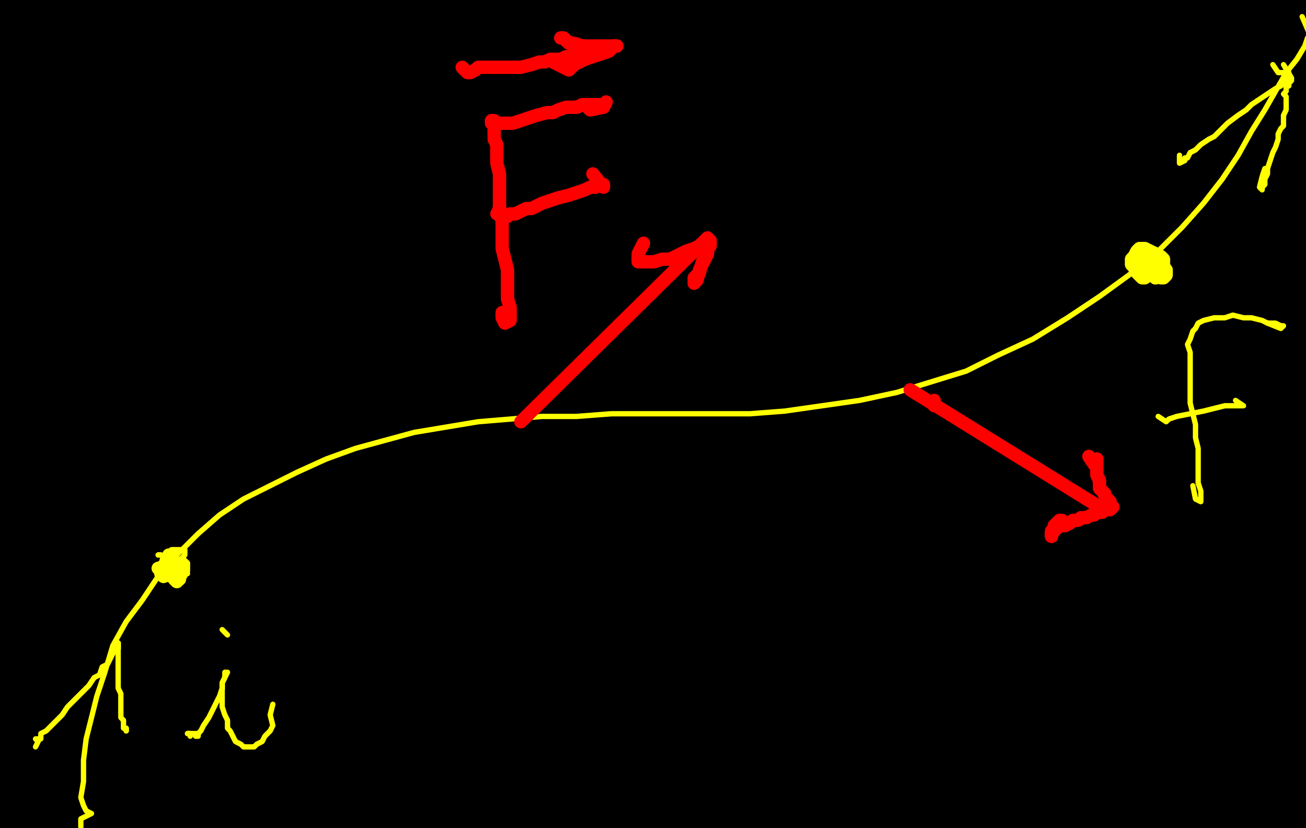
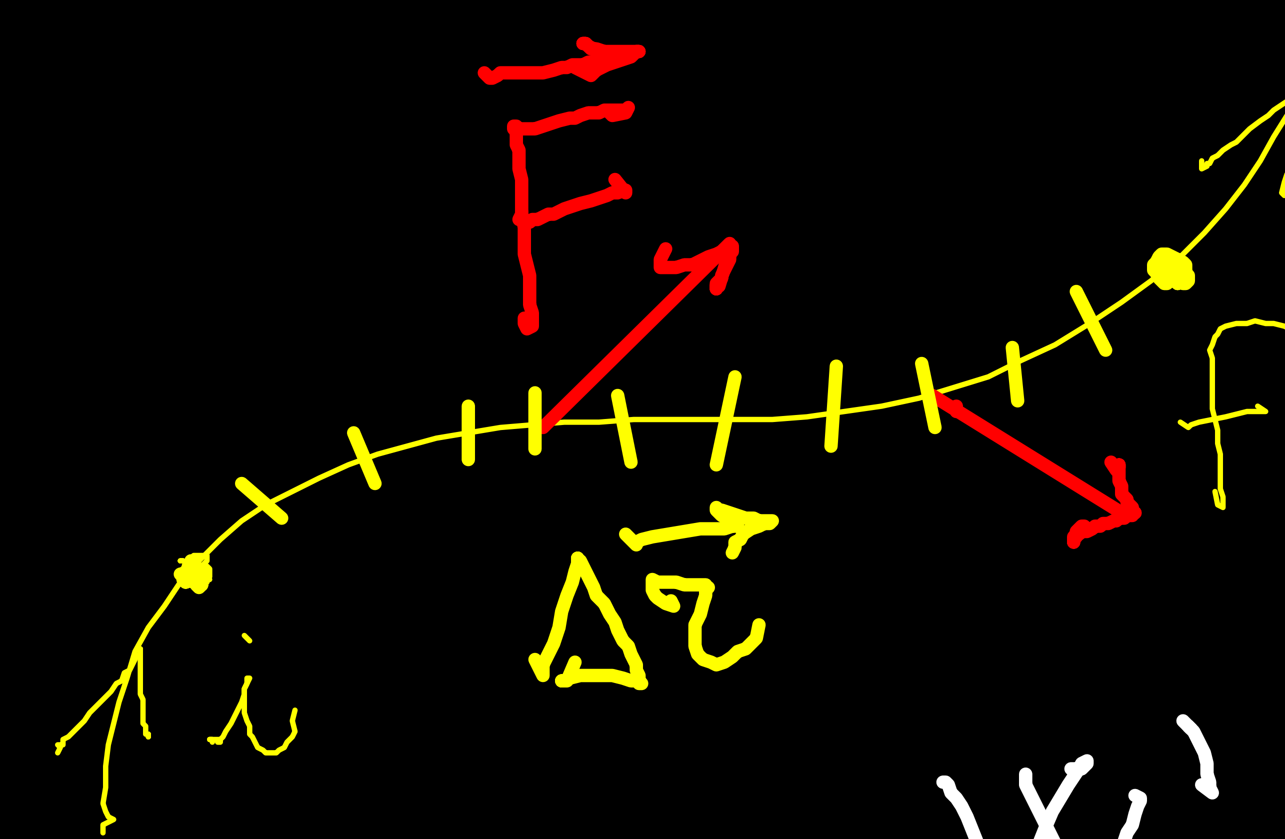
