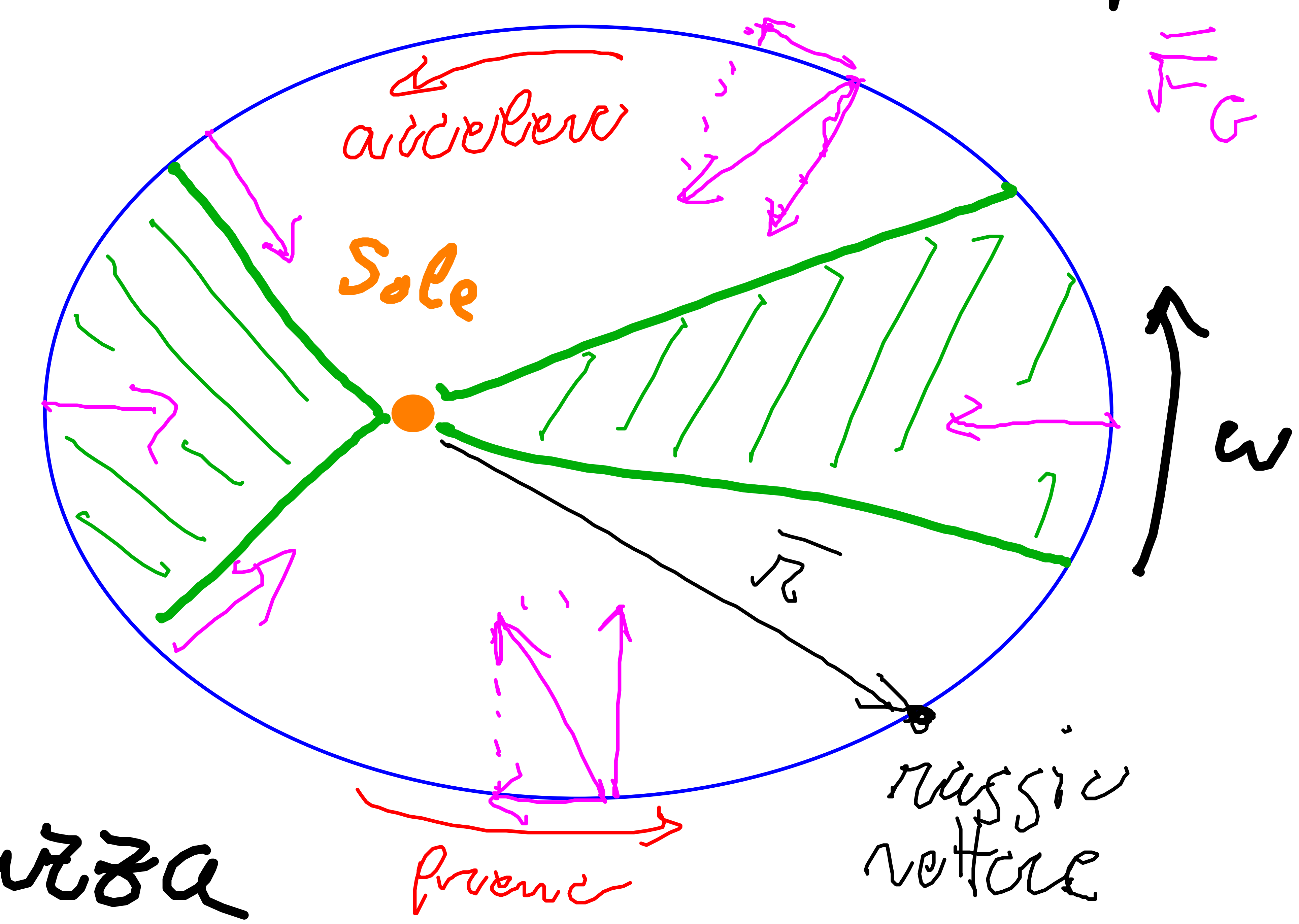
Keplero

1) L'orbita dei pianeti è un'ellisse di cui il Sole occupa uno dei due fuochi



