Cinematica

- Sistema di riferimento fisso
- Vettori posizione $\vec{r}$ e spost. $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
- vettori velocità $\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$ $\quad \vec{v} = \lim_{\Delta t \to 0} \langle \vec{v} \rangle = \frac{d\vec{r}}{dt}$
- vettori accelerazione $\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t}$

$$\vec{a} = \lim_{\Delta t \to 0} \langle \vec{a} \rangle = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Legge oraria

$$x = x(t)$$

1) caso particolare

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

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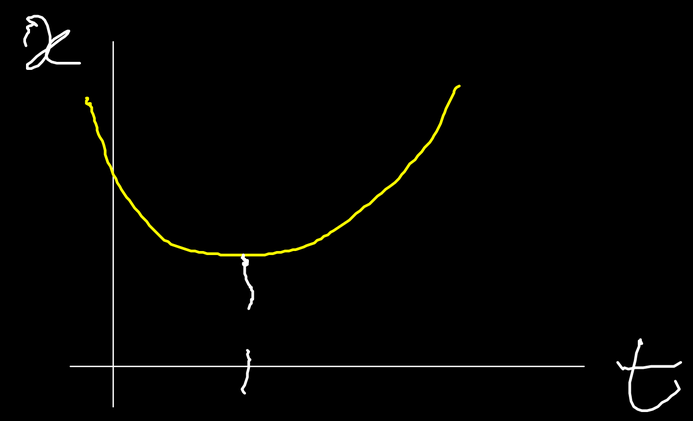
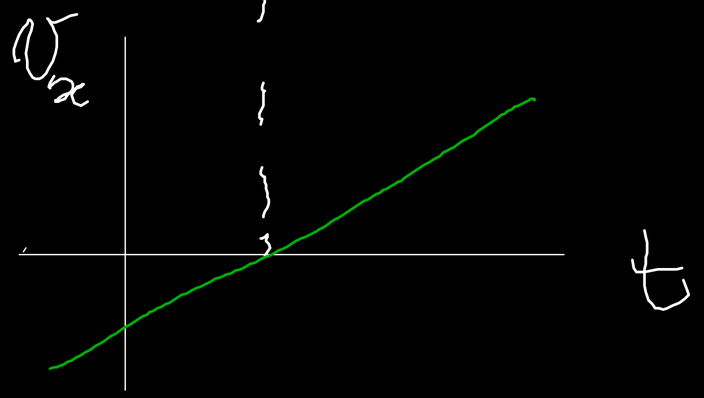
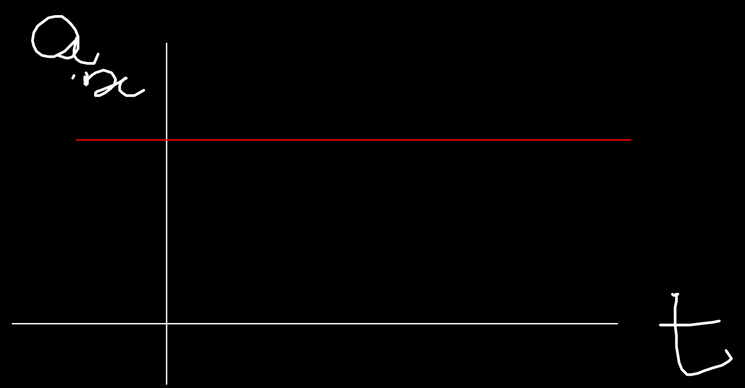
2) $y = y(t)$

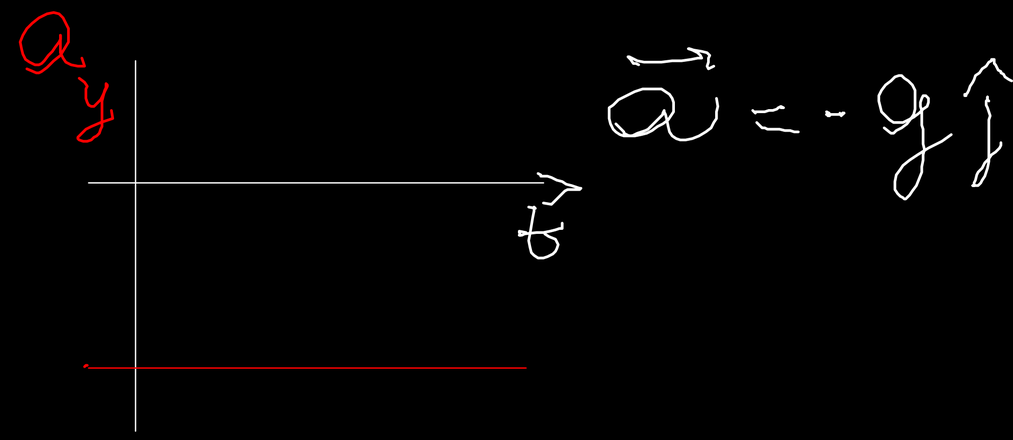
3) $\vec{r} = \vec{r}(t)$ $\quad x(t) \quad y(t) \quad z(t)$