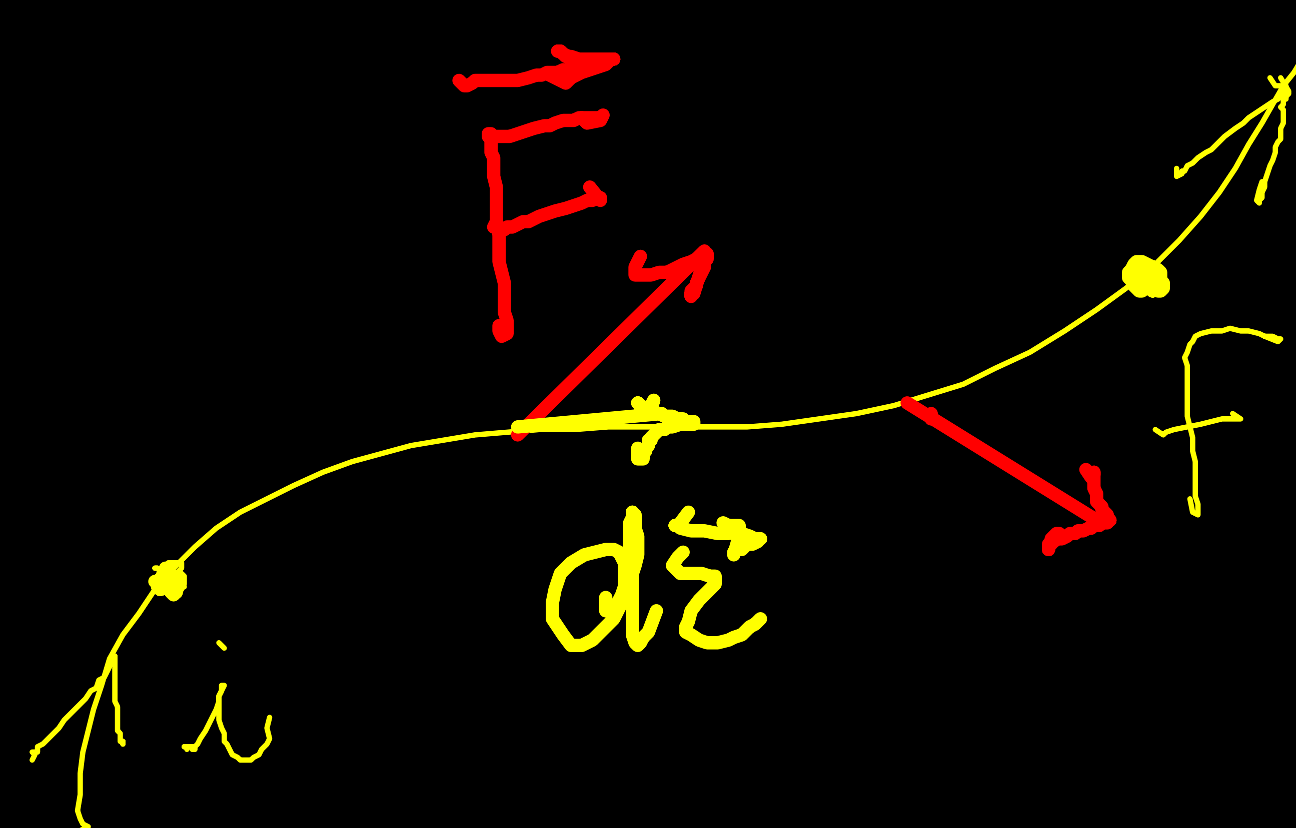




