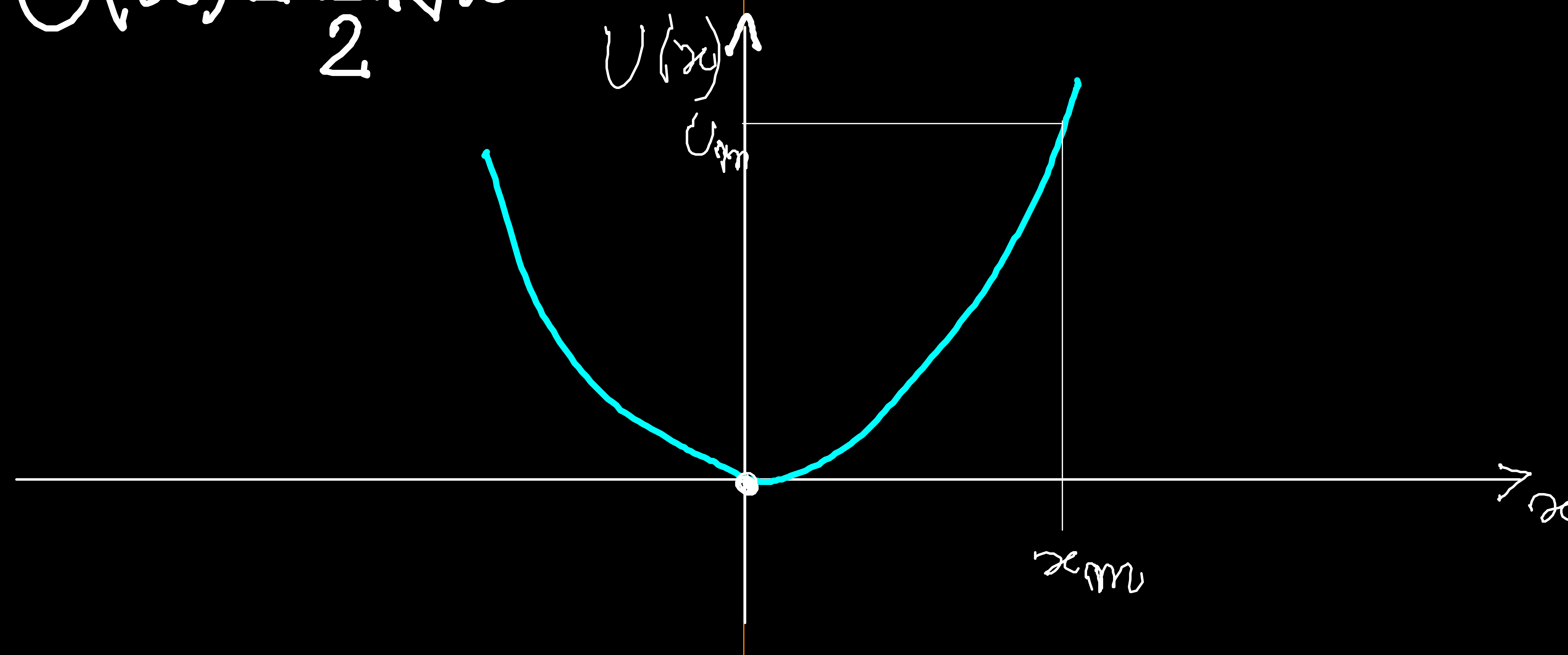
2) $\overset{\text{el.}}{U}(x) = \frac{1}{2}Kx^2 + C \longrightarrow F_x(x) = -\frac{dU}{dx} = -Kx$