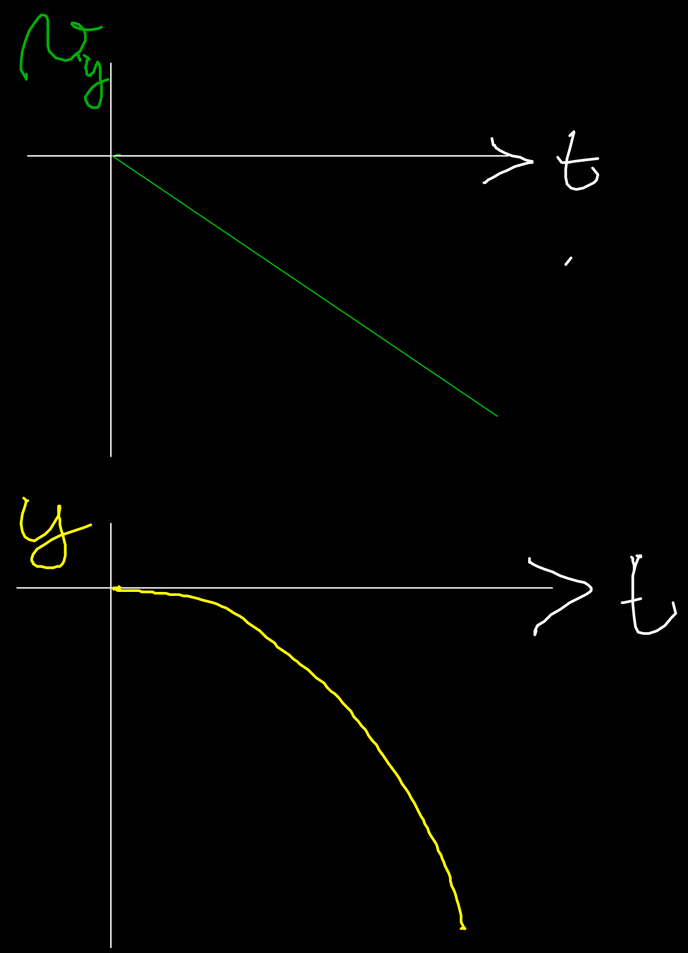
4) $\vec{v} = \vec{v}(t)$ $\quad v_x(t) \quad v_y(t) \quad v_z(t)$