$$W' = \vec{F} \cdot \Delta\vec{r}$$



$\overline{}$

dW

$$W = \int_i^f \vec{F} \cdot d\vec{r}$$

integrale di linea

1^a possibilità

Se conosco comp. cart.

$$\vec{F}(x, y, z) = F_x(x, y, z)\,\hat{i} + F_y(x, y, z)\,\hat{j} + F_z(x, y, z)\,\hat{k}$$

$$d\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

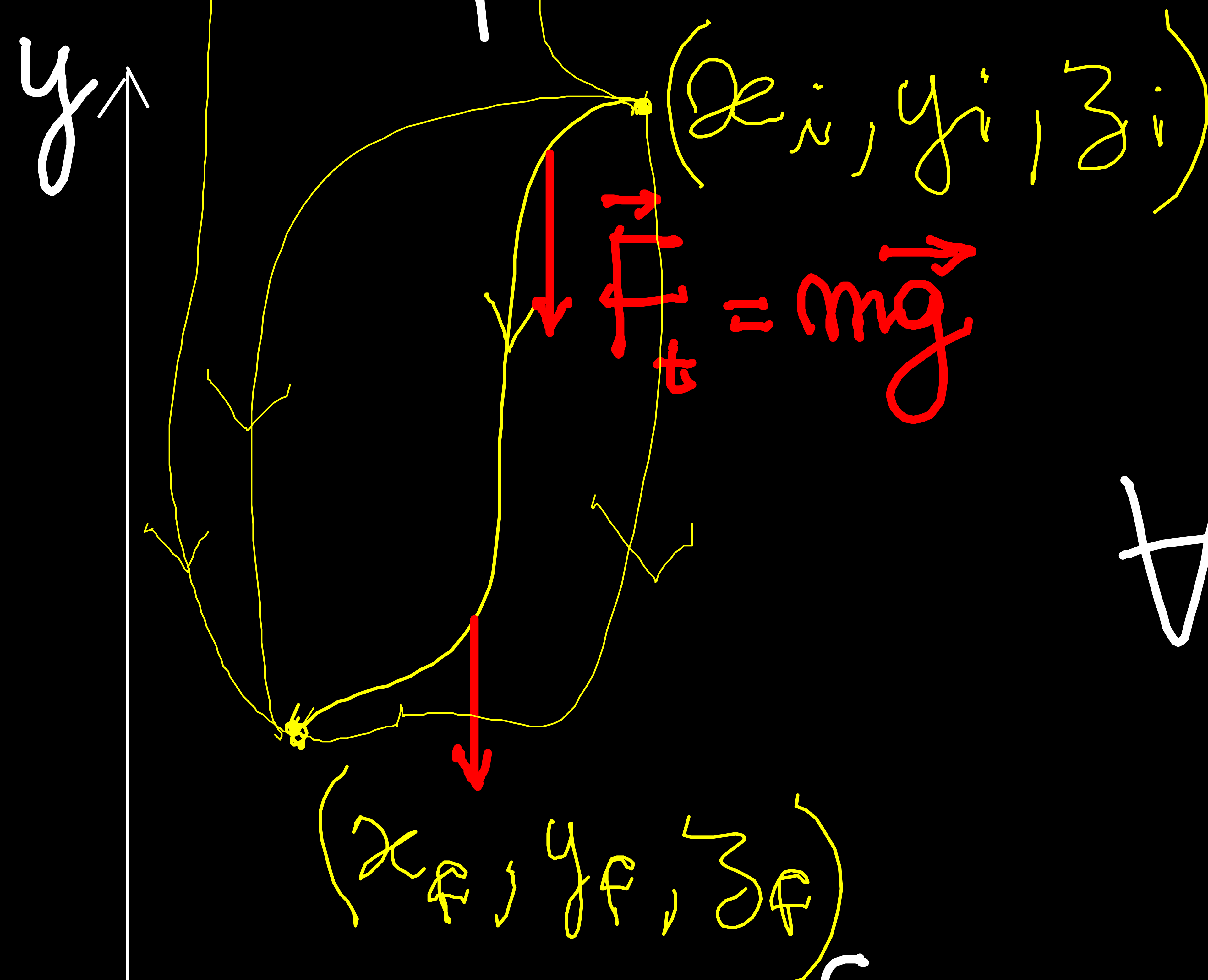
$$dW = \vec{F} \cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz$$