$$U(x) = \frac{1}{2} k x^2$$



$$U(x) = \frac{1}{2} k x^2$$

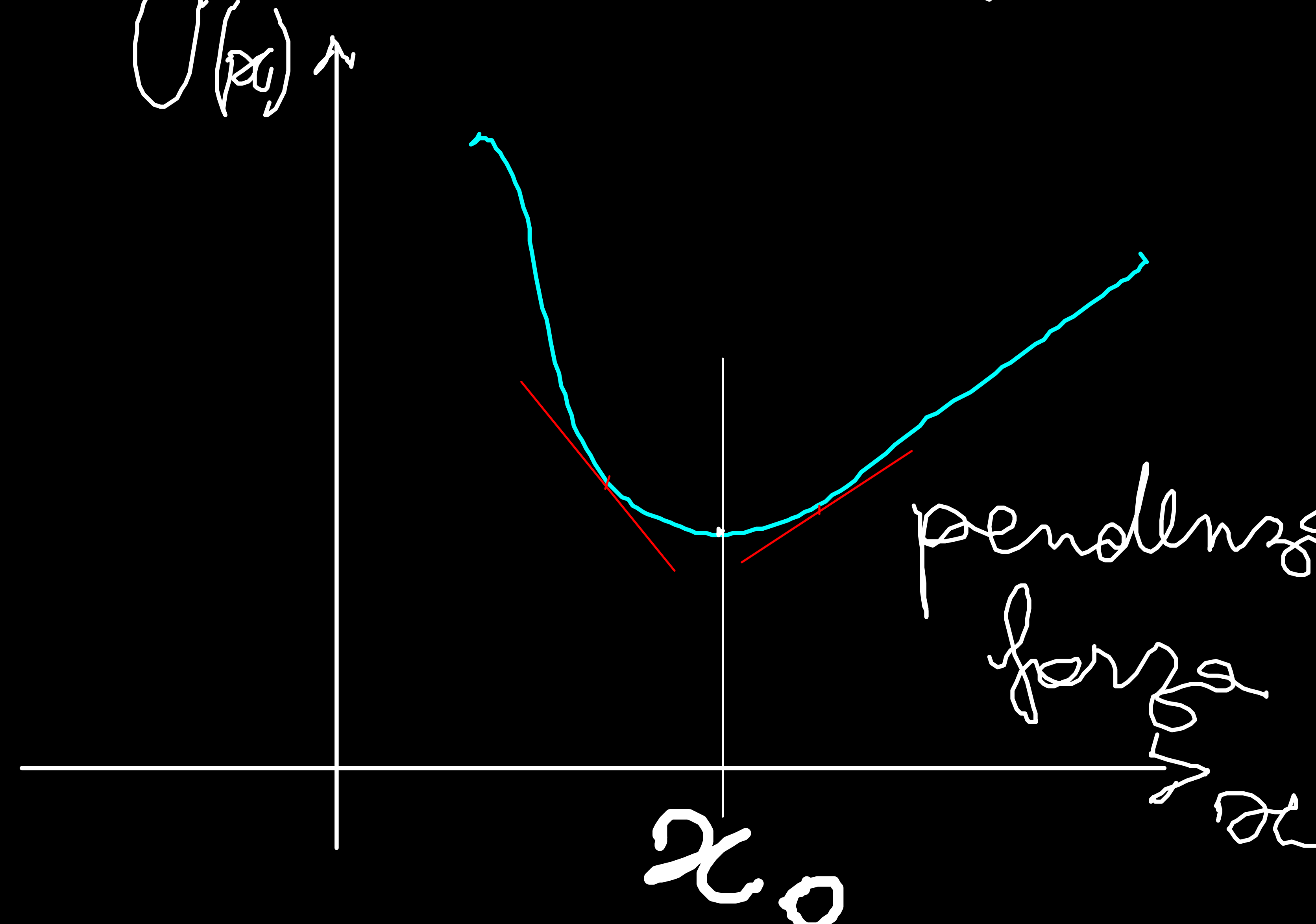


Equilibrio e stabilità

→ Posizione di equilibrio
punto in cui forza risultante nulla

Analisi grafica

1)