x
t


v_x
t


a_x
t


a_y
t
$\vec{a} = -g\hat{j}$


v_y
t

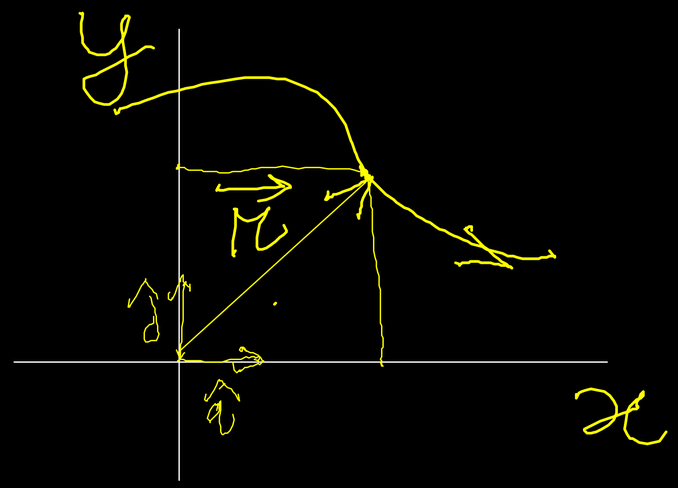
y
t

Moti 2D cap 4

x y

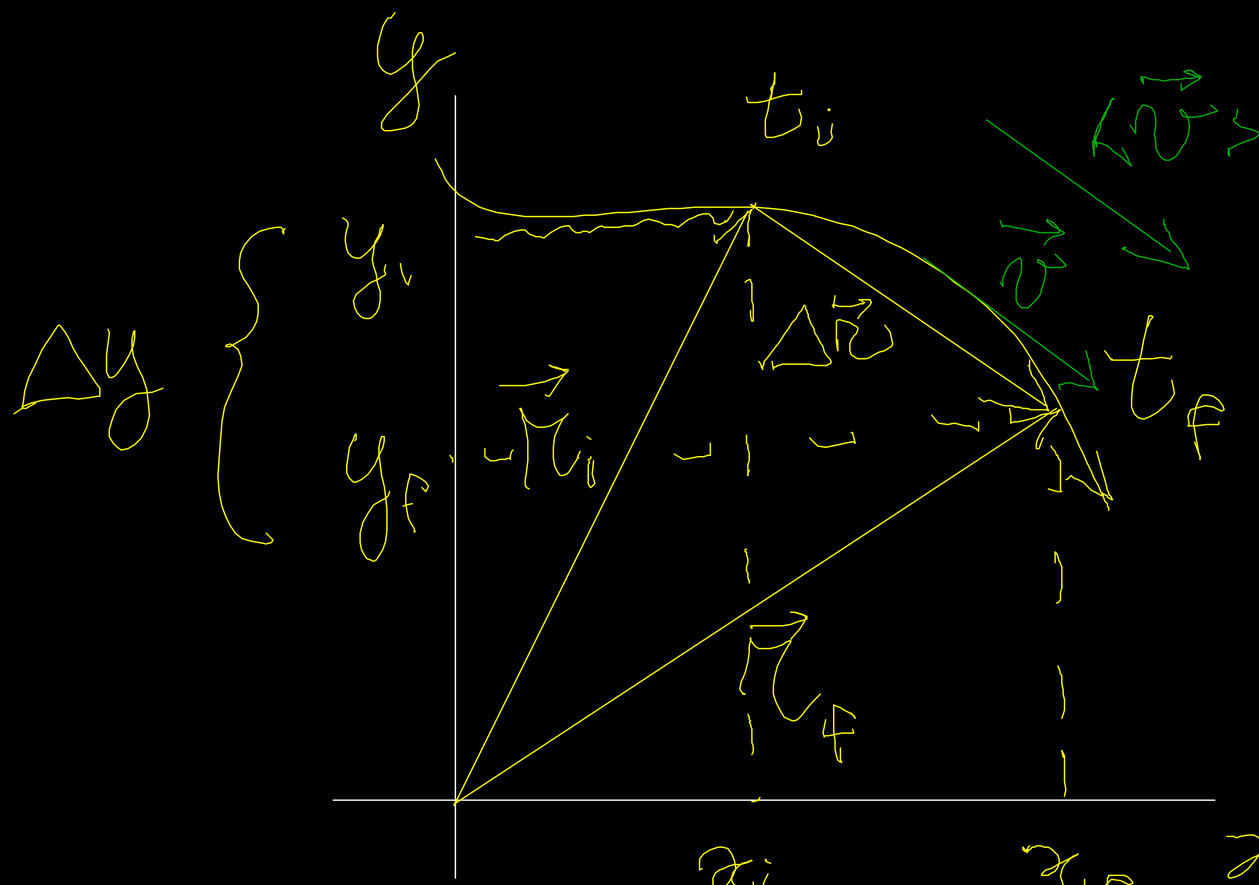
$\vec{r} = x\hat{i} + y\hat{j}$

percorso o traiettoria



$x(t)$

$y(t)$



$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$(x_f\hat{\imath} + y_f\hat{\jmath}) - (x_i\hat{\imath} + y_i\hat{\jmath})$$

$$= \underbrace{(x_f - x_i)}_{\Delta x}\hat{\imath} + \underbrace{(y_f - y_i)}_{\Delta y}\hat{\jmath}$$