$$W = \int_i^f \left(F_x\,dx + F_y\,dy + F_z\,dz\right)$$

$$= \int_i^f F_x\,dx + \int_i^f F_y\,dy + \int_i^f F_z\,dz$$

W compiuto dalla $\vec{F}_t$ sul

Corpo con traiettoria arbitraria sup. terra



$$\vec{F}_t = F_{ty}\,\hat{j} \qquad F_{ty} = -mg$$

$$\forall d\vec{r} \qquad \vec{F}_t \cdot d\vec{r} = 0 + F_{ty}\,dy + 0 = -mg\,dy$$

$$W = \int_i^f \vec{F}_t \cdot d\vec{r} = \int_{y_i}^{y_f} F_{ty}\,dy = \ldots = -mg\int_{y_i}^{y_f} dy = -mg(y_f - y_i)$$

8.5 Teorema Lavoro-energia
energia cinetica

uniforme

1) moto 1D con $\vec{F}$ risultante costante
agente sul corpo in moto linea retta

$\sum \vec{F} = m\vec{a}$ $\quad x \quad F_x \quad i \longrightarrow f$

$x_i \longrightarrow x_f$

$v_i \longrightarrow v_f$

$$\begin{cases} a_x = \dfrac{F_x}{m} \\ v_f^2 - v_i^2 = 2\,\boxed{a_x (x_f - x_i)} \end{cases} \longleftarrow$$

$$W = F_x (x_f - x_i) = m\, a_x (x_f - x_i) =$$

$$= m\left[a_x(x_f - x_i)\right] = m\left[\frac{1}{2}(v_f^2 - v_i^2)\right]$$

$$\boxed{W_{TOT} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2}$$

Teorema Lavoro-energia

$$K = \frac{1}{2} m v^2$$

energia cinetica

$$W_{tot} = \Delta K$$

2) Caso generale $\begin{cases} \text{3D traiettorie} \\ \sum \vec{F} \text{ non uniforme} \\ \text{non costante} \end{cases}$

$$\sum \vec{F} = m\vec{a}$$

$$W_{tot} = \int_i^f \left(\sum \vec{F}\right) \cdot d\vec{r} = \int_i^f m \underbrace{\vec{a} \cdot d\vec{r}}_{\underbrace{a_x dx}_{} + a_y dy + a_z dz} =$$

$$= \int_i^f \left[m \frac{d}{dt}\left(\frac{1}{2} v_x^2\right) dt + \dots \right]$$

