# Cap. 8

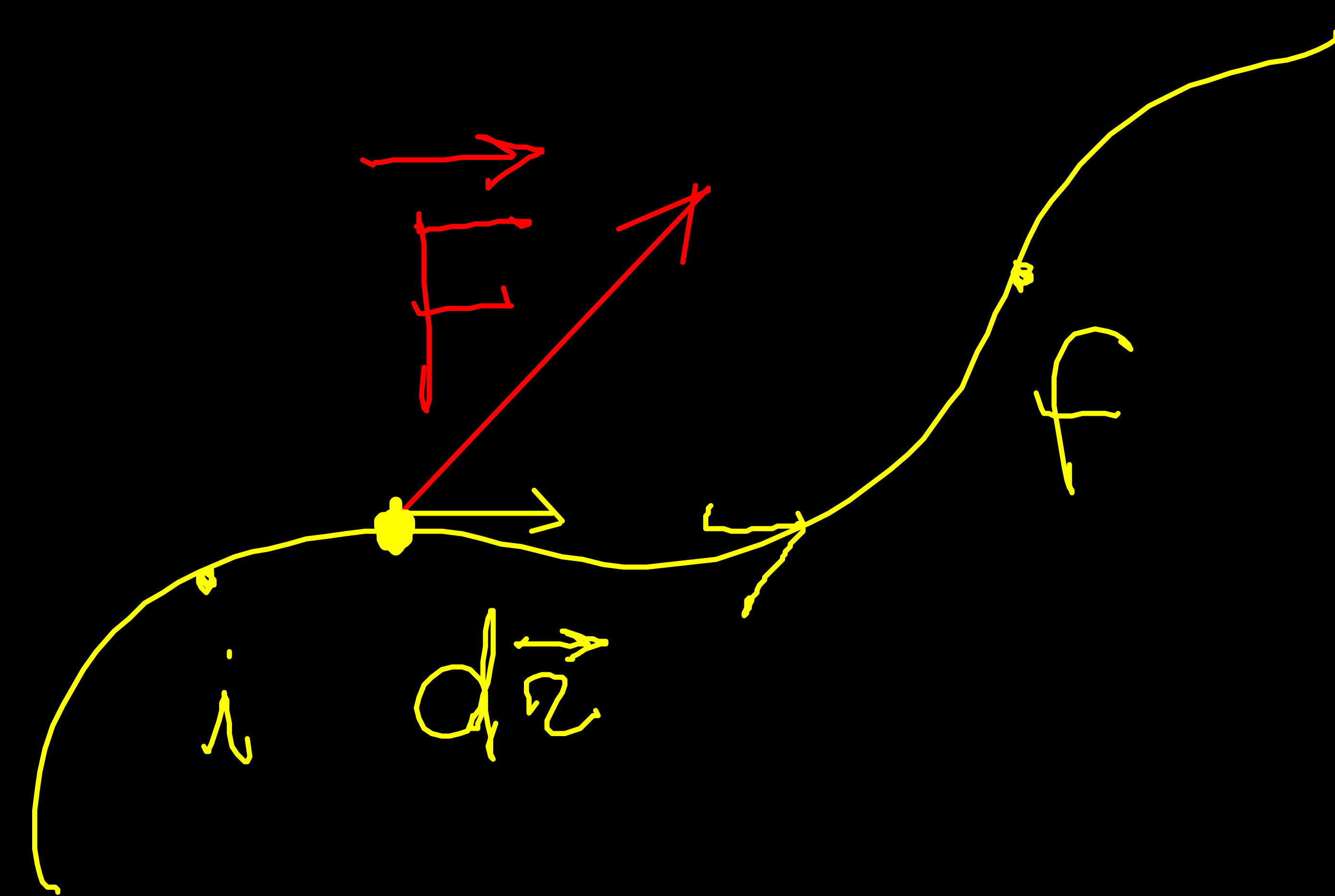
Lavoro

Energia

grandezze

scalari

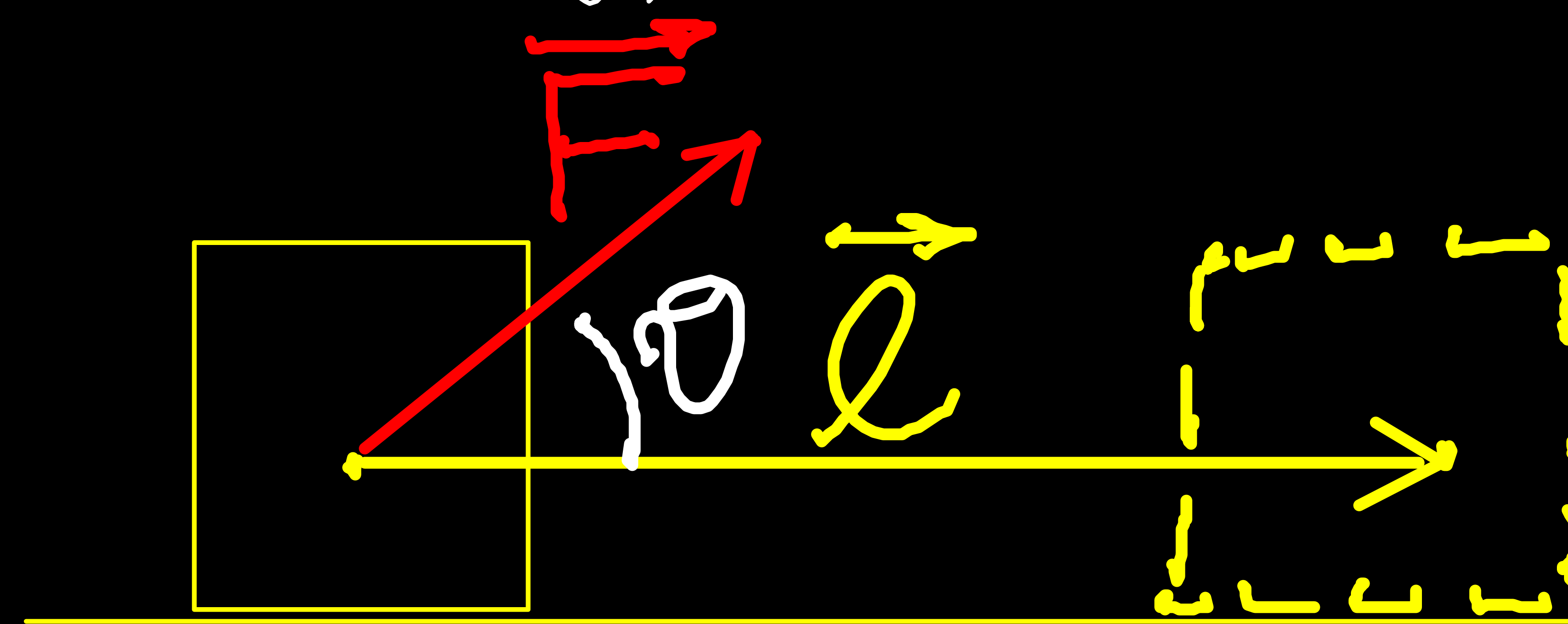
Scopo: Definire il lavoro di una forza $\vec{F}$




- traiettoria 3D di un punto materiale
- agiscono varie forze sul punto materiale tra cui $\vec{F}$ (non cost.)