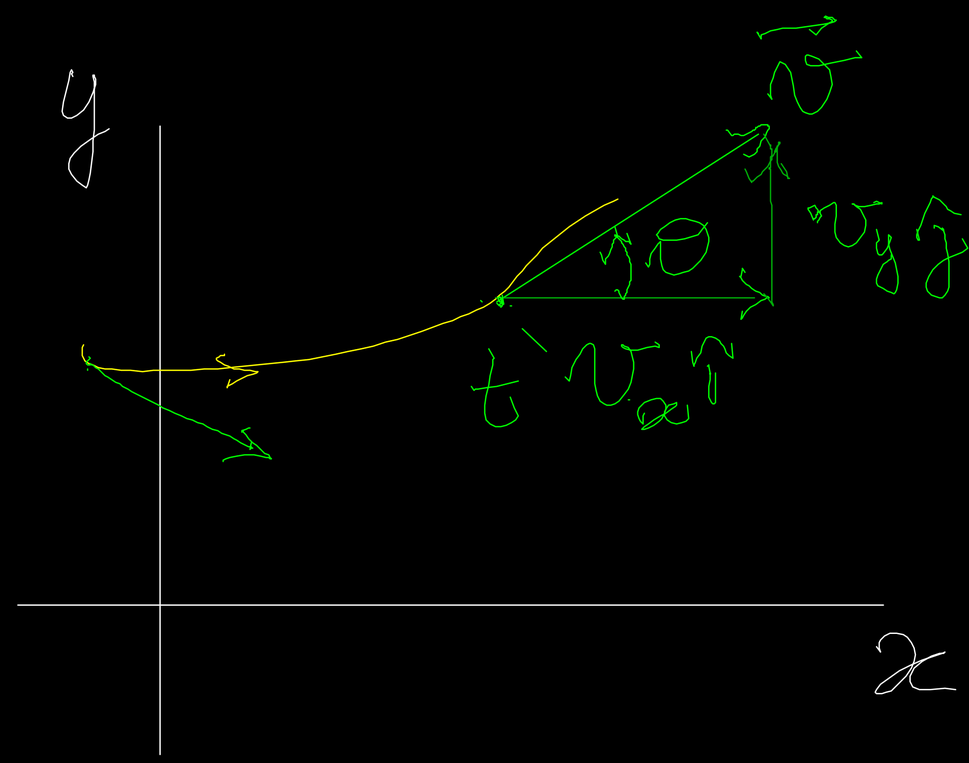
$$\Delta x \qquad \Delta y$$

$$\Delta x = x_f - x_i$$

velocità

$$\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$

$$\vec{v} = \lim_{\Delta t \to 0} \langle\vec{v}\rangle = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



$$v = \sqrt{v_x^2 + v_y^2}$$

$$tg\,\theta = \frac{v_y}{v_x}$$

$$\begin{cases} v_x = v\cos\theta \\ v_y = v\,\mathrm{sen}\,\theta \end{cases}$$

$$\vec{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \right)$$

$$= \left(\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right)\hat{i} + \left(\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \right)\hat{j}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

Es. 1

Motoscafo

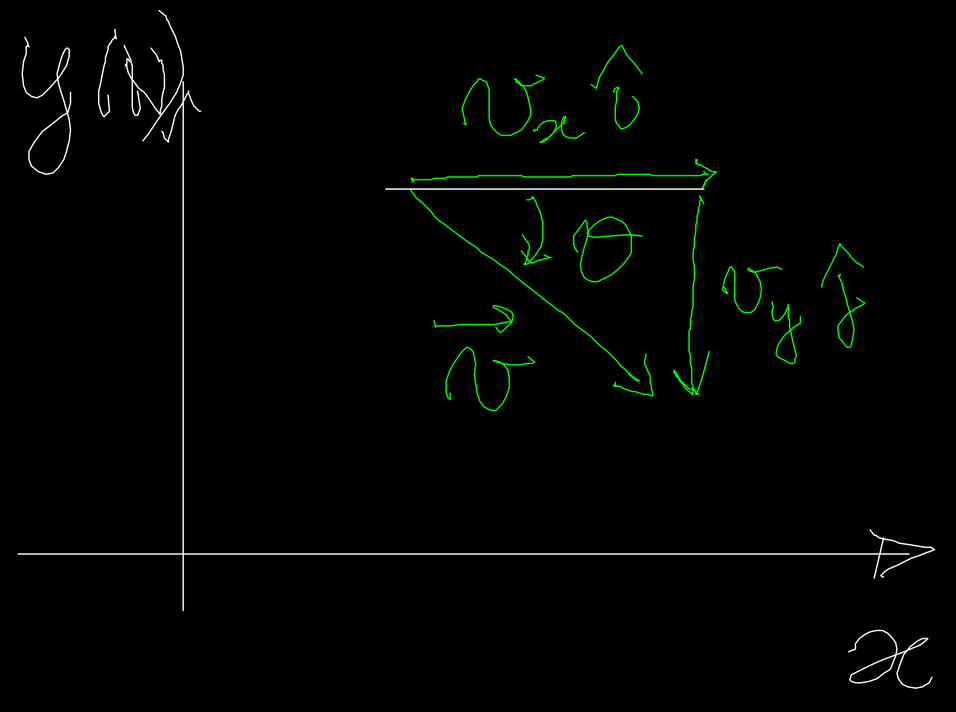
52°

vel. 18 nodi

$1\,M = 1852\,m$

$1\,nt = \frac{1\,M}{1\,h} = \frac{1852\,m}{3600\,s} = 0.514\,m/s$

$v = 18\,nt = 18\,\cancel{nt} \cdot \frac{0.514\,m/s}{1\,\cancel{nt}} = 9.26\,\frac{m}{s}$



$\theta = -52^\circ$

$$v_x = v\cos\theta = 9.26\frac{m}{s}\cos(-52^\circ)$$

$$= 5.70\ m/s = 5.7\frac{m}{s}$$

$$v_y = v\sin\theta = 9.26\frac{m}{s}\sin(-52^\circ)$$

$$= -7.297\frac{m}{s} = -7.3\ m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = 9.26\frac{m}{s} = 9.3\frac{m}{s}$$