$$a_x dx = \frac{dv_x}{dt} \overbrace{\frac{dx}{dt}}^{v_x} dt$$

$$= v_x \frac{dv_x}{dt} dt$$

$$= \frac{d}{dt}\left(\frac{1}{2} v_x^2\right) dt$$

$$= \int_{t_i}^{t_f} \frac{1}{2} m \frac{d}{dt} \left(\underbrace{v_x^2 + v_y^2 + v_z^2}_{v^2}\right) dt$$

$$W_{tot} = \int_i^f \left(\sum \vec{F}\right) \cdot d\vec{r} = \int_{t_i}^{t_2} \frac{1}{2} m \frac{d}{dt}\left(v^2\right) dt$$

$$t \rightarrow v^2$$

$$= \int_{v_i^2}^{v_f^2} \frac{1}{2} m \, d\left(v^2\right) = \frac{1}{2} m \int_{v_i^2}^{v_f^2} d\left(v^2\right)$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

$$f(x)$$

$$\frac{d}{dx}\left[f^2(x)\right] = \frac{1}{2} f(x) \underbrace{f'(x)}_{\frac{d}{dx} f(x)}$$

$$v_x(t)$$

8.6 Potenza

rapidità con cui il lavoro delle forze è compiuto

Potenza media Δt

$$\langle P \rangle = \frac{W}{\Delta t}$$

SI watt W

$$1\,W = \frac{1\,J}{1\,s}$$

hp

$$1\,hp = 746\,W$$

Potenza istantanea $P = \frac{dW}{dt}$

definita istante per istante $P(t)$

$$dW = \vec{F} \cdot d\vec{r}$$

$$P = P(t) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

- Comunemente $KWh = 10^3 W \cdot 3.6 \cdot 10^3 s = 3{,}6 \cdot 10^6 J$

Es. 8.8 Ordini di grandezza K

a) Terra orbita Sole

$$M_T = 5.97 \cdot 10^{24}\,Kg$$

$$R_{TS} = 150 \cdot 10^{6}\,Km = 1{,}5 \cdot 10^{11}\,m$$

$$T \simeq 365\,gg = 365 \cdot 24 \cdot 3600 = 3.16 \cdot 10^{7}\,s$$

$$v_T = \frac{2\pi R_{TS}}{T} = 2.99 \cdot 10^{4}\,m/s$$

$$K_T = \frac{1}{2} M_T v_T^2 = 2.67 \cdot 10^{33}\,J$$

b) auto utilitaria

$$m_a = 1{,}2\,t = 1200\,Kg = 1{,}2 \cdot 10^3\,Kg$$

$$v = 50\,\frac{Km}{h} = \frac{50}{3{,}6}\,m/s = 13{,}9\,m/s$$

$$K_a = \frac{1}{2} \cdot 1{,}2 \cdot 10^3\,Kg \cdot (13{,}9)^2\,\frac{m^2}{s^2} = 1{,}2 \cdot 10^5\,J$$

c) Usain Bolt 94 Kg

100 m 9'' 58

$$\langle v \rangle = \frac{100 \text{ m}}{9''.58} = 10.4 \text{ m/s}$$

$$\langle K_B \rangle = \frac{1}{2} 94 \text{ Kg} (10.4)^2 \frac{\text{m}^2}{\text{s}^2} = 5.1 \cdot 10^3 \text{ J}$$

d) Proiettile 10 g 1000 m/s

$$K_P = \frac{1}{2} 10^{-2} \cdot 10^6 \text{ Kg} \frac{\text{m}^2}{\text{s}^2} = 5 \cdot 10^3 \text{ J}$$