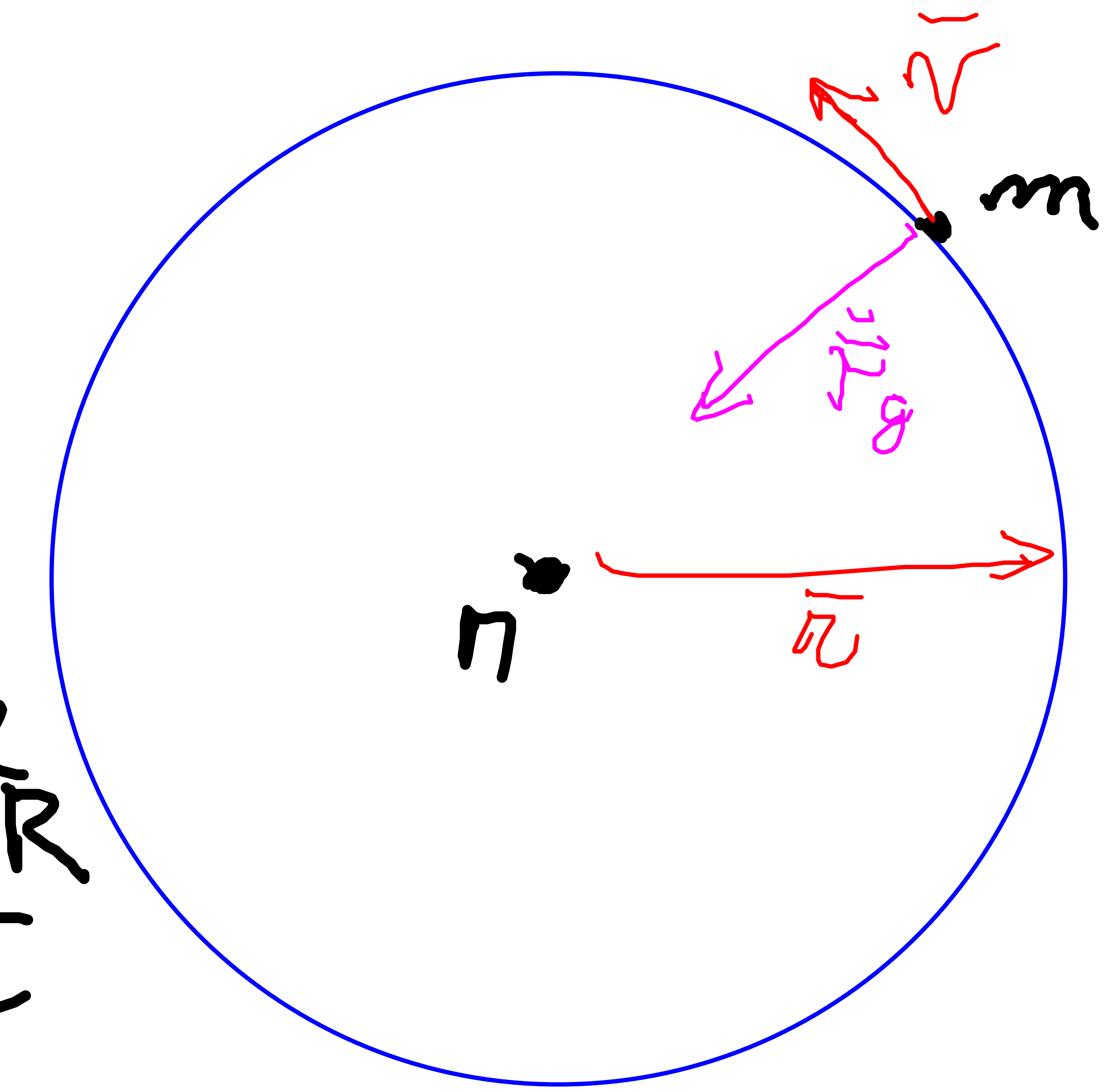
2) Il raggio vettore di un'orbita spazza aree uguali in tempi uguali