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

$$= v_x \hat{i} + v_y \hat{j}$$

$$|\vec{v}| = v$$

Accelerazione

media $\langle \vec{a} \rangle = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\Delta v_x}{\Delta t}\hat{\imath} + \dfrac{\Delta v_y}{\Delta t}\hat{\jmath}$

$\langle a_x \rangle \hat{\imath} + \langle a_y \rangle \hat{\jmath}$

istantanea $\vec{a} = \lim_{\Delta t \to 0} \langle \vec{a} \rangle = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$

$\Delta \vec{v} = \Delta v_x \hat{\imath} + \Delta v_y \hat{\jmath}$

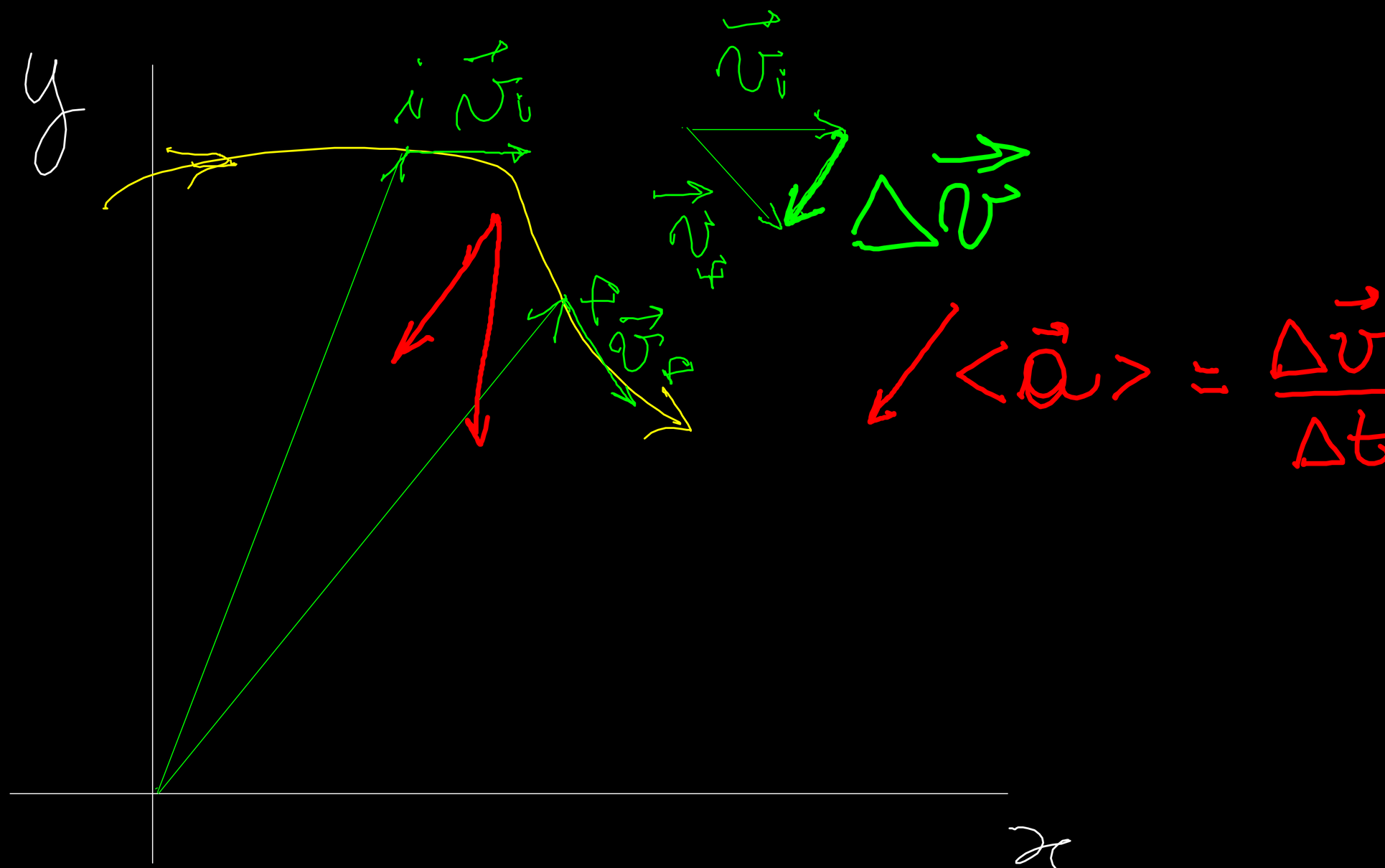
$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} \right)$$

$$= \left(\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \right) \hat{i} + \left(\quad \right) \hat{j}$$