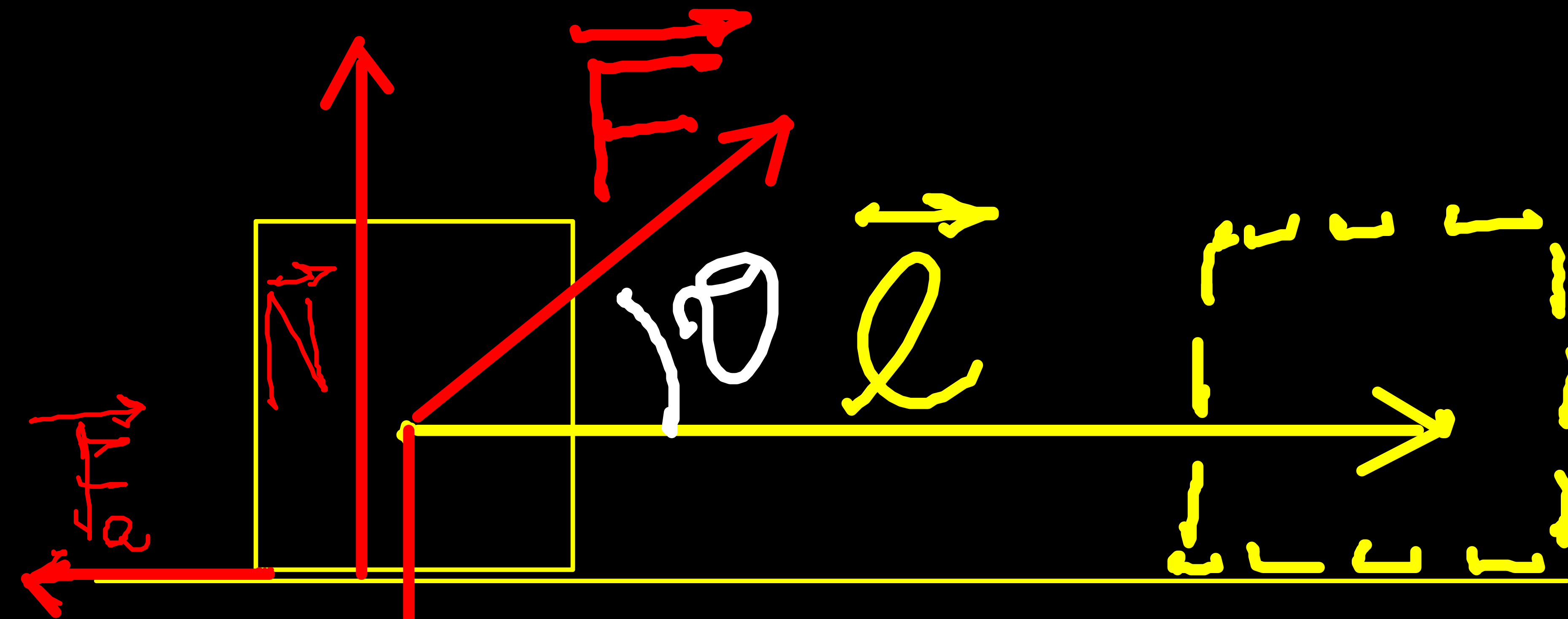
$$W = \int_i^f \vec{F} \cdot d\vec{r}$$

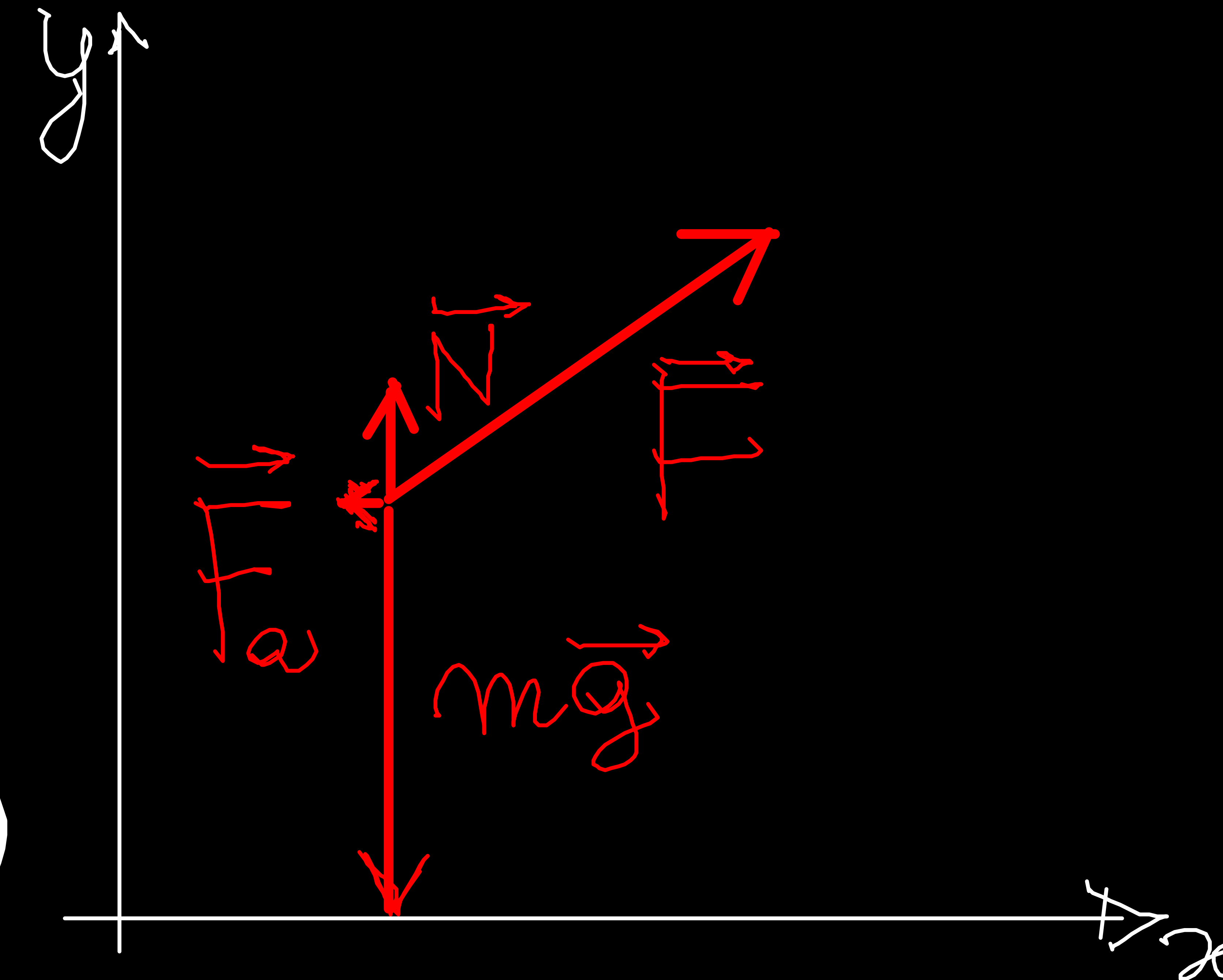
## 8.2 Lavoro di una forza costante agente su un corpo che fa un moto in linea retta. $\Delta\vec{r} = \vec{\ell}$




$$W = F\ell\cos\theta$$

Unità nel SI $\quad J = Nm$




$m = 41\,kg$

$\mu_K = 0{,}45$

$l = 6{,}5\,m$

$F = 460\,N$

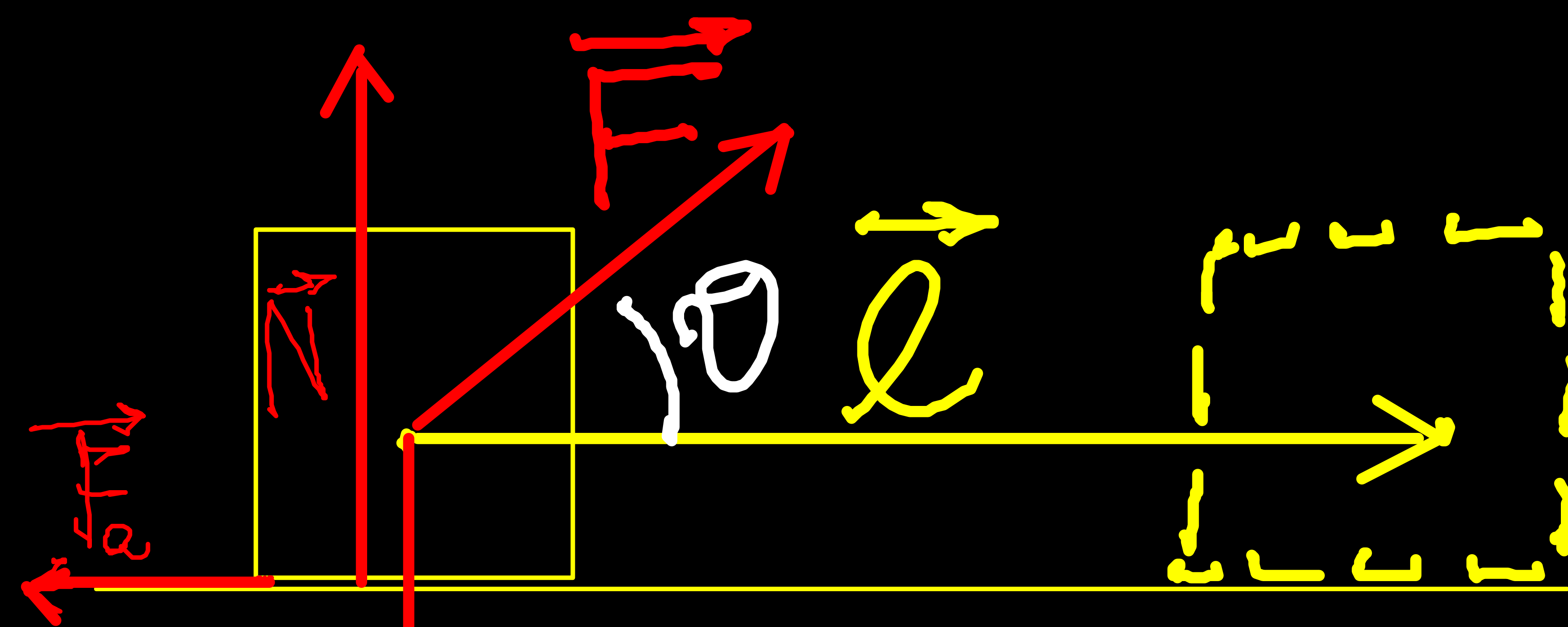
$\theta = 30$

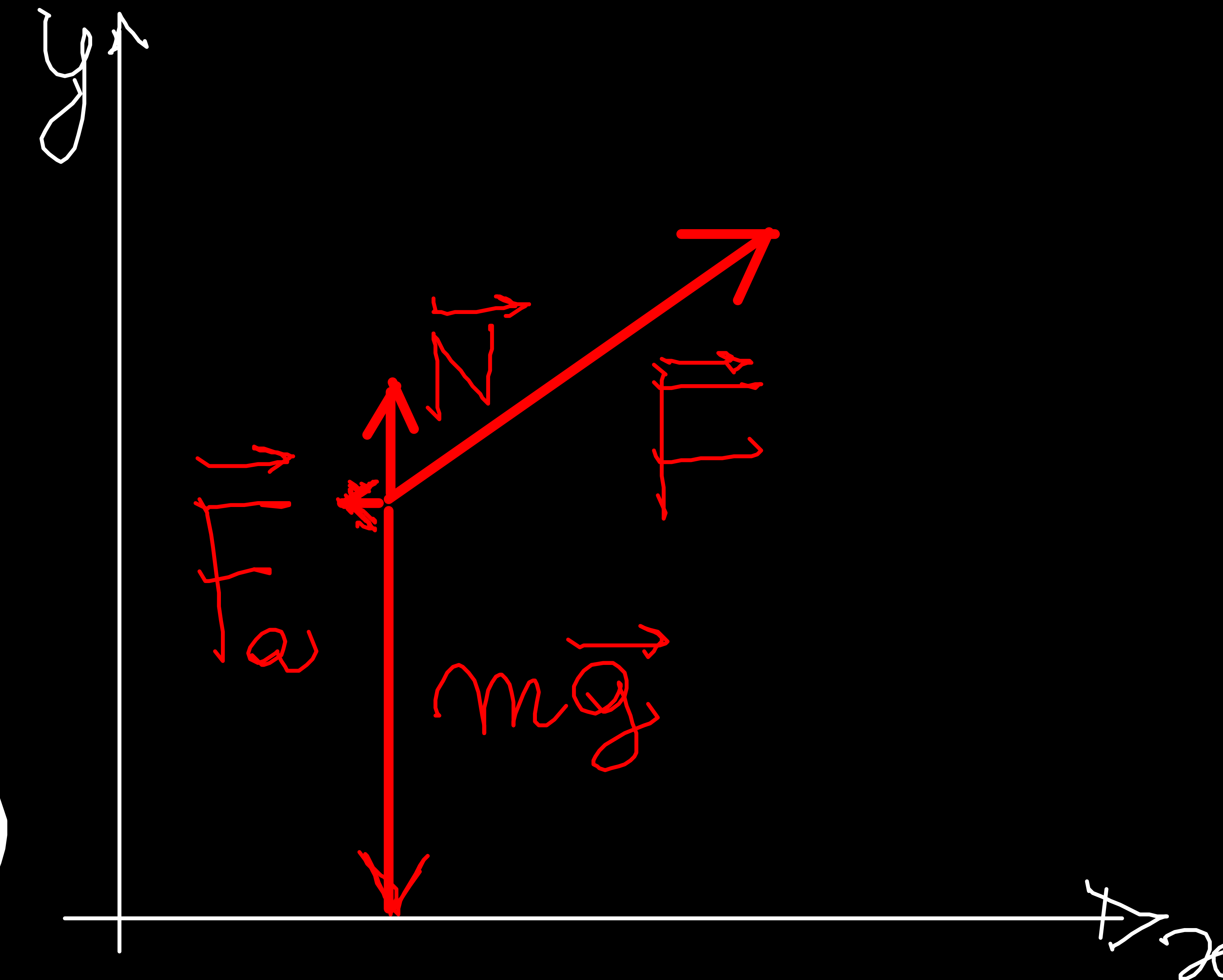
Calcoli W:

$$W = F\,l\cos\theta$$

1) $m\vec g$ $\quad W_{mg} = 0,\ m\vec g \perp \vec l$

2) $\vec N$ $\quad W_N = 0\ ,\ \vec N \perp \vec l$

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$\theta = 30$

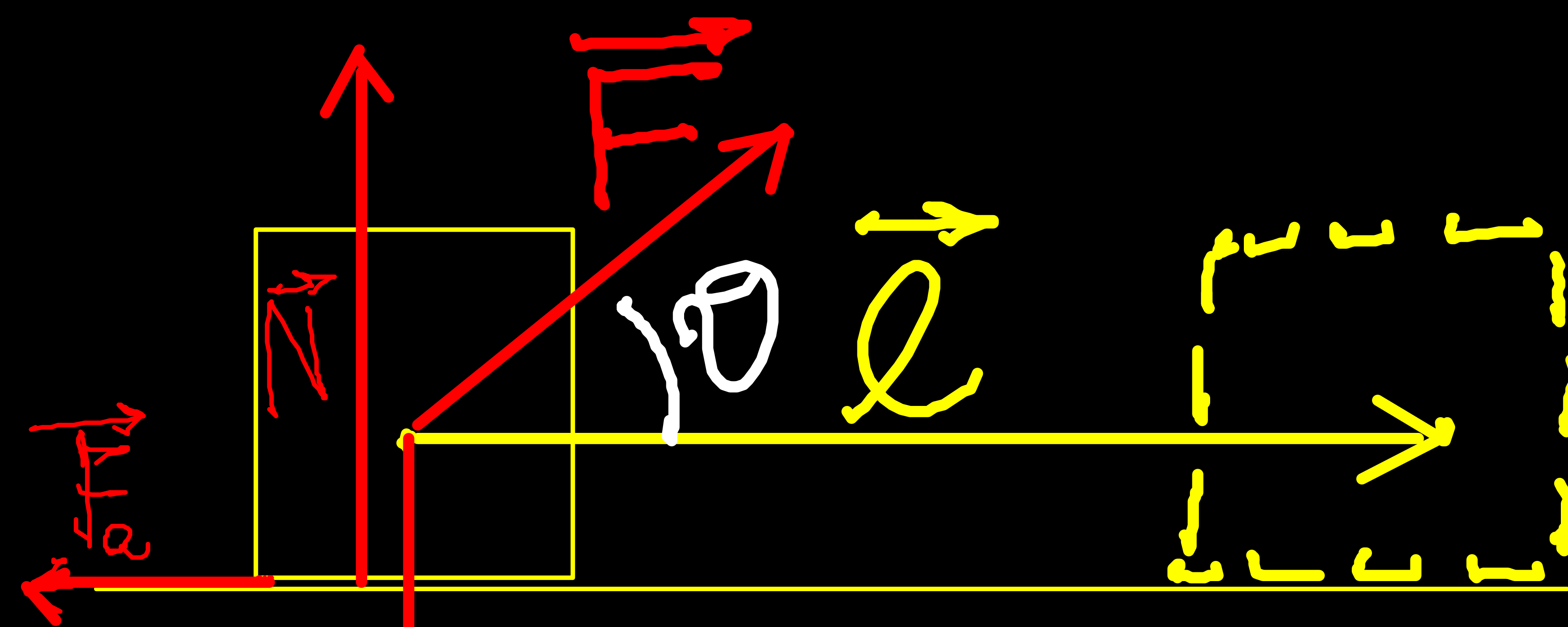
Calcolare $W_i$

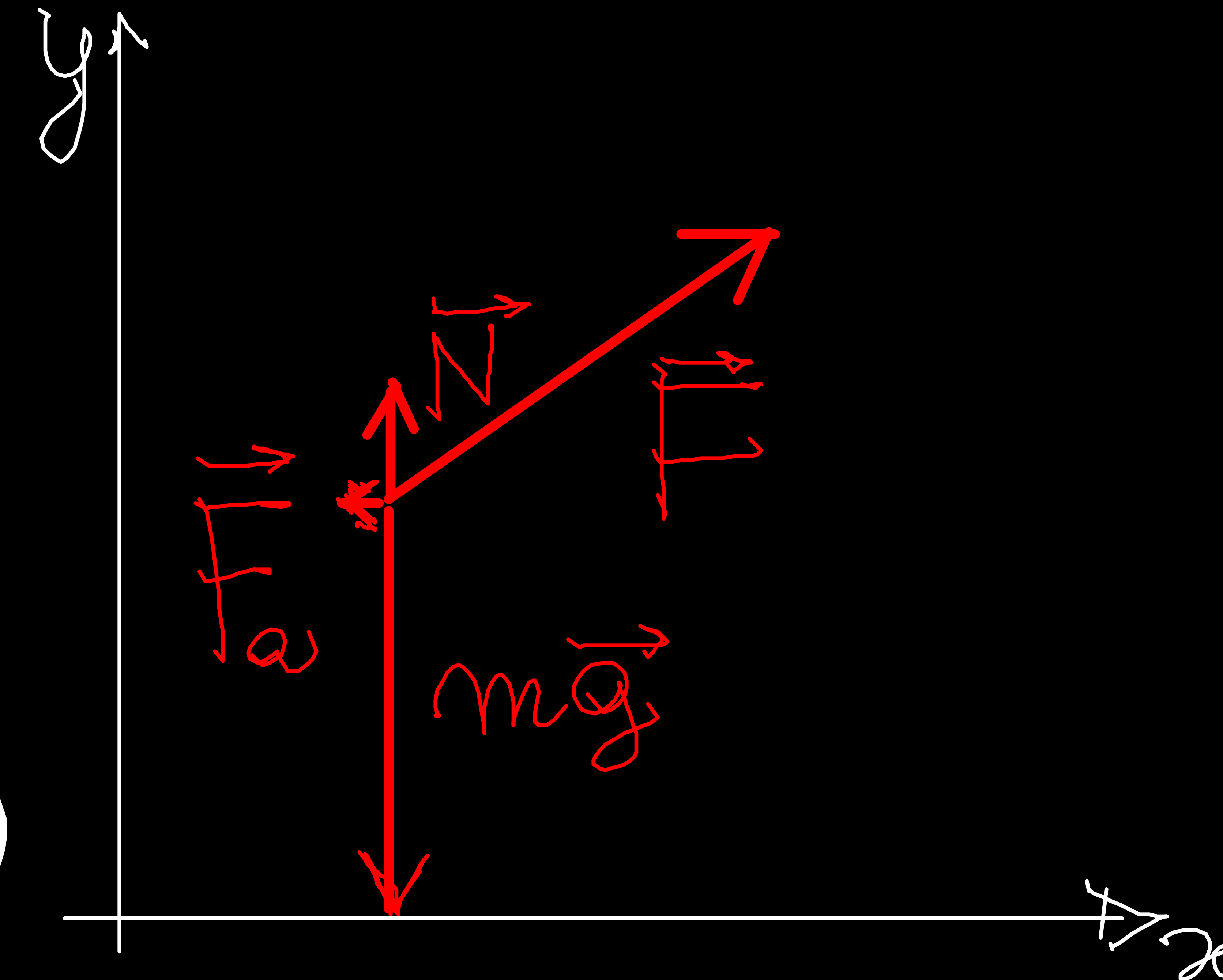
$$W = F\,l\cos\theta$$

1) $m\vec{g}$ $\quad W_{mg} = 0,\ m\vec{g} \perp \vec{l}$

2) $\vec{N}$ $\quad W_N = 0,\ \vec{N} \perp \vec{l}$

3) $\vec{F}$ $\quad W_F = 460\,N\; 6.5\,m\;\frac{\sqrt{3}}{2} = 2.6\,kJ$

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$F = 460\ N$

$\theta = 30^\circ$

Cercari W:

$$W = F\, l \cos\theta$$

1) $m\vec{g}$ $\quad W_{mg} = 0,\ m\vec{g} \perp \vec{l}$

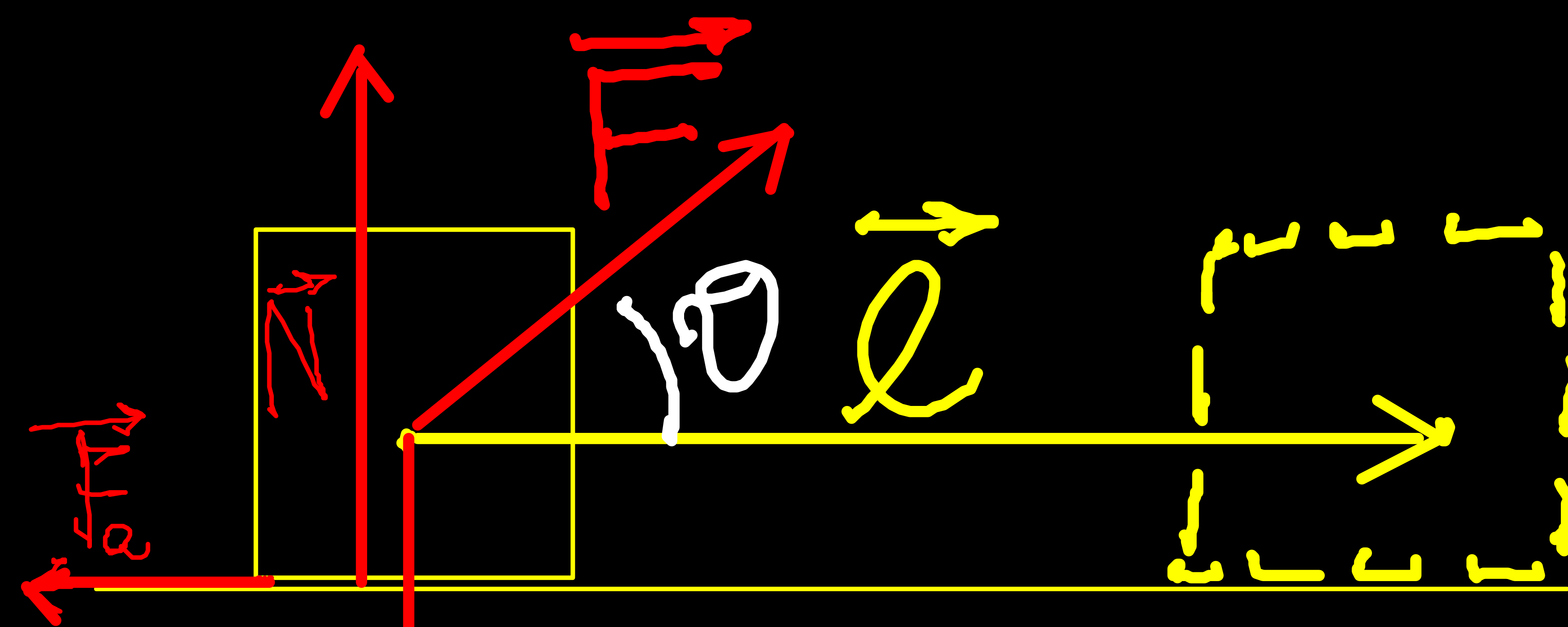
2) $\vec{N}$ $\quad W_N = 0,\ \vec{N} \perp \vec{l}$