Keplero

3) Il quadrato del periodo di un'orbita è proporzionale al cubo del raggio medio

orbita circolare

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi R}{T}$$



$$F_g = F_{CEN}$$

$$\frac{GmM}{R^2} = m\frac{v^2}{R} = m\frac{(2\pi R)^2}{T^2}\frac{1}{R} = m\frac{4\pi^2 R}{T^2}$$

$$T^2 GM = 4\pi^2 R^3$$

$$\boxed{T^2 = \frac{4\pi^2}{GM}R^3}$$

$$\bar{F}_G = -\frac{GmM}{r^2}\hat{r}$$

$$\bar{F}_c = \bar{\omega}\times(\bar{\omega}\times\bar{r}) = -\omega^2 r\,\hat{r}$$

Raggio orbita Marte

$$R_{TERRA} = 149.6 \cdot 10^6 \text{ Km}$$

$$T_{TERRA} = 365.24 \text{ giorni}$$

$$T_{MARTE} = 686.98 \text{ giorni}$$

$$R_{MARTE} = ?$$

III Kepler

$$\frac{T_T^2}{R_T^3} = \frac{4\pi^2}{GM} = \frac{T_M^2}{R_M^3}$$

$$R_M^3 = \frac{T_M^2}{T_T^2} R_T^3 =$$

$$R_M = R_T \sqrt[3]{\frac{T_M^2}{T_T^2}} = 228.0 \times 10^6 \text{ Km}$$

Orbita satellite geostazionario

$M_T = 5.97 \times 10^{24}\ Kg$

$T = 23h\ 56' = 1436' = 86160\ s$

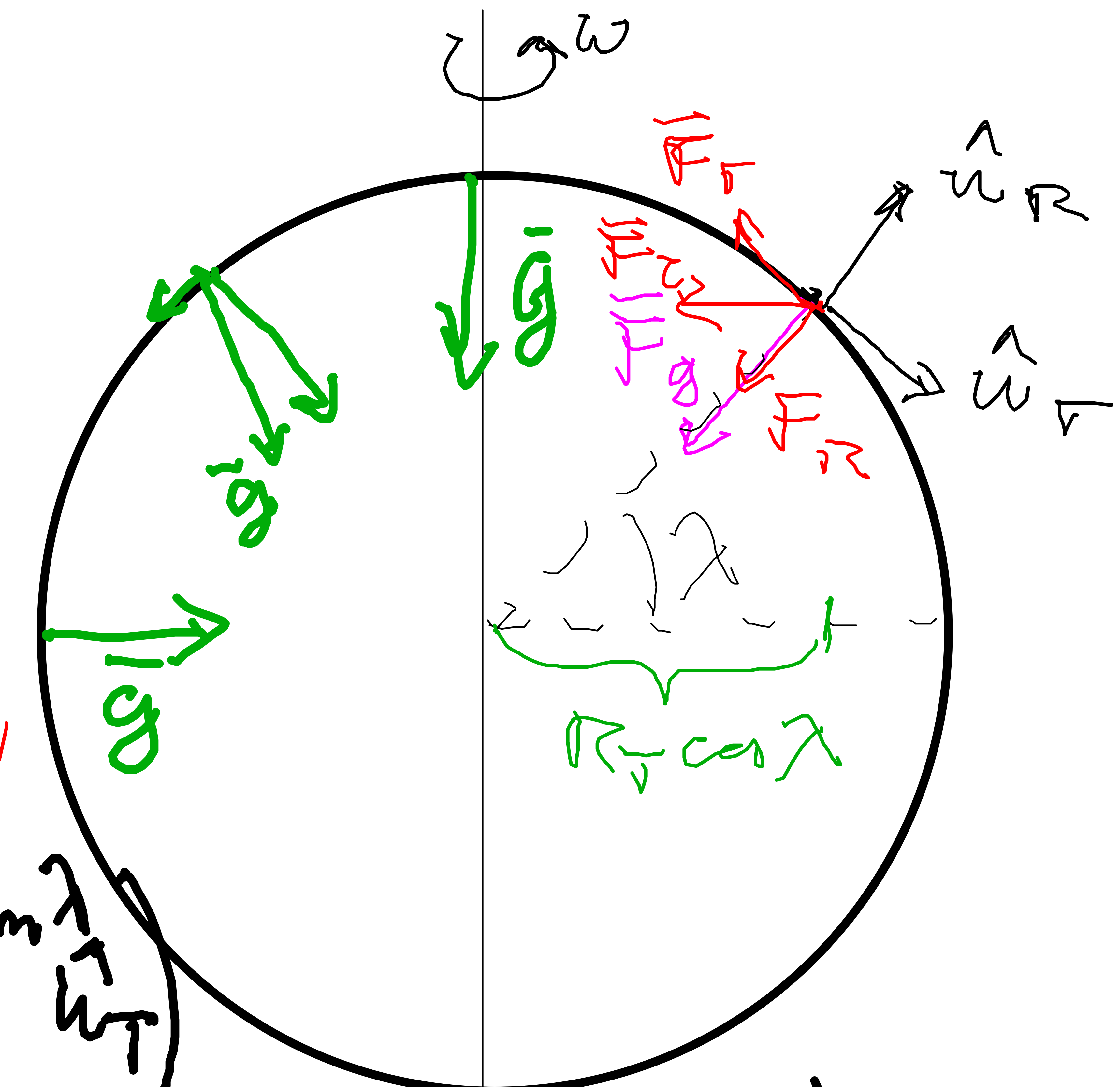
$$T^2 = \frac{4\pi^2}{GM_T} R^3$$

$$R = \sqrt[3]{\frac{GM_T T^2}{4\pi}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \cdot 5.97 \times 10^{24} \cdot (8.616 \times 10^4)^2}{4\pi^2}}$$

$$= 4.21 \times 10^7\ m = 42100\ Km$$

Forze di marea

TERRA



Forza su superficie

gravità + centripeta

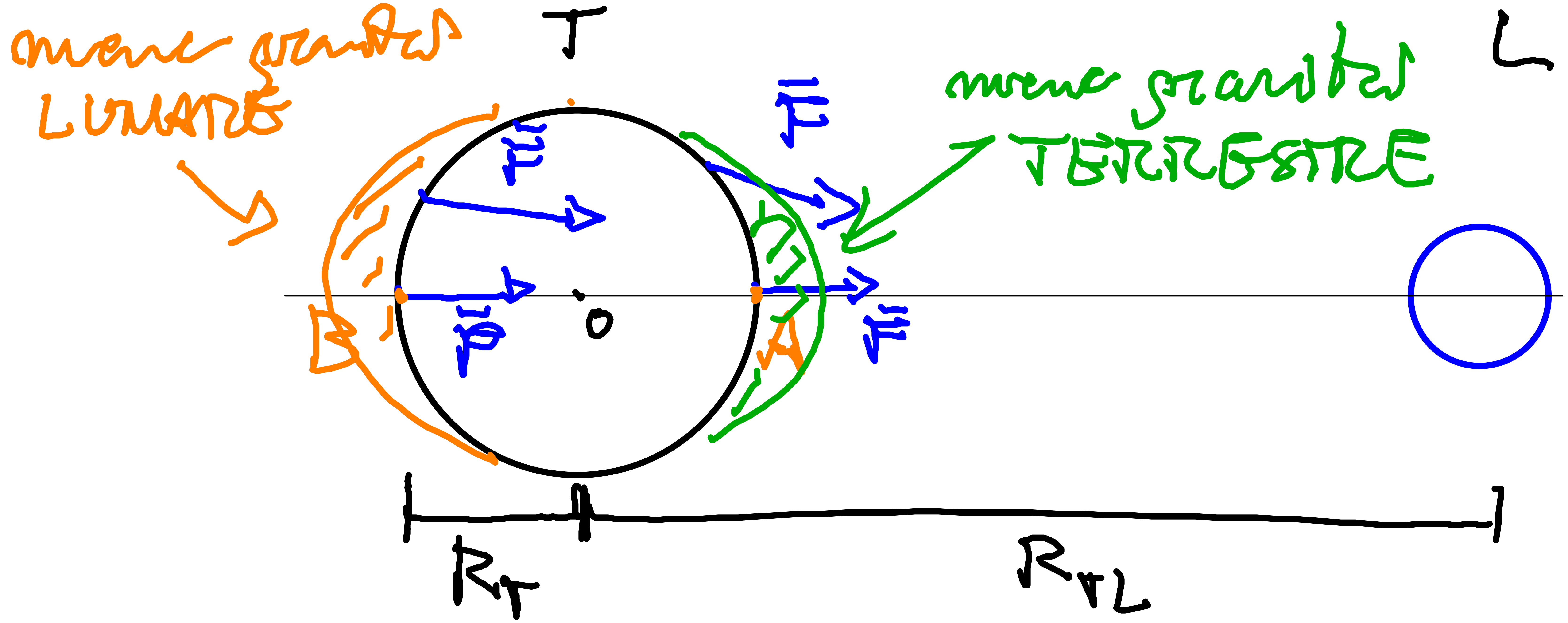
modulo F_c $\quad$ $\hat F_R$ $\quad$ $\hat F_T$