e) Molecola di N_2

$$M_{N_2} = 4{,}6 \cdot 10^{-26}\ Kg$$

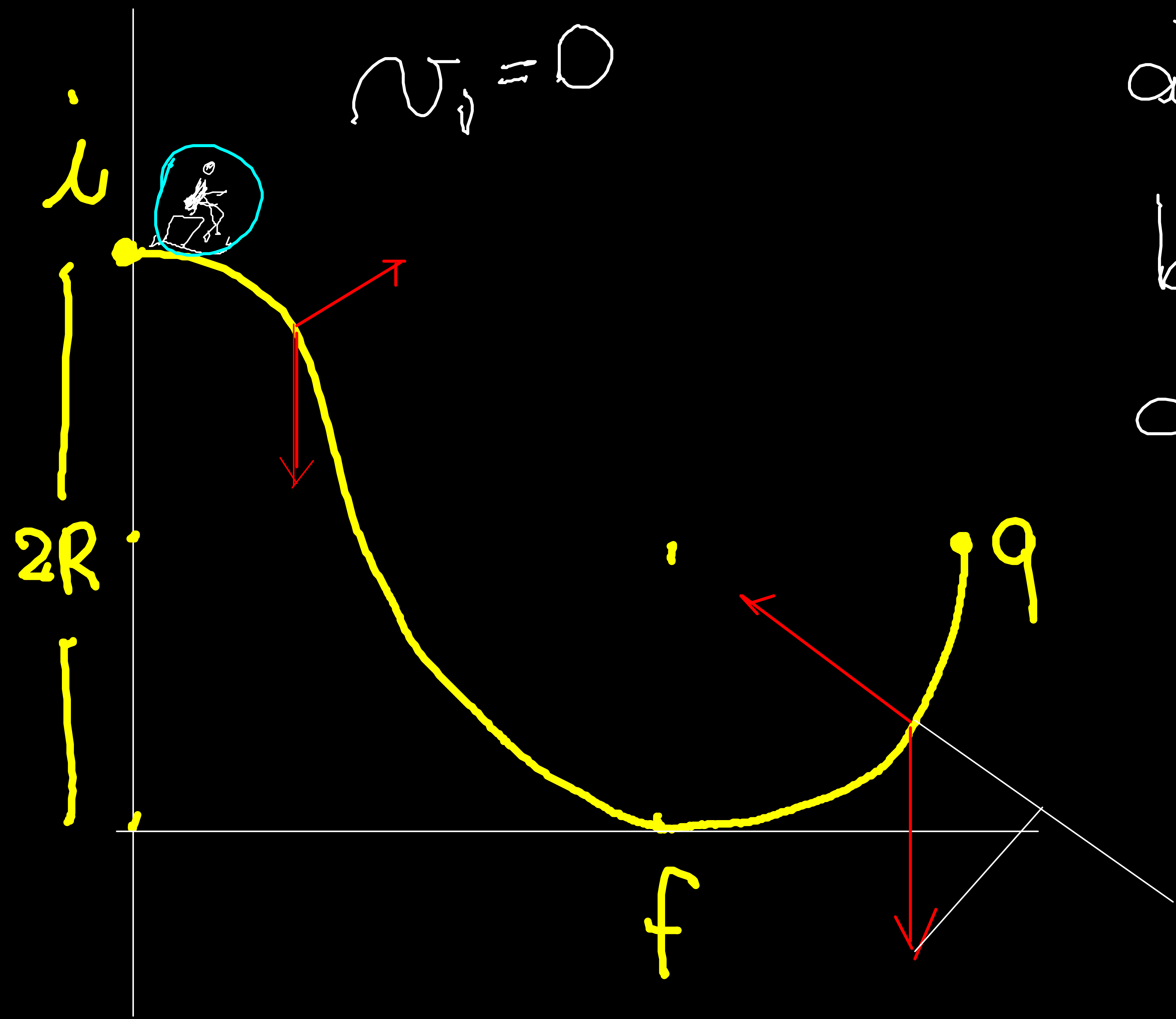
$$v = 500\ \frac{m}{s}$$

$$K_{N_2} = 5.8 \cdot 10^{-21}\ J$$

Esempio 8.11 slittino ghiaccio

$\downarrow$

attrito trascurabile



a) v_f