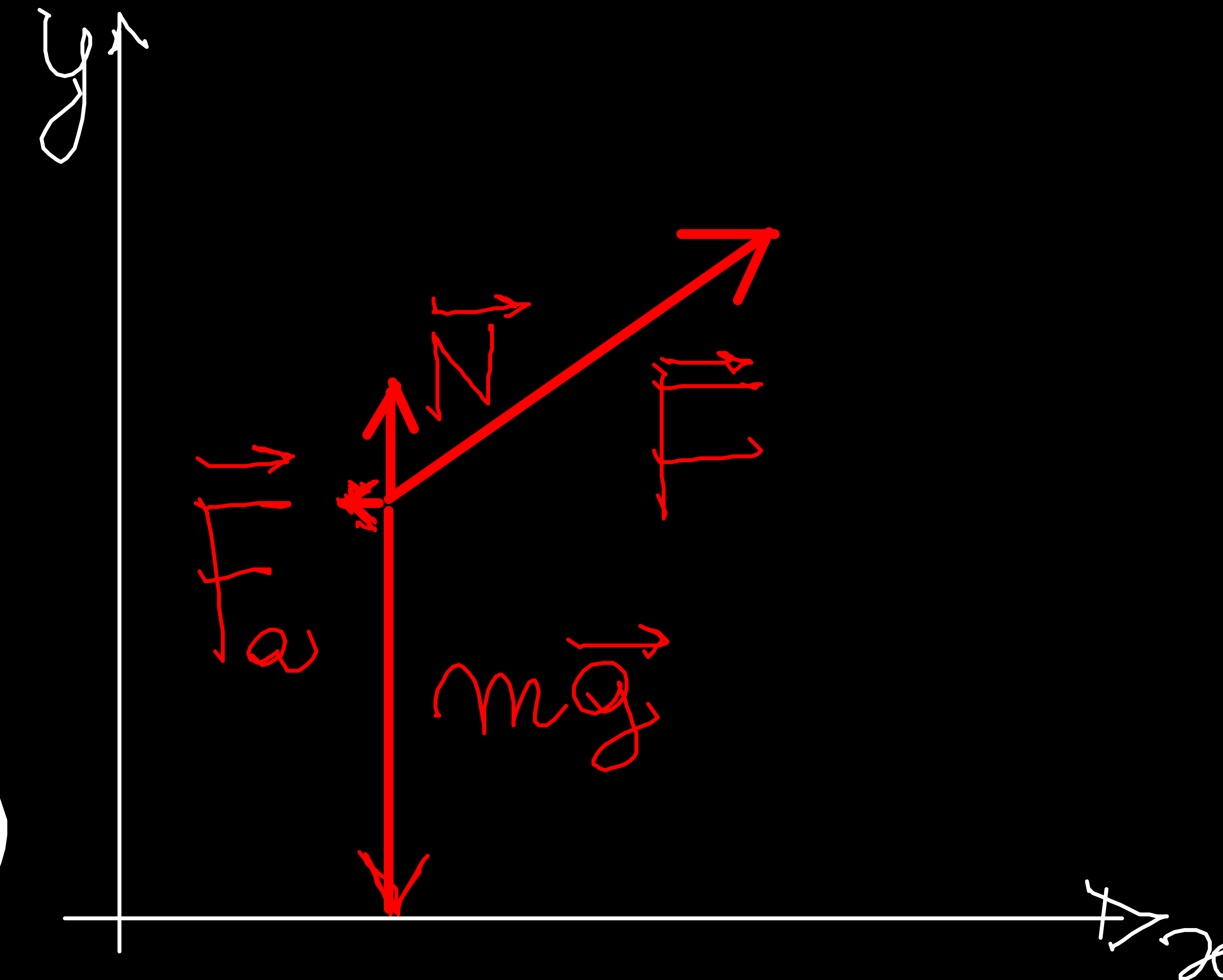
Proietto y

$$F_y + N_y + F_{P_y} = 0$$

$$F \operatorname{sen}\theta + N - mg = 0$$

$$N = mg - F \operatorname{sen}\theta = 402 - 230 = 172\ N$$

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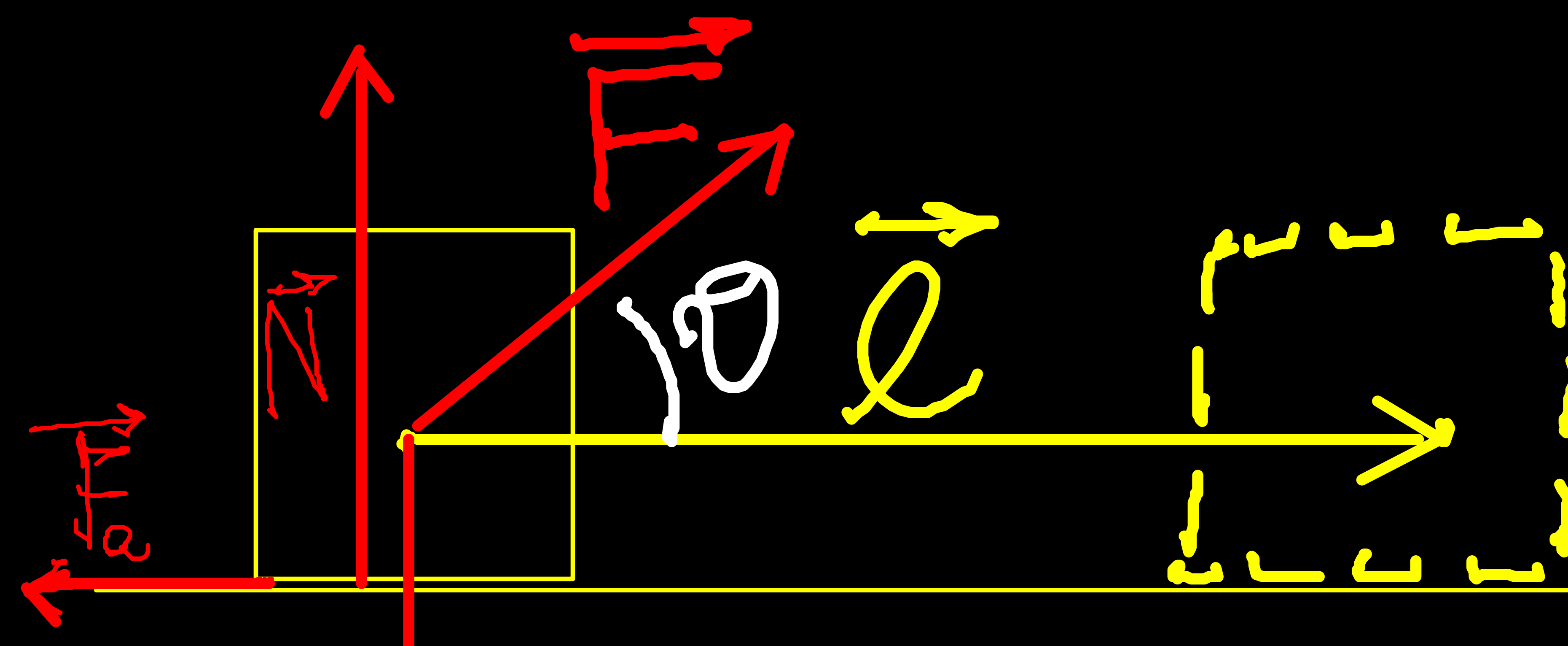
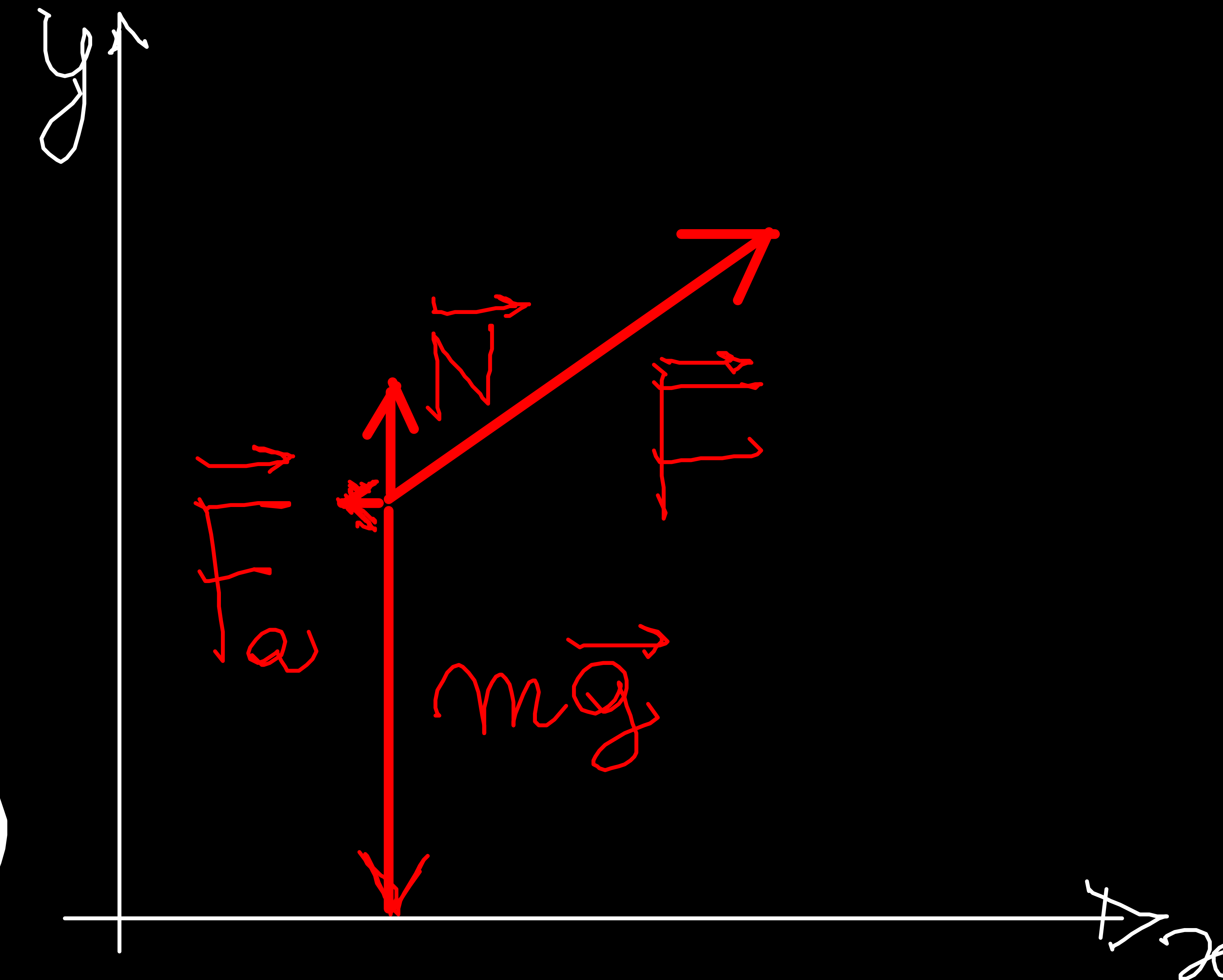
$$\boxed{\vec{F}_a = -\mu_K N\,\hat{\imath}}$$