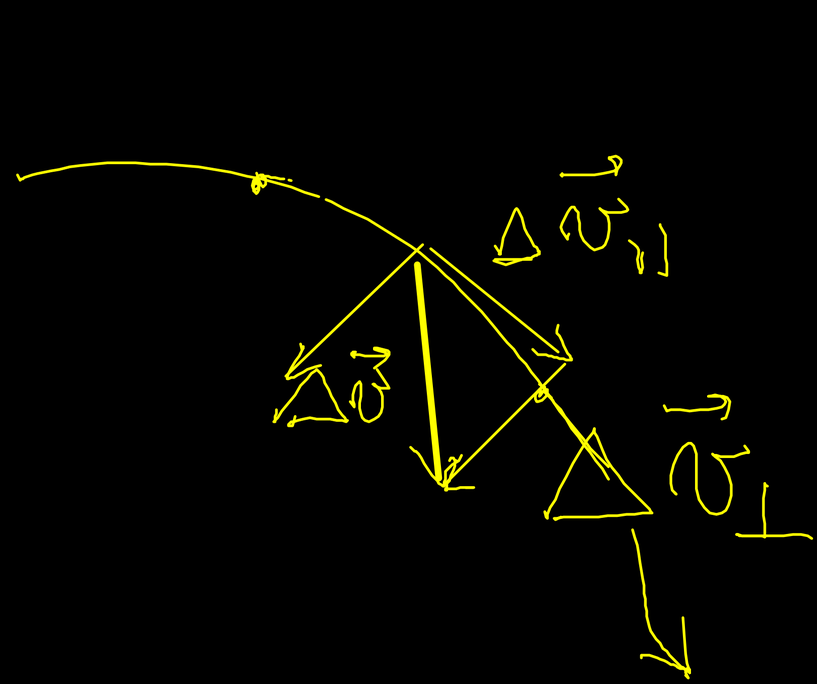
$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

$$= a_x \hat{i} + a_y \hat{j}$$

$$\begin{cases} a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2} \\ a_y = \dfrac{dv_y}{dt} = \dfrac{d^2y}{dt^2} \end{cases}$$