equilibrio stabile x_0
minimo locale

pendenza positiva nello spostamento $\Delta x > 0$
forza $F_x < 0$

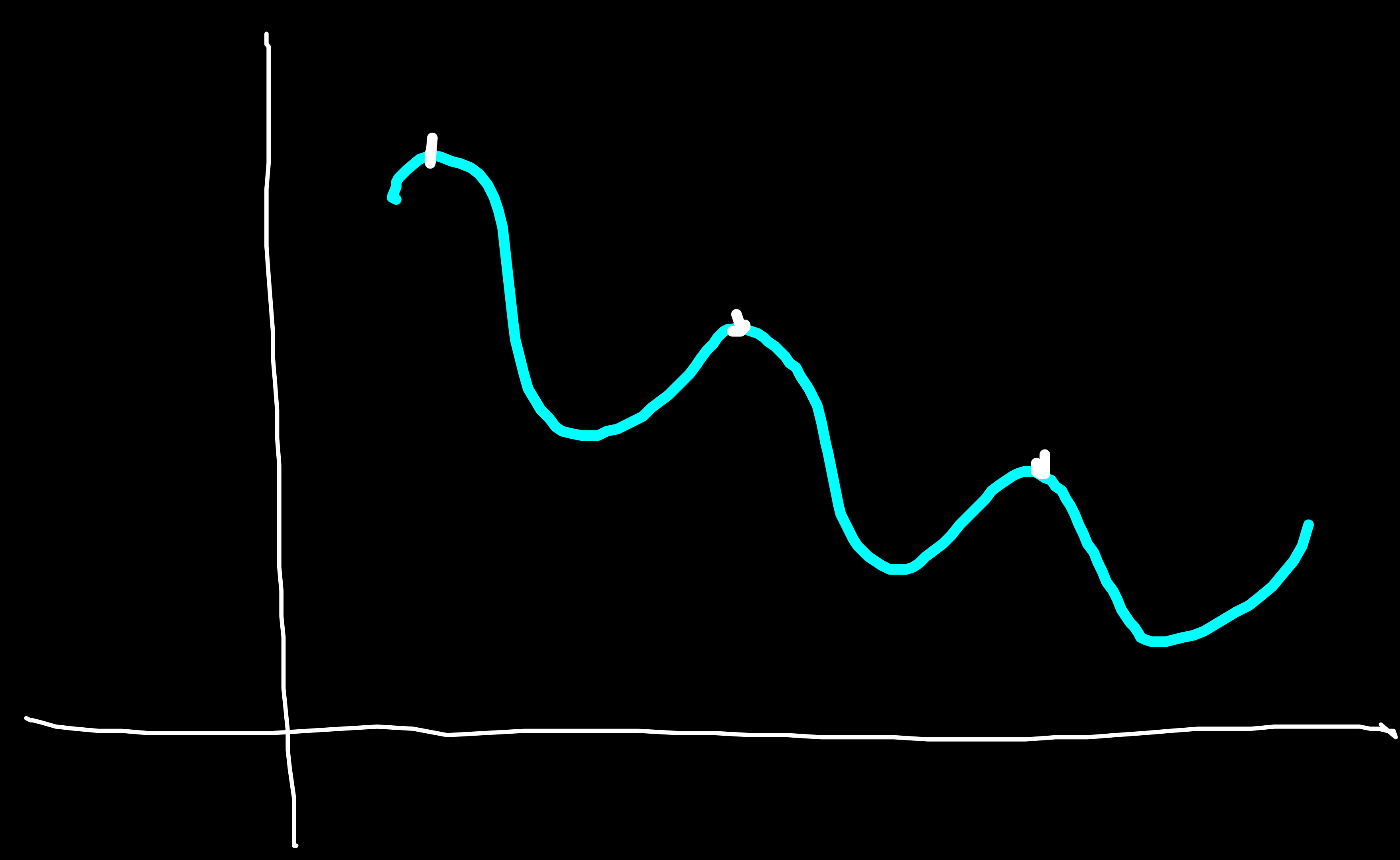
$$F_x(x_0) = 0$$

Equilibrio e stabilità

→ Posizione di equilibrio
punto in cui forza risultante nulla

Analisi grafica

1)bis
2)bis



3 punti di eq. stabile

3 punti di eq. instabile

Equilibrio e stabilità

→ Posizione di equilibrio
punto in cui forza risultante nulla

Analisi grafica

2)