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Calcolare $W$:

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1) $m\vec g$  $W_{mg} = 0,\ m\vec g \perp \vec l$

2) $\vec N$  $W_N = 0,\ \vec N \perp \vec l$

3) $\vec F$  $W_{\vec F} = 460\,N\;6.5\,m\,\frac{\sqrt{3}}{2} = 2.6\,KJ$

4) $\vec F_a$  $W_{F_a} = F_a\,l\,\underbrace{\cos 180^\circ}_{-1} = -503\,J$

Prodotto scalare

$$W = \vec{F} \cdot \vec{l}$$

gode della proprietà

- commutativa $\vec{F} \cdot \vec{l} = \vec{l} \cdot \vec{F}$
- distributiva $\vec{F} (\vec{l}_1 + \vec{l}_2) = \vec{F} \cdot \vec{l}_1 + \vec{F} \cdot \vec{l}_2$
- inoltre $\vec{F} \cdot (s \vec{l}) = s (\vec{F} \cdot \vec{l})$

$\vec{F}$ e $\vec{l}$ in coordinate cartesiane

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{l} = l_x \hat{i} + l_y \hat{j} + l_z \hat{k}$$

$$W = \vec{F} \cdot \vec{l} = \left(F_x \hat{i} + \dots\right)\left(l_x \hat{i} + \dots\right) =$$

$$= F_x l_x \underbrace{\hat{i} \cdot \hat{i}}_{1} + F_x l_y \underbrace{\hat{i} \cdot \hat{j}}_{=0} + \dots\dots =$$

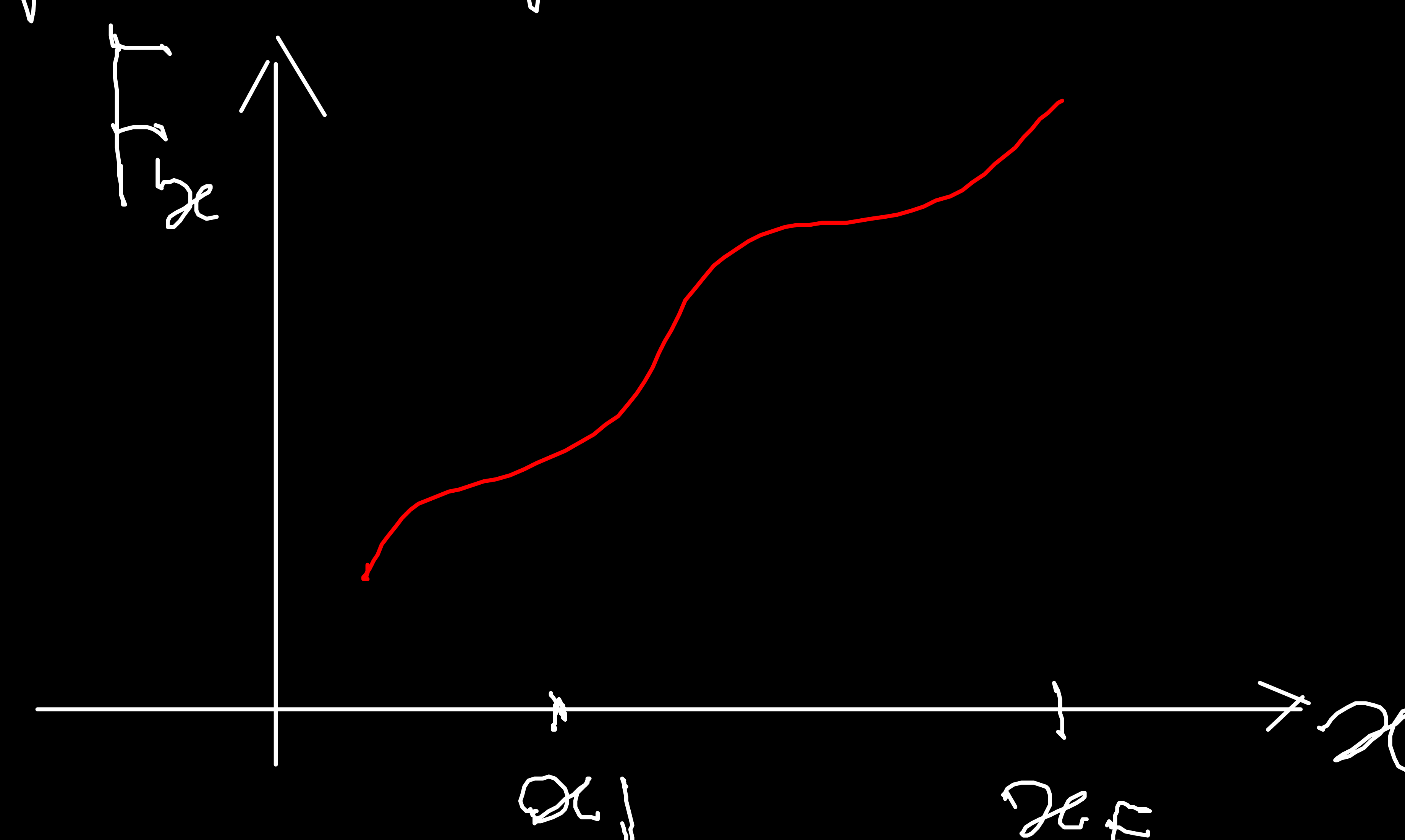
$$= F_x l_x + F_y l_y + F_z l_z$$

## 8.4 forza variabile

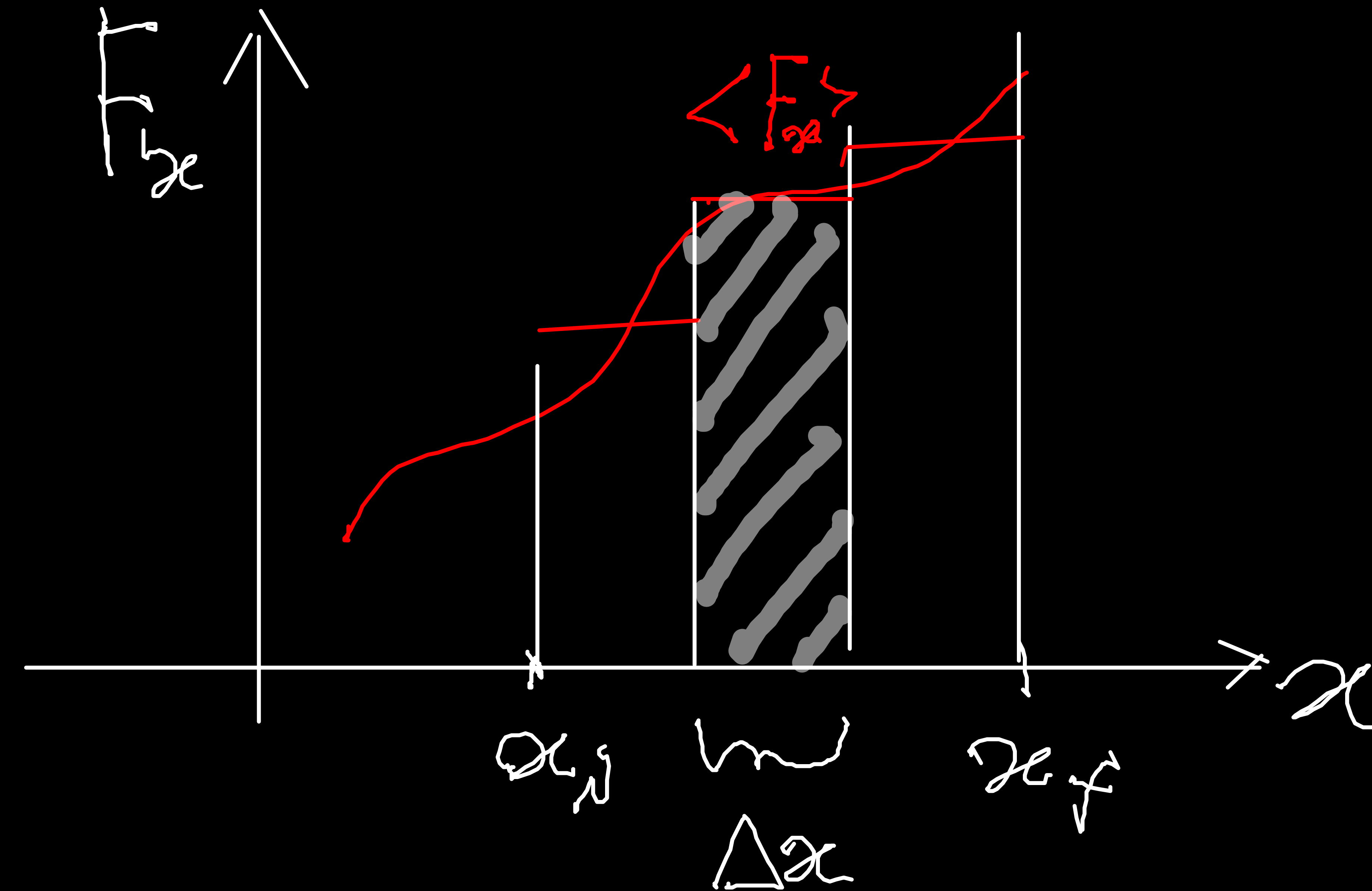
$\Delta\vec{r} = \vec{l}$ avviene nell'intervallo $\Delta t$