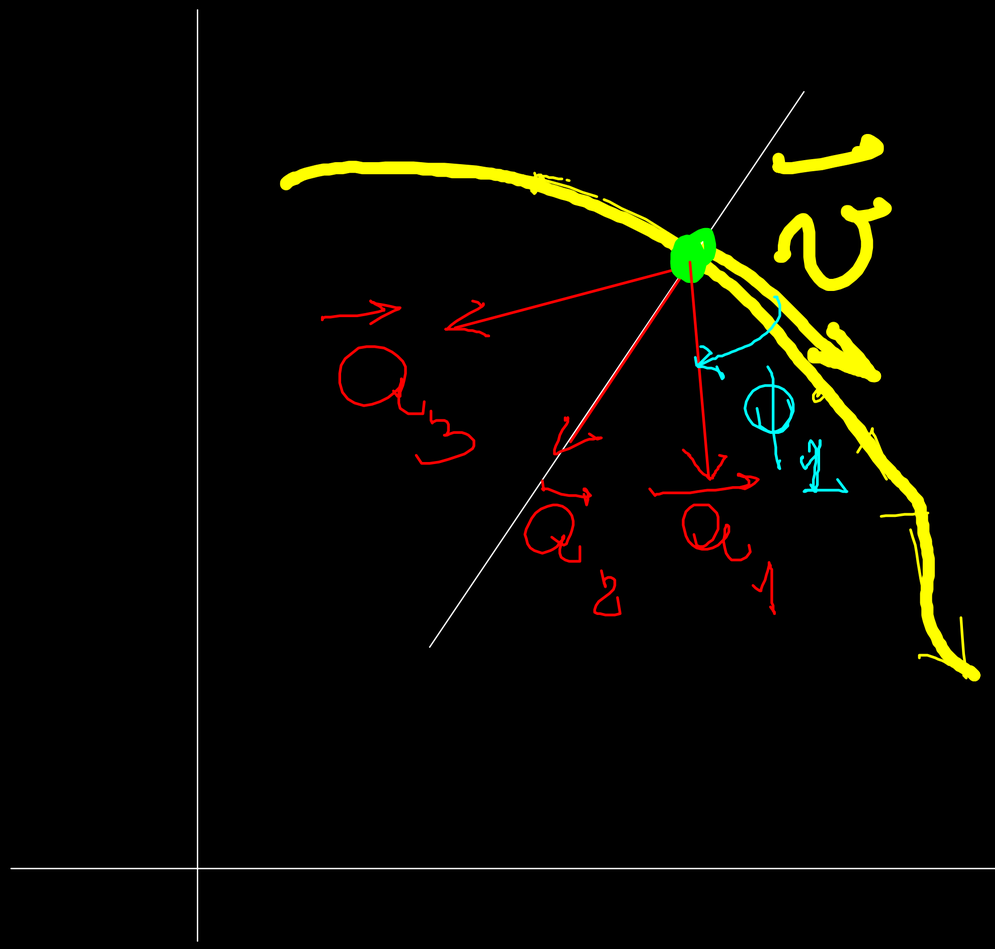


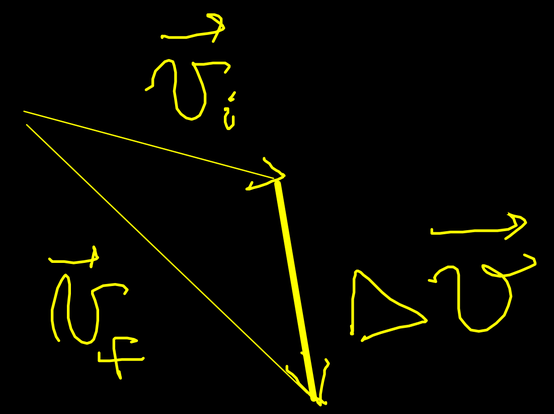
$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t}$$



$$\vec{a} = \lim_{\Delta t \to 0} \langle a \rangle = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta\vec{v} = \Delta\vec{v}_{\parallel} + \Delta\vec{v}_{\perp}$$





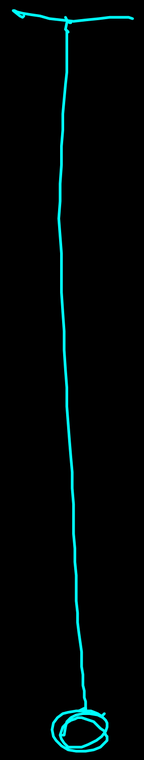
$\vec{a}_1$ modulo sta aumentando $0° < \phi_1 < 90°$

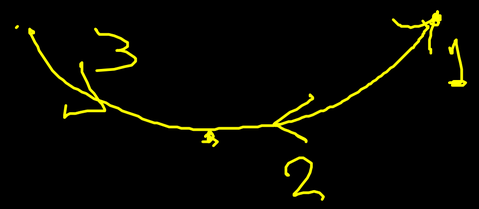
$\vec{a}_2$ modulo rel. costante $\phi_2 = 90°$

$\vec{a}_3$ " sta diminuendo $90° < \phi_3 < 180°$

$\Delta\vec{v} =$

$\Delta\vec{v}_{//} + \Delta\vec{v}_{\perp}$




3
1
2

Esempio 4.2

$$x = x(t) = \left(2.3\ \frac{m}{s^3}\right) t^3 - 12\ m \qquad t \text{ in } s$$

$$y = y(t) = \left(5.1\ \frac{m}{s}\right) t$$

$$v_x = \frac{dx(t)}{dt} = \frac{d}{dt}\left[\left(2.3\ \frac{m}{s^3}\right) t^3 - 12m\right]$$

$$= \left(2.3\ \frac{m}{s^3}\right) 3t^2 = \left(6.9\ \frac{m}{s^3}\right) t^2$$

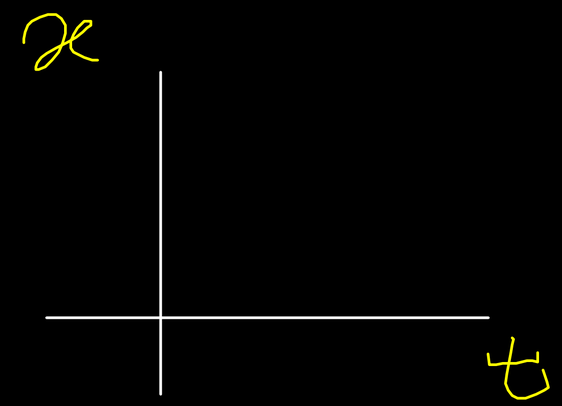
$$v_y = \frac{dy(t)}{dt} = 5.1\ \frac{m}{s}$$

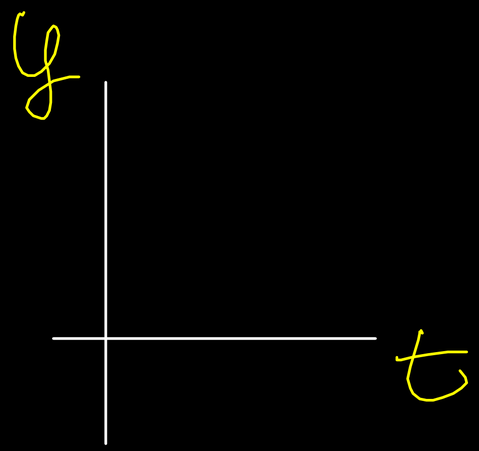
$$\vec{v} = \left[\left(6.9\,\frac{m}{s^3}\right)t^2\right]\hat{i} + \left(5.1\,\frac{m}{s}\right)\hat{j}$$