Equilibrio instabile

$$F_x(x = x_0) = 0$$

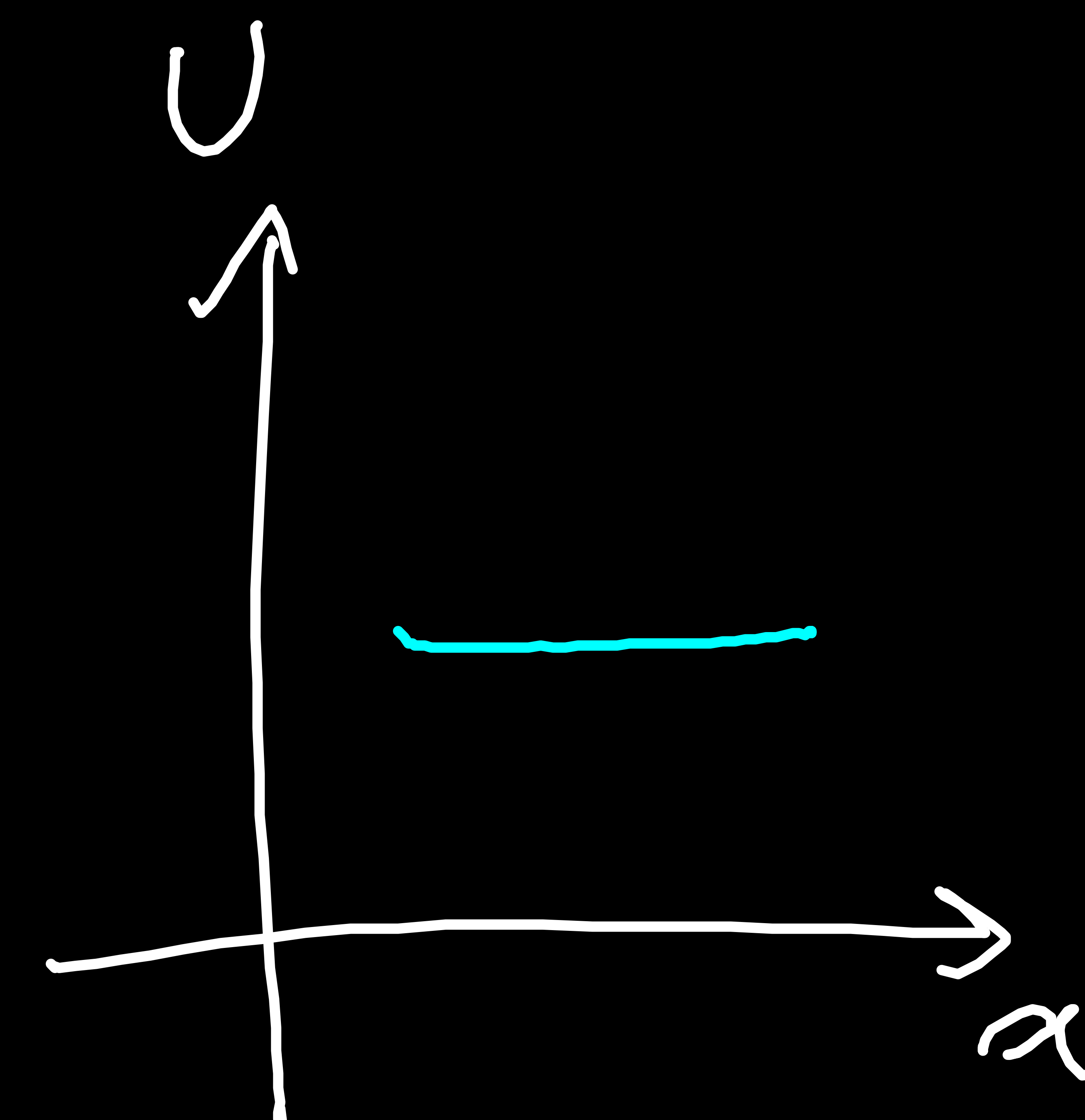
massimo locale

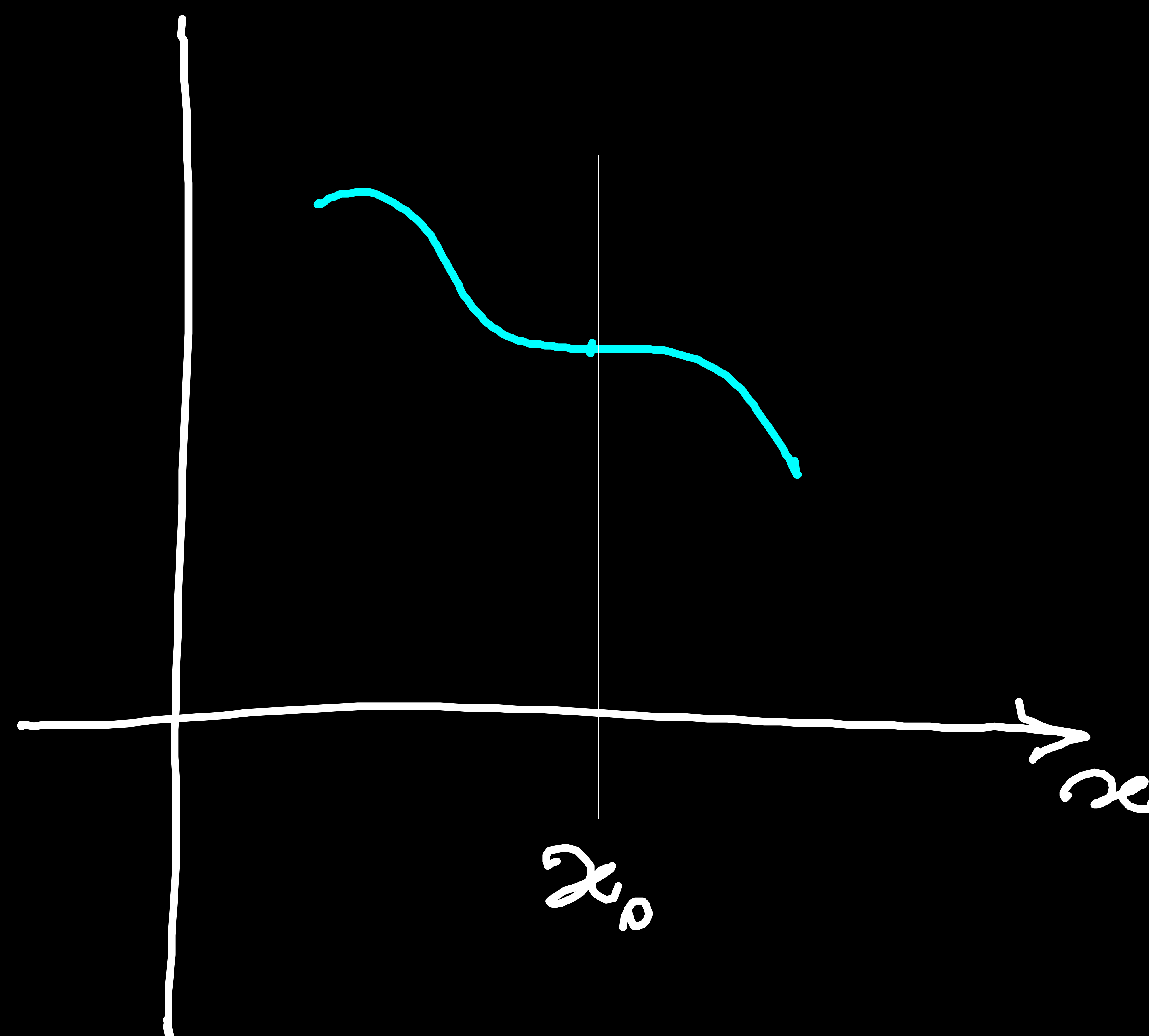
Equilibrio e stabilità

→ Posizione di equilibrio
punto in cui forza risultante nulla

Analisi grafica

3) Equilibrio indifferente
per piccoli spostamenti Δx
mantiene la sua posizione