$$\bar g_{TOT} = -\frac{GM_T}{R_T^2}\hat u_R + (\omega^2 R_T \cos\lambda)(\cos\lambda\,\hat u_R - \sin\lambda\,\hat u_T)$$

ai poli o all'equatore $\bar g$ diretta verso il centro

in mezzo c'è una componente tangenziale

Forze di marea



$$g_A = \frac{GM_L}{(R_{TL} - R_T)^2}$$

$$g_B = \frac{GM_L}{(R_{TL} + R_T)^2}$$

$$g_O = \frac{GM_L}{R_{TL}^2} \quad \text{al centro}$$

serie

$$\Delta g_A = g_A - g_O = \frac{GM_L}{(R_{TL} - R_T)^2} - \frac{GM_L}{R_{TL}^2} \simeq \frac{2R_T}{R_{LT}^3} GM_L$$

$$\Delta g_B = g_B - g_O = \frac{GM_L}{(R_{TL} + R_T)^2} - \frac{GM_L}{R_{TL}^2} \simeq -\frac{2R_T}{R_{LT}^3} GM_L$$

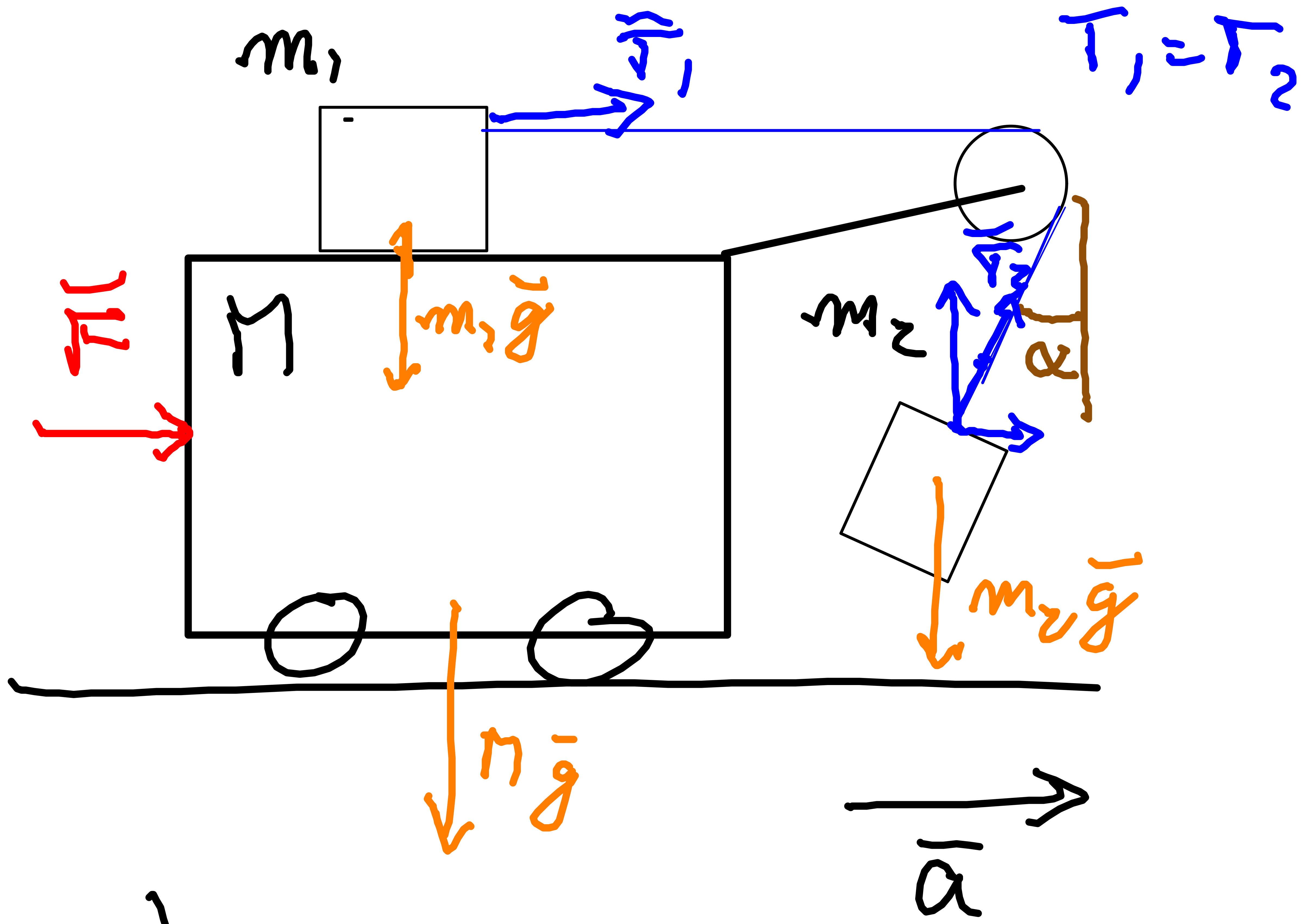
$$\Delta g_{TOT} = \frac{4R_T}{R_{LT}^3} GM_L$$

$$\Delta g_{sole} = 0.45\,\Delta g_L$$

Problema I 13.02.20

F costante sul sistema
m_1 non si muove rispetto a M
$m_1 = 5.0\ Kg$; $m_2 = 4.0\ Kg$
no attrito!

1) $\alpha = ?$ 2) accel. $= ?$
3) diagramma forze $M = ?$



$(T_1 = T_2)$

TOT $\quad F = M_{TOT} \cdot a$

1x $\quad T_1 = m_1 a$

2x $\quad T_2 \sin\alpha = m_2 a$

2y $\quad T_2 \cos\alpha - m_2 g = 0 \quad — \quad T_2 = \dfrac{m_2 g}{\cos\alpha}$

$m_1 \cancel{a} \sin\alpha = m_2 \cancel{a} \;\rightarrow\; \sin\alpha = \dfrac{m_2}{m_1}$

$\rightarrow \boxed{\alpha = \arcsin\left(\dfrac{m_2}{m_1}\right) = 53^\circ}$

$\dfrac{\cancel{m_2} g}{\cos\alpha} \sin\alpha = \cancel{m_2} a \;\rightarrow\; \boxed{a = g \tan\alpha = 13\ m/s^2}$