Ipotesi iniziale semplificativa
- spostamento solo lungo $x$ da $x_i$ a $x_f$
- forza con componente $\neq 0$ solo lungo $x$

$F_x(x)$



$F_x(x)$




N sotto intervalli

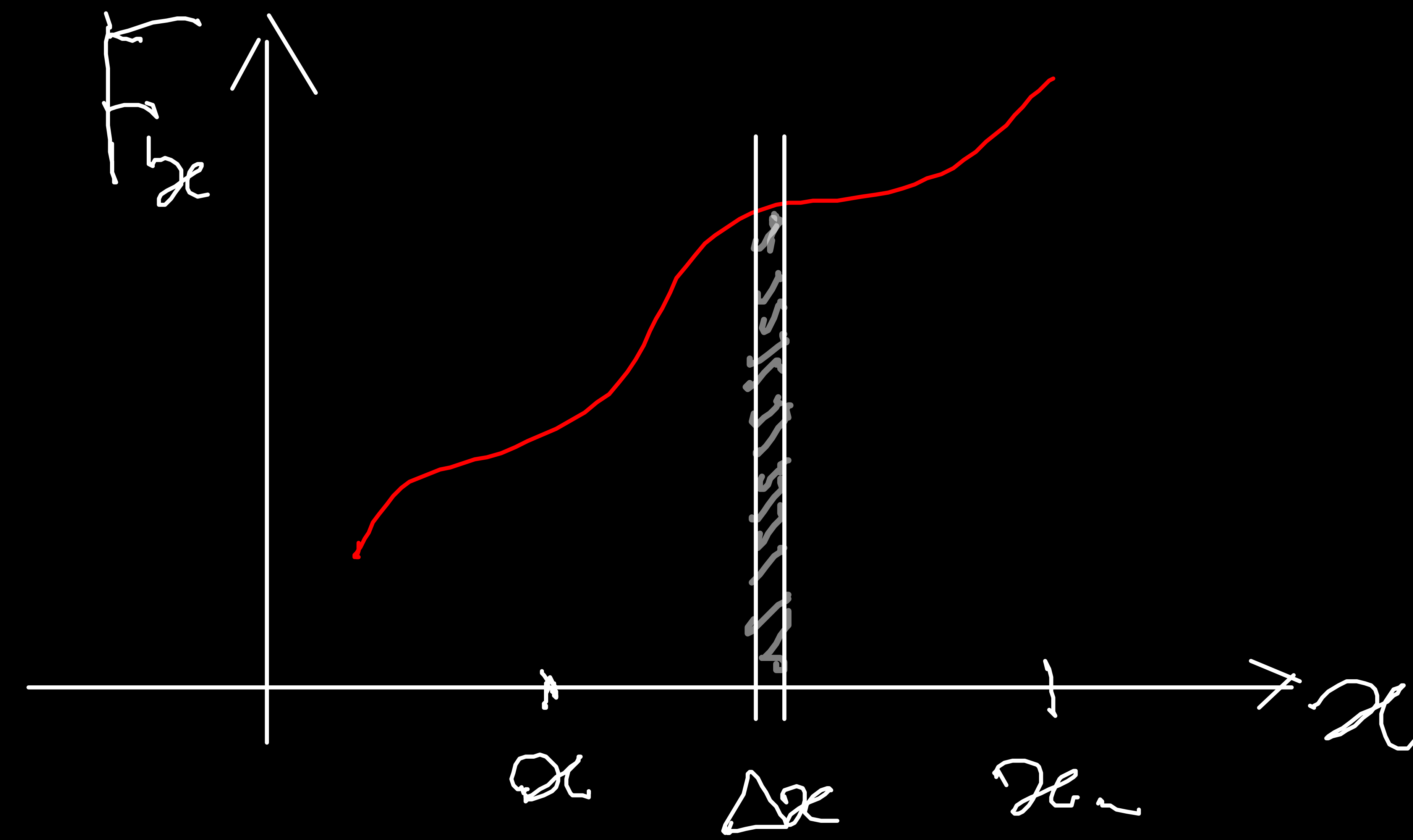
$$\Delta x = \frac{x_f - x_i}{N}$$

$$W' = \langle F_x \rangle \Delta x$$

Se aumento N, allora $\Delta x$ diminuisce

$$\langle F_x \rangle \approx F_x(\langle x \rangle) = F_x \langle x \rangle$$

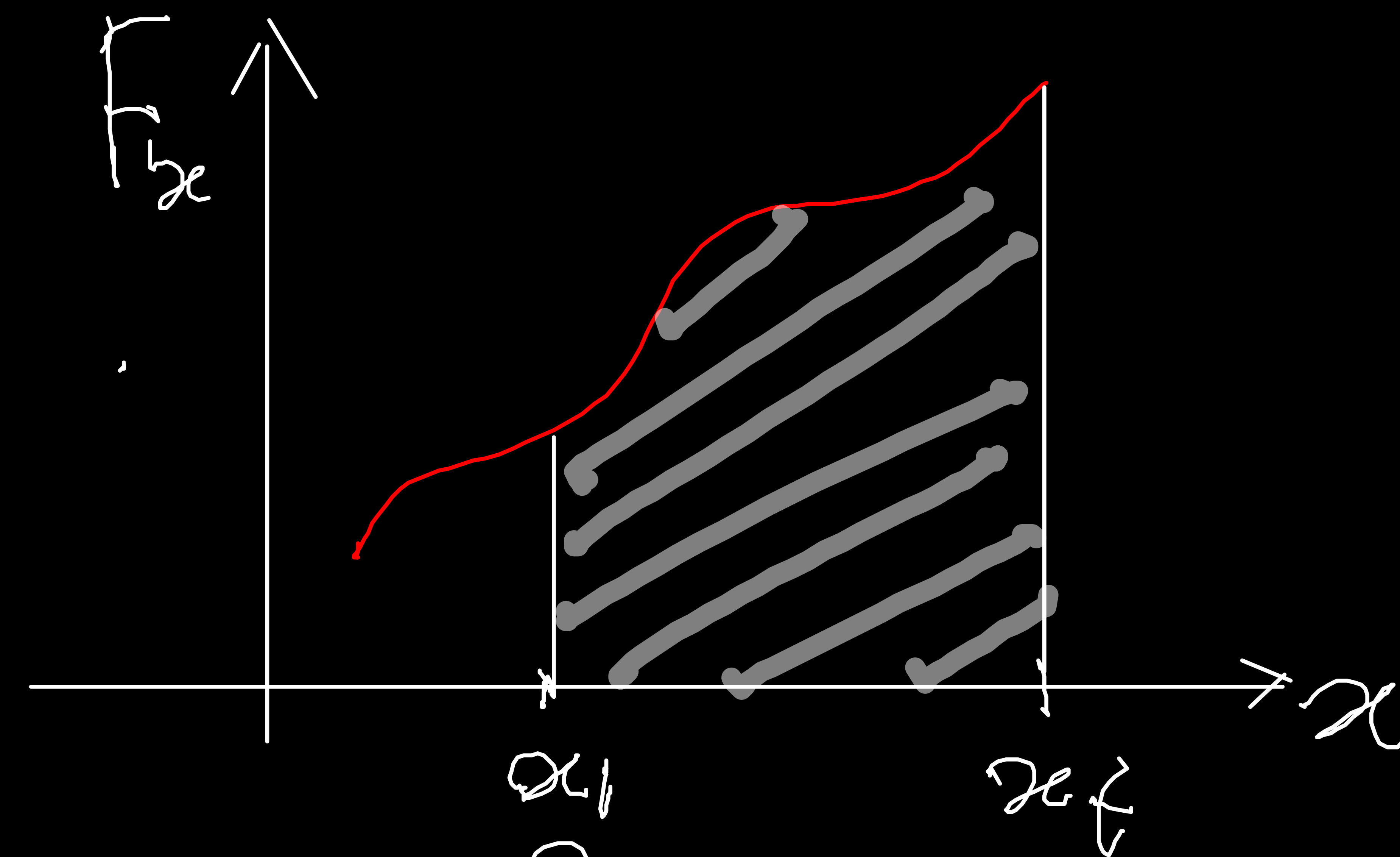
$F_x(x)$




$$W_{x_i \to x_f} \approx \sum F_x\langle x\rangle \Delta x$$

Per $N \to \infty$ $\quad \Delta x \to 0 \quad \approx \;\longrightarrow\; =$

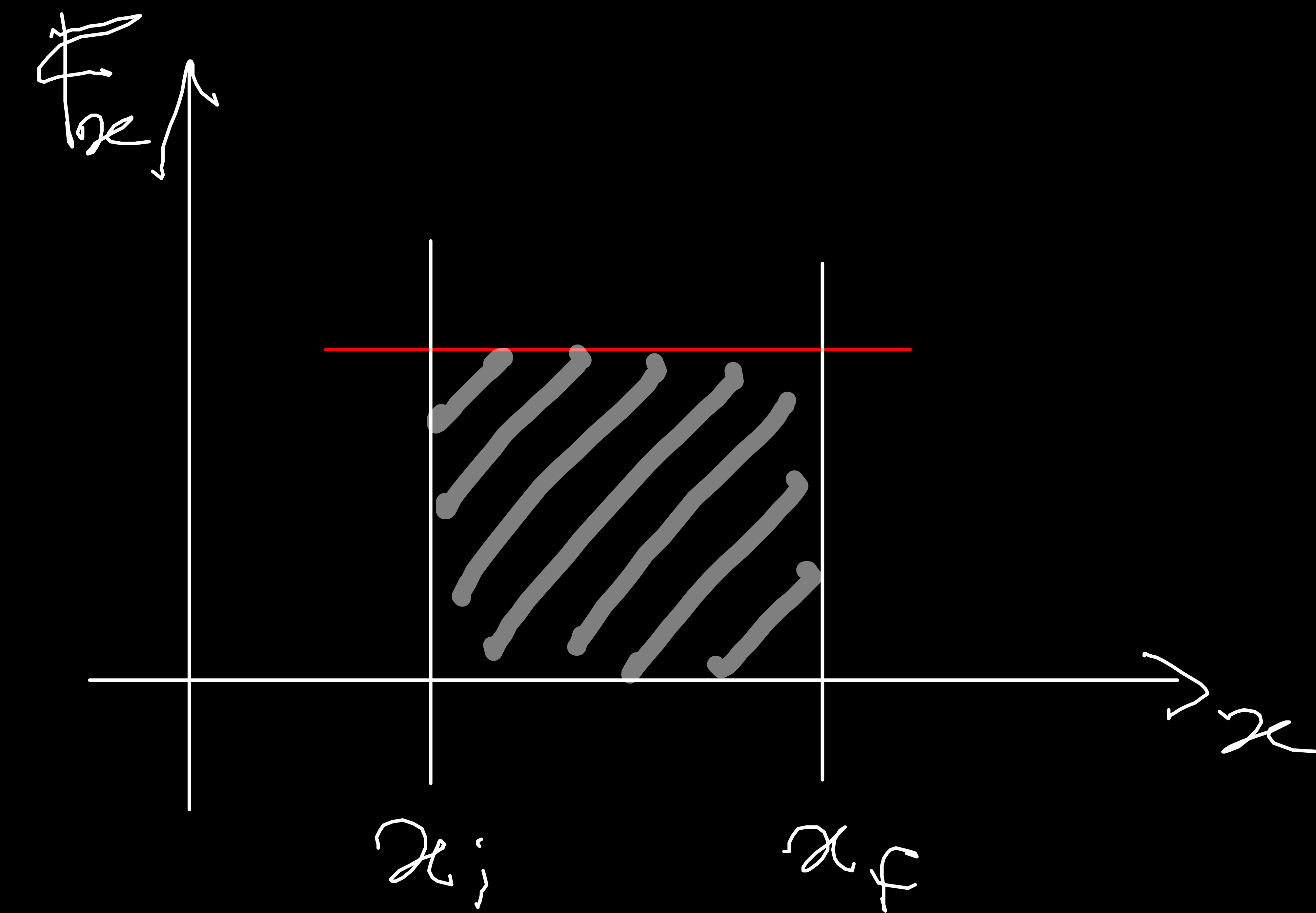
$F_x(x)$




$$W = \int_{x_i}^{x_f} F_x(x)\,dx$$

pari all'area sottesa dalla funzione $F_x(x)$

Corollario: Se $F_x$ uniforme



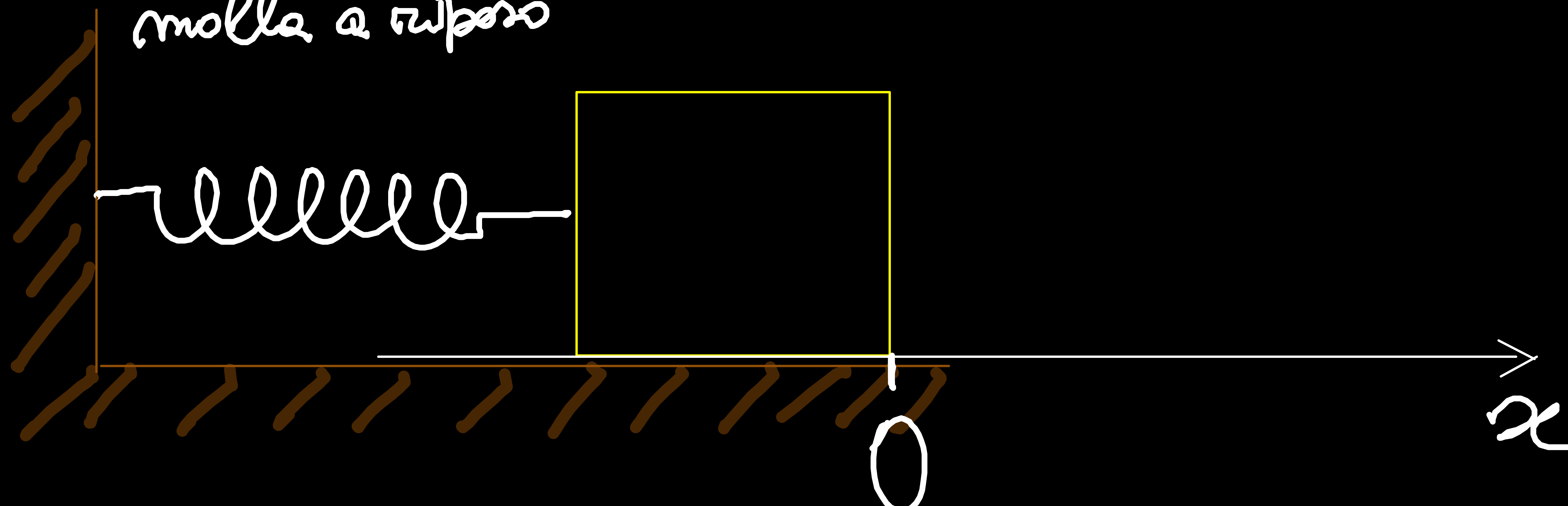

$$W = \int_{x_i}^{x_f} F_x(x)\,dx = \int_{x_i}^{x_f} F_x\,dx = F_x \int_{x_i}^{x_f} dx = F_x\left(x_f - x_i\right)$$

area del rettangolo
spazio disegnato

Esempio (importante)