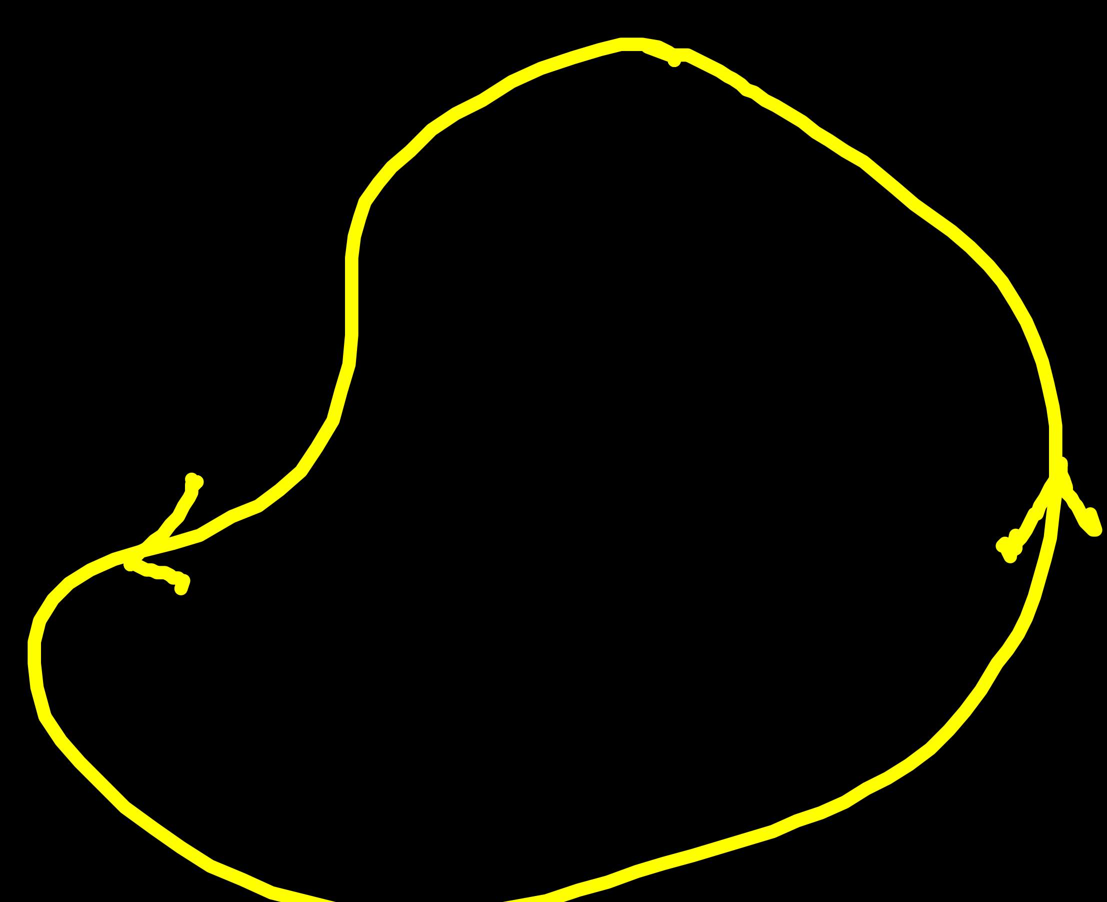


x_0 punto di flesso

Forze conservative ed energia Potenziale 3D

percorso chiuso nello spazio

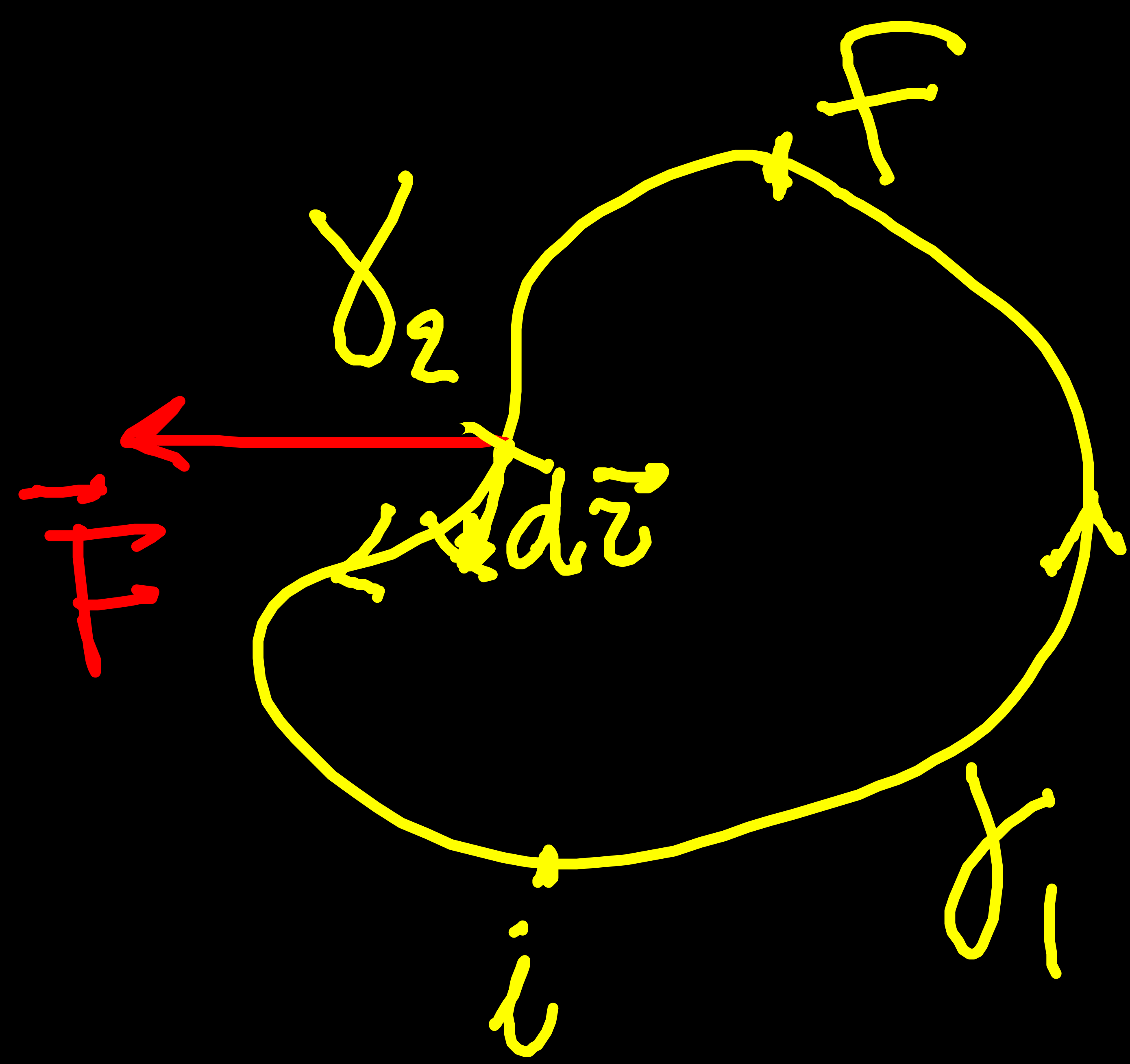
"si chiude su se stesso"



$W = 0 \quad \forall$ percorso chiuso

$$\oint \vec{F} \cdot d\vec{z} = 0$$

circuitazione



$$\oint \vec{F}\cdot d\vec{r} = \int_{i(\gamma_1)}^{f} \vec{F}\cdot d\vec{r} + \int_{f(\gamma_2)}^{i} \vec{F}\cdot d\vec{r}$$

$$= \int_{i(\gamma_1)}^{f} \vec{F}\cdot d\vec{r} - \int_{i(-\gamma_2)}^{f} \vec{F}\cdot d\vec{r}$$

faccio γ_2 alla rovescia :

inverto $f \leftrightarrow i$ $-d\vec{r}$

dato che $\oint \vec{F} \cdot d\vec{r} = 0$ $\forall$ per. ch.

γ_A γ_B

generici

A $i \to f$

B $i \to f$

$$\int_{i\,(\gamma_A)}^{f} \vec{F} \cdot d\vec{r} = \int_{i\,(\gamma_B)}^{f} \vec{F} \cdot d\vec{r}$$

Esempi di forze cons. 3D

1) Forza peso $\vec{F}_p = m\vec{g}$

$$\int_i^f \vec{F}_p \cdot d\vec{r} = -mg(y_f - y_i)$$

$$= -(mgy_f - mgy_i)$$

$$U_f - U_i = -\int_i^f \vec{F} \cdot d\vec{r}$$