Problema I 13.02.20

$\boxed{T_1 = T_2}$

F costante sul sistema

m_1 non si muove rispetto a M

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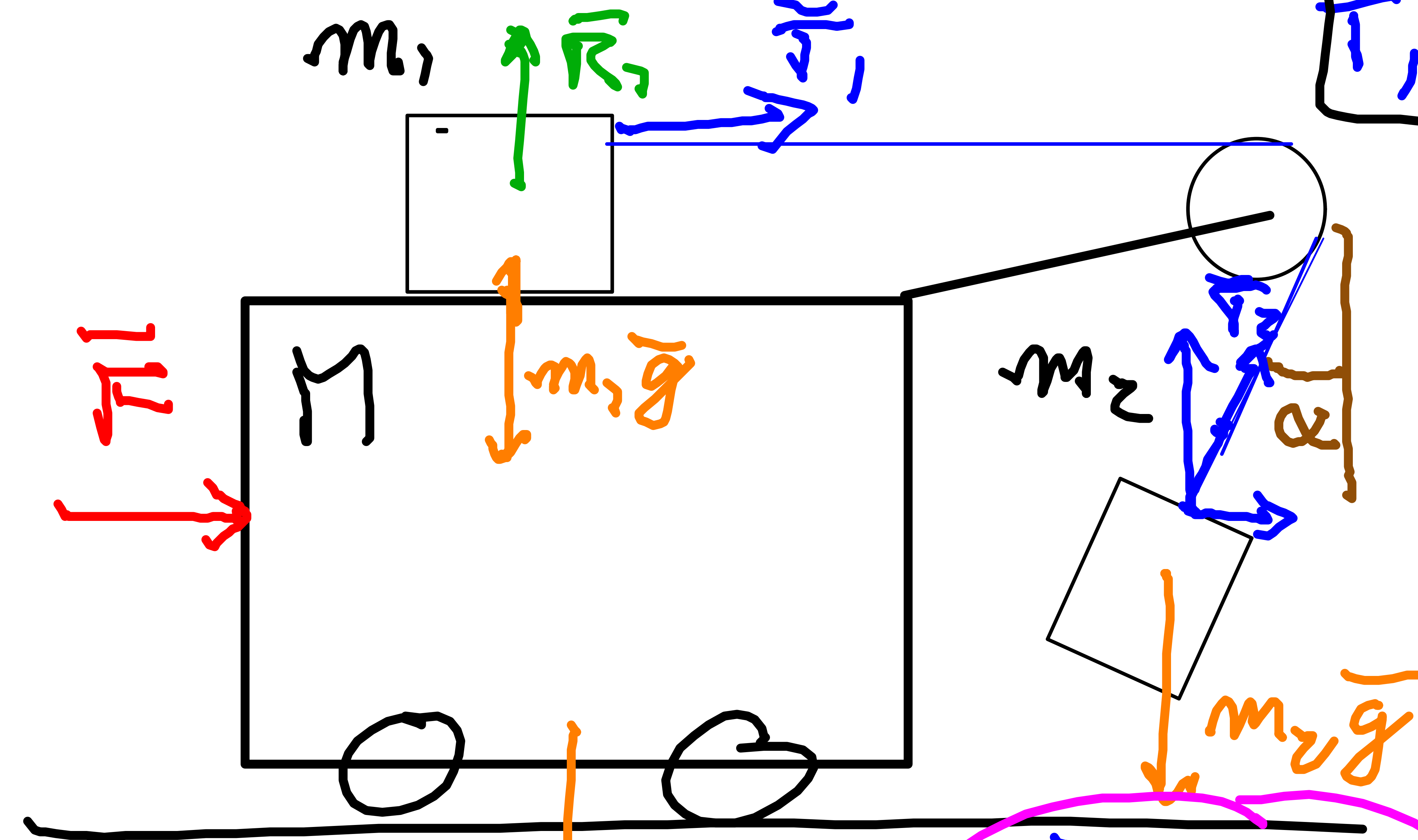
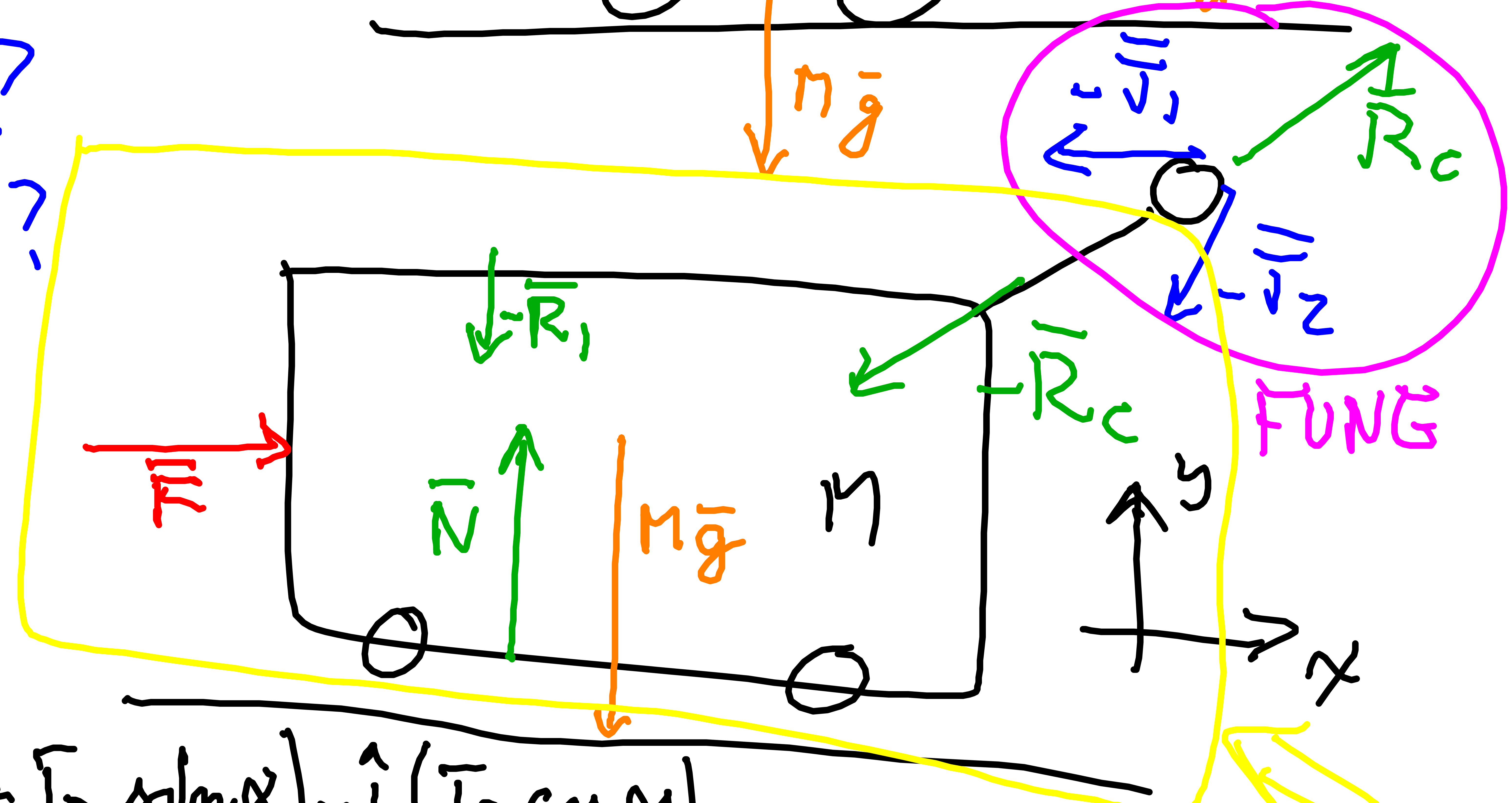
1) $\alpha = ?$ 2) accel. = ?

3) diagramma forze M = ?





$$\bar{N} = -(m_1 + m_2 + M)\,\bar{g}$$

$$-\bar{R}_1 = m_1\bar{g}$$

$$\bar{R}_c = -(-\bar{T}_1 - \bar{T}_2)$$

$$-\bar{R}_c = -(\bar{T}_1 + \bar{T}_2) = -\hat{i}\,(T_1 + T_2 \sin\alpha) - \hat{j}\,(T_2\cos\alpha)$$