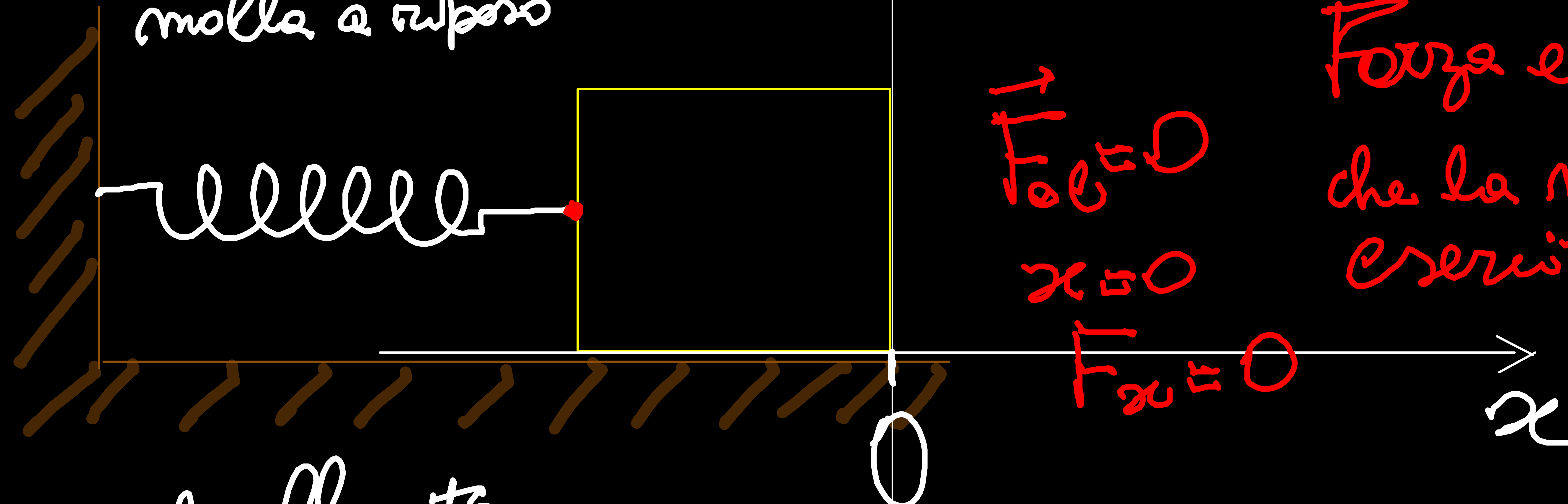
Lavoro forza elastica

1) molla a riposo



1) molla a riposo

Forza el $\vec{F}_{el}$ che la molla esercita sul blocco

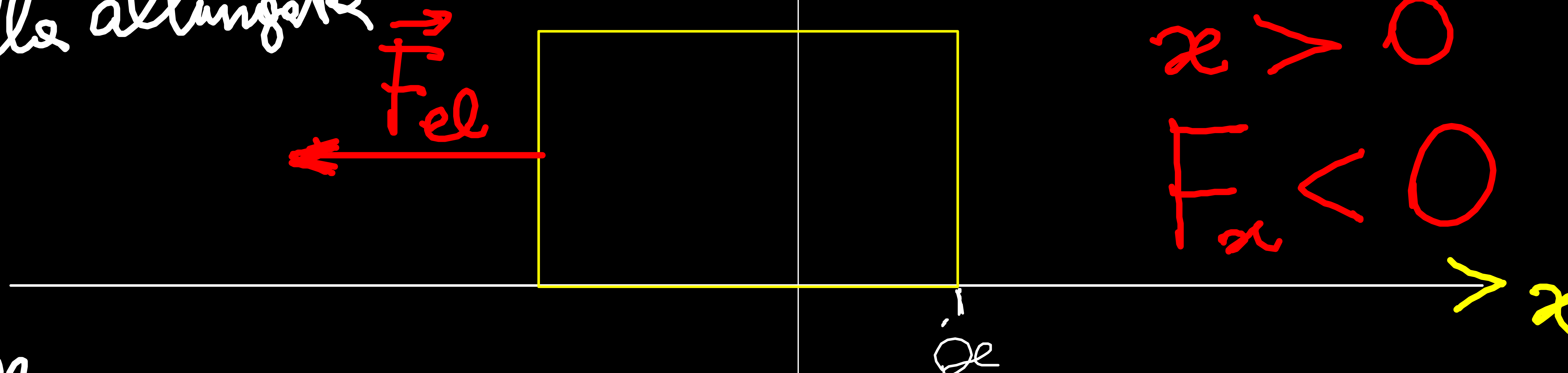



$\vec{F}_{el}=0$

$x=0$

$F_x=0$

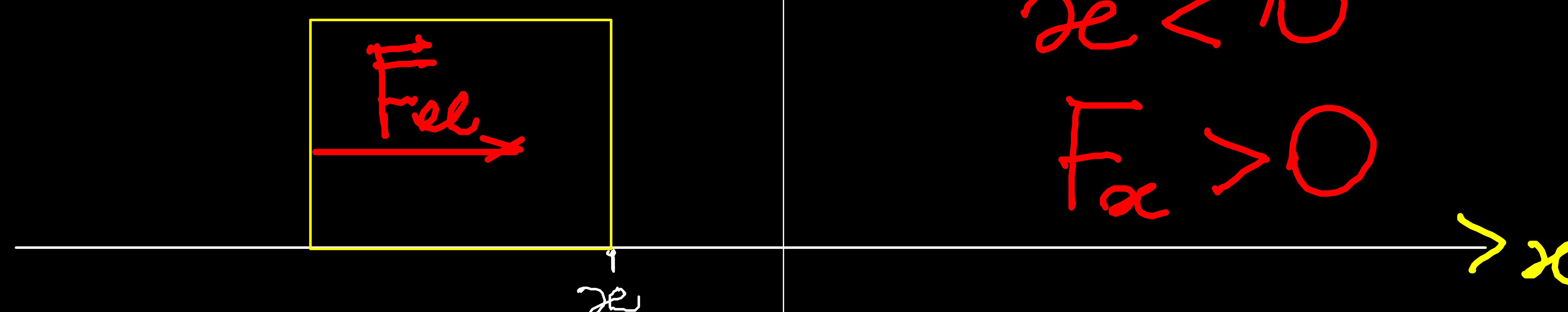
2) molla allungata




$x>0$

$F_x<0$

3) molla compressa




$x<0$

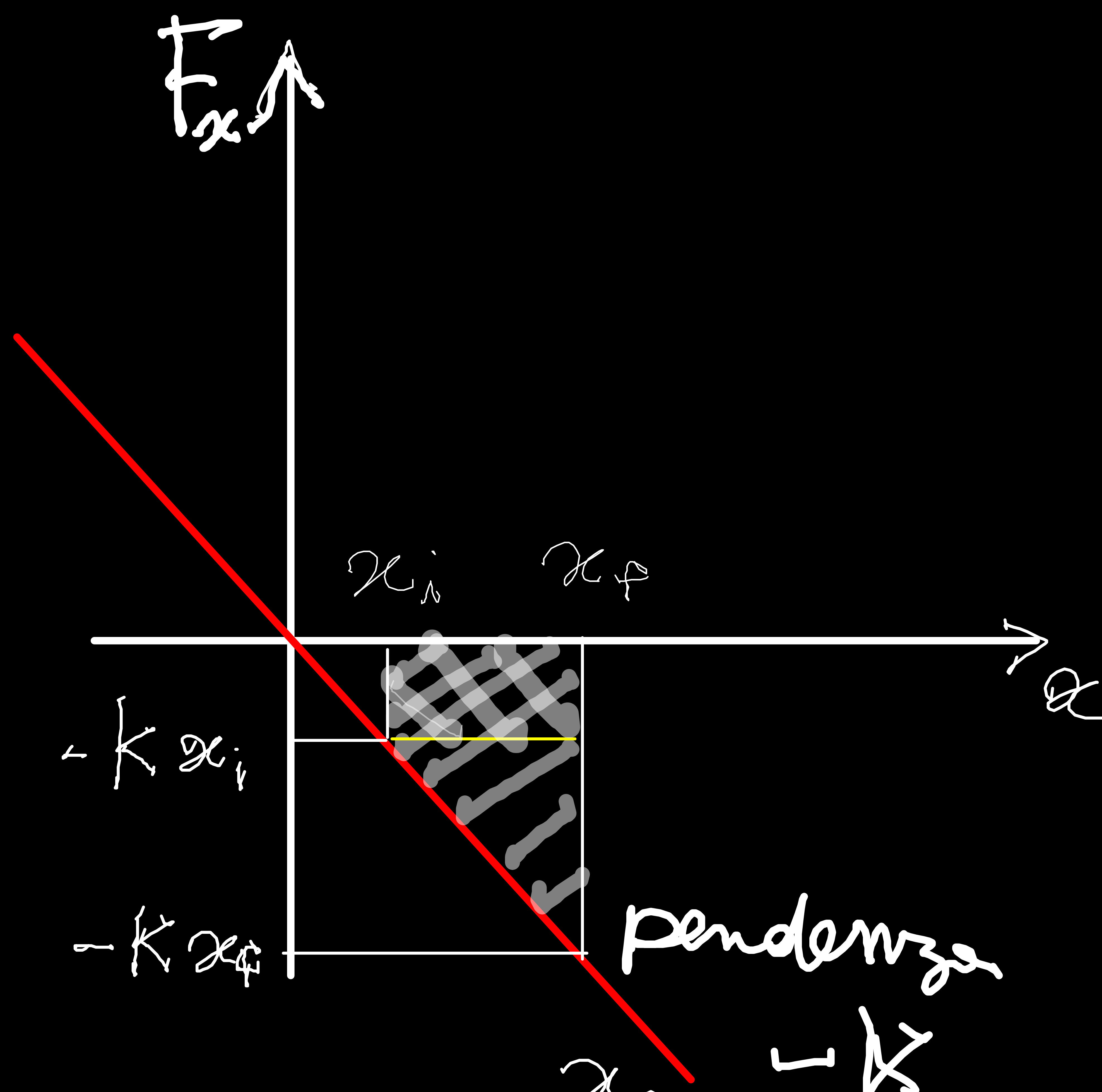
$F_x>0$

# Legge di Hooke

Vale per $x$ piccoli
(def. reversibili)

$$F_x(x) = -Kx$$

- $K$ costante elastica della molla
  nel SI si misura in $N/m$
- Segno "—" forza di richiamo




$$W = A_{\text{tot tratteggi}}$$

$$= \underset{\text{rett.}}{\text{area}} \quad -Kx_i(x_f - x_i) +$$

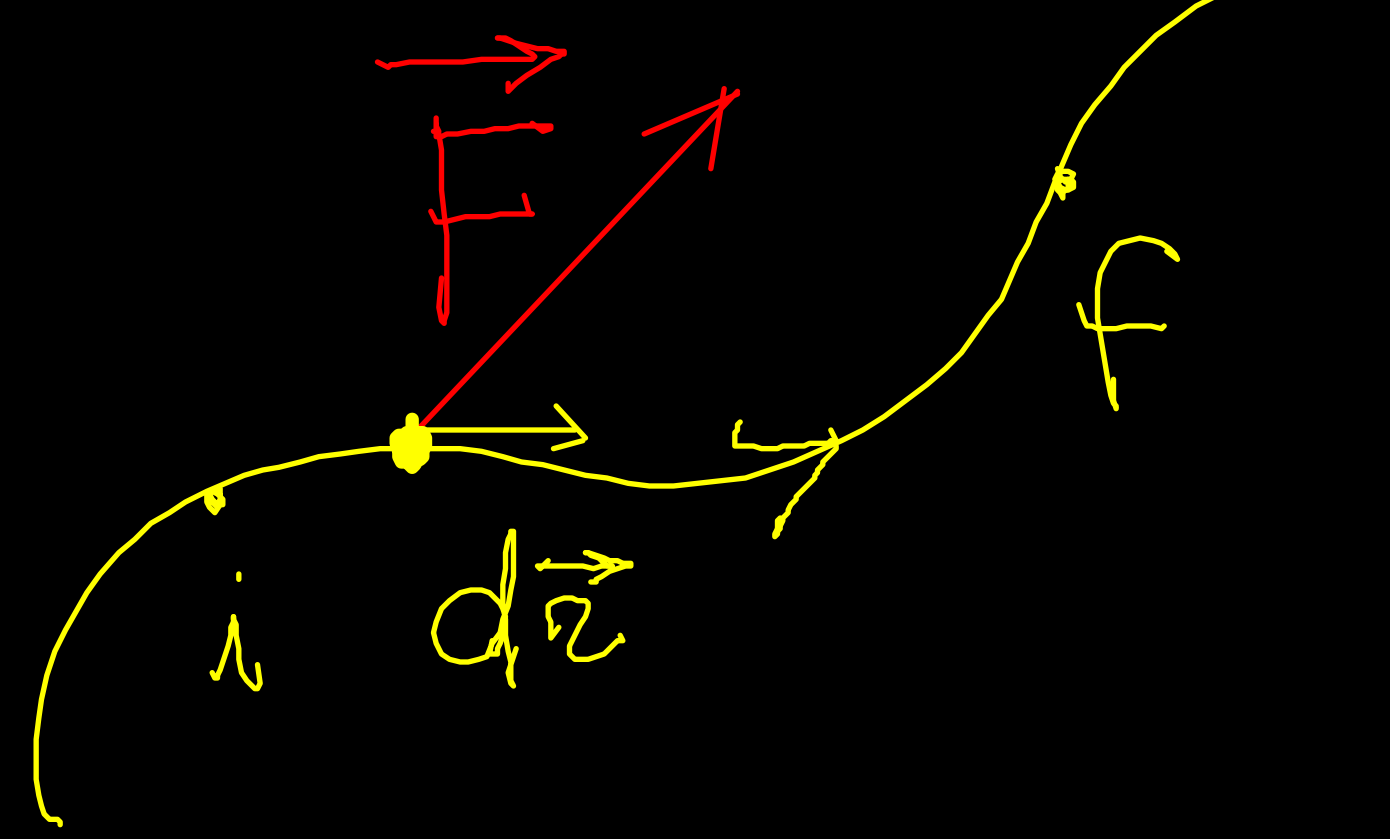
$$\underset{\text{triang.}}{\text{area}} \quad -\frac{1}{2}(Kx_f - Kx_i)(x_f - x_i)$$

$$= -\frac{1}{2}K(x_f^2 - x_i^2)$$

$$W = \int_{x_i}^{x_f} F_x(x)\,dx = \int_{x_i}^{x_f} -Kx\,dx =$$

$$= -K\int_{x_i}^{x_f} x\,dx = -K\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = -\frac{1}{2}K(x_f^2 - x_i^2)$$

Scopo: Definire il lavoro di una forza $\vec{F}$




- traiettoria 3D di un punto materiale
- agiscono varie forze sul punto materiale tra cui $\vec{F}$ (non cost.)

$$W = \int_i^f \vec{F} \cdot d\vec{r}$$


$$N \to \infty$$

$$W' = \vec{F} \cdot \Delta\vec{r}$$

$$W_{\underset{i \to f}{tot}} \approx \sum W' = \sum \vec{F} \cdot \Delta\vec{r}$$

$\lim N \to \infty$ integrale di linea $W = \int_i^f \vec{F} \cdot d\vec{r}$