b) $|\vec{F}_N|$ in f

c) v_q $|\vec{F}_N|$ in q

Analisi preliminare

$$\vec{F} \xrightarrow{2} \vec{F}_P \quad \vec{F}_N$$

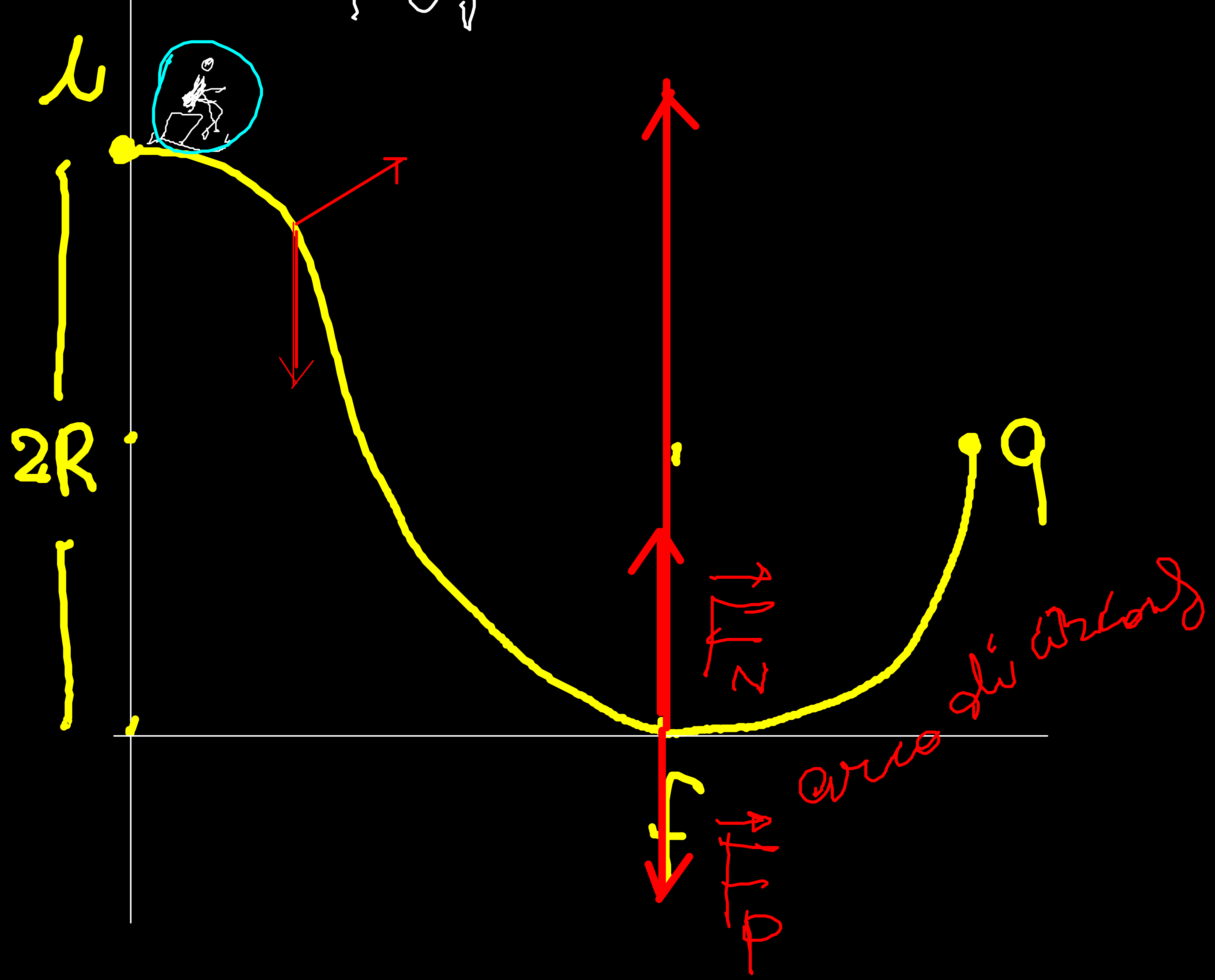
$$W_{F_N} = 0 \quad \forall d\vec{r} \quad \vec{F}_N \perp d\vec{r}$$

$$W_{tot} = W_g = -mg\underbrace{(y_F - y_i)}_{-2R} = 2mgR$$

$$\frac{1}{2}mv_f^2 - \underbrace{\frac{1}{2}mv_i^2}_{=0} = 2mgR$$

parte da fermo

a) $$v_f = \sqrt{4gR}$$

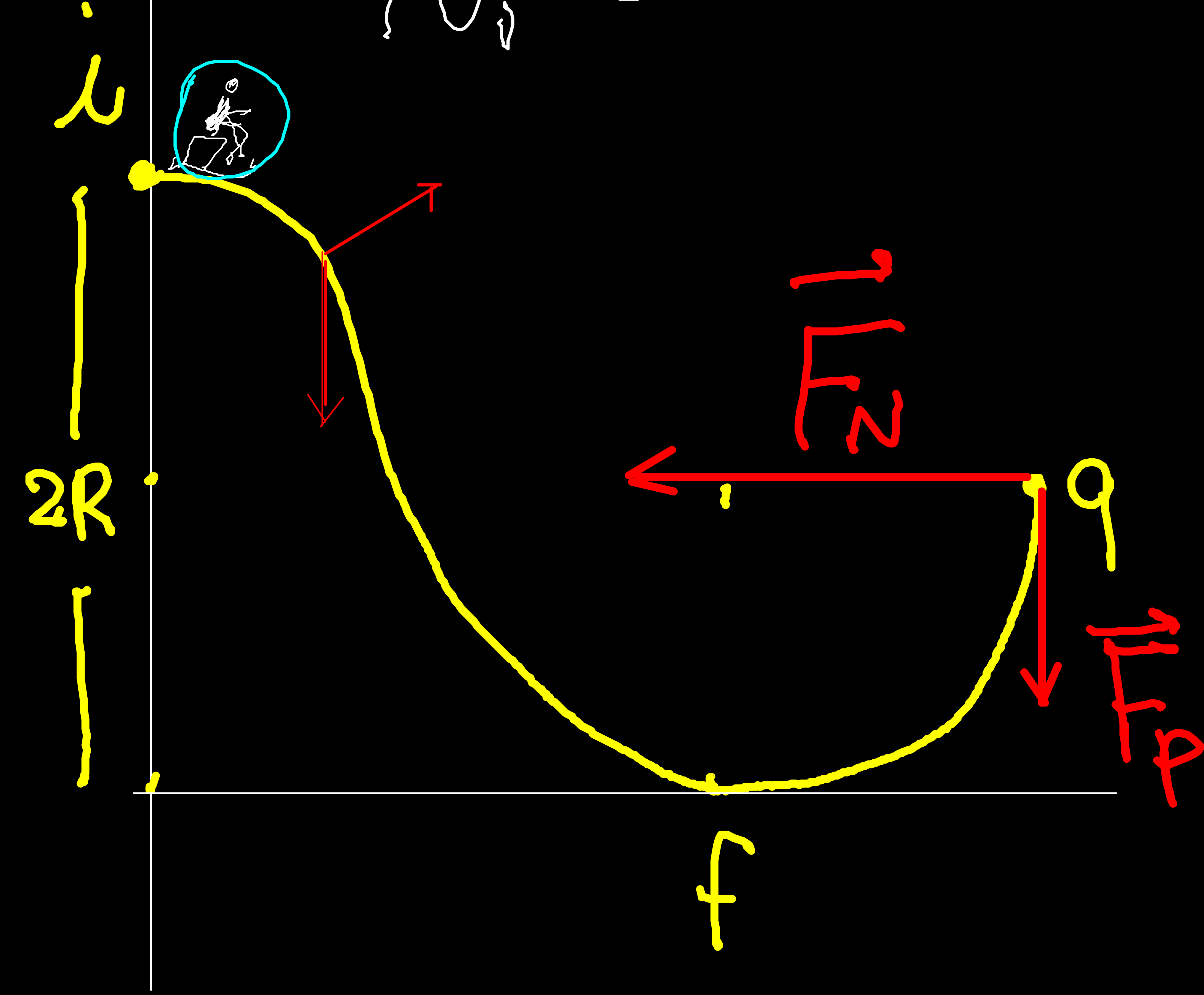


in f

$$\sum \vec{F} = \underbrace{\left(F_{Ny} + F_{Py}\right)\hat{j}}_{\text{forza centripeta}}$$

$$F_N - mg = m\,\frac{v_f^2}{R}$$

$$F_N = mg + m\,\frac{v_f^2}{R} = 5mg$$



nel punto q
solo $\vec{F}_N$
fornisce la forza
centripeta

$$v_q = \sqrt{2gR}$$

Scompongo
$a_c \longrightarrow$
a_t

$$F_N = \frac{m v_q^2}{R} = 2mg$$

Problema 1 29/01/2019

Treno $M = 7.3 \cdot 10^5$ Kg

accelerato potenza costante $P = 2.1 \cdot 10^3$ KW

$t_0 = 0$ s $v_0 = 32$ Km/h

a) negli istanti succ. a t_0 a ? costante? aumenta? diminuisce

Potenza ist. $P = \vec{F} \cdot \vec{v} = F v$

II legge Newt. $F = m a$

$$a = \frac{P}{m} \frac{1}{v}$$

v aumenta → a diminuisce

b) $t_1 = 75\,s$ $\quad W = ?$

$$W = P\Delta t = 1.6 \cdot 10^8\,J$$

c) $v_1 = ?$ $\quad a_1 = ?$

$$W = \Delta K = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

$$v_1 = \sqrt{v_0^2 + \frac{2}{m} W} = 23\,m/s$$

$$a_1 = \frac{P}{m}\frac{1}{v_1} = 0.13\,m/s^2$$