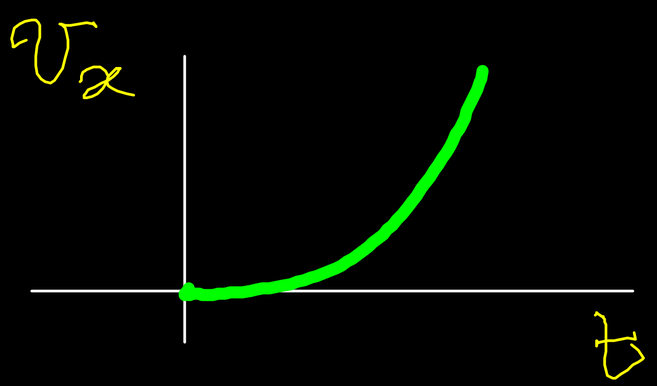
$$a_x = \frac{dv_x}{dt} = \left(13.8\,\frac{m}{s^3}\right)t$$

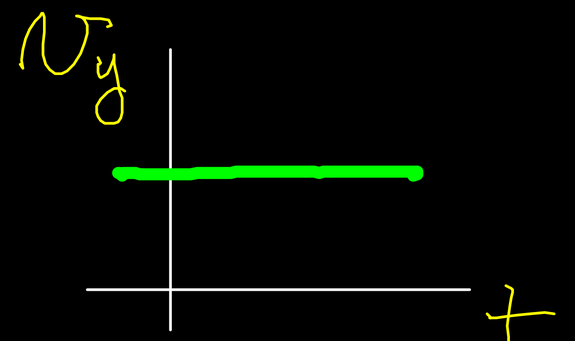
$$a_y = \frac{dv_y}{dt} = 0$$

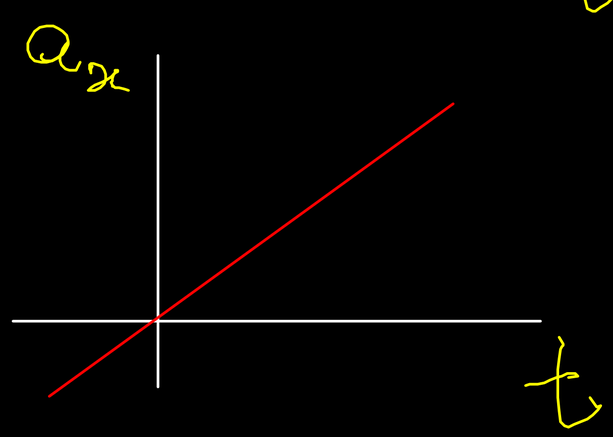
$$\vec{a} = \left[\left(13.8\,\frac{m}{s^3}\right)t\right]\hat{i}$$

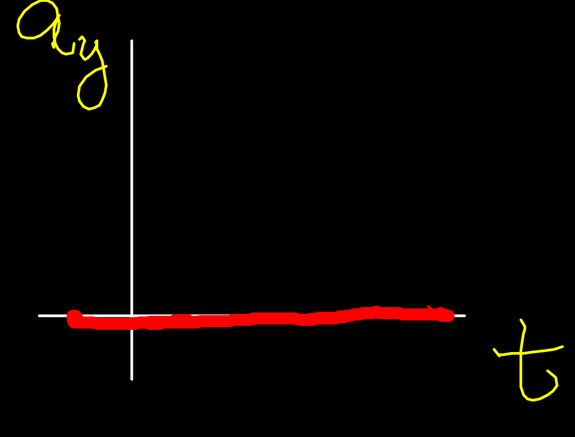

x
t


y
t


v_x
t


v_y
t


a_x
t


a_y
t

4.3 accel. cost. 2D

Ip. $\vec{a} = \text{cost}$ $\vec{v}_0$ $\vec{r}_0$

$t = 0$

Solo in questo caso $\boxed{\vec{a} = \langle \vec{a} \rangle}$

$\vec{v}_f = \vec{v}$ $t_f = t$

$\vec{v}_i = v_0$ $t_i = 0$

$$= \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \boxed{\frac{\vec{v} - \vec{v}_0}{t}}$$

$$\boxed{\vec{v}(t) = \vec{v}_0 + \vec{a}\,t}$$

$$\vec{v} = \left(v_{x0}\,\hat{i} + v_{y0}\,\hat{j}\right) + \left(a_x\,\hat{i} + a_y\,\hat{j}\right) t$$

$$= \underbrace{\left(v_{x0} + a_x t\right)}_{v_x(t)}\,\hat{i} + \underbrace{\left(v_{y0} + a_y t\right)}_{v_y(t)}\,\hat{j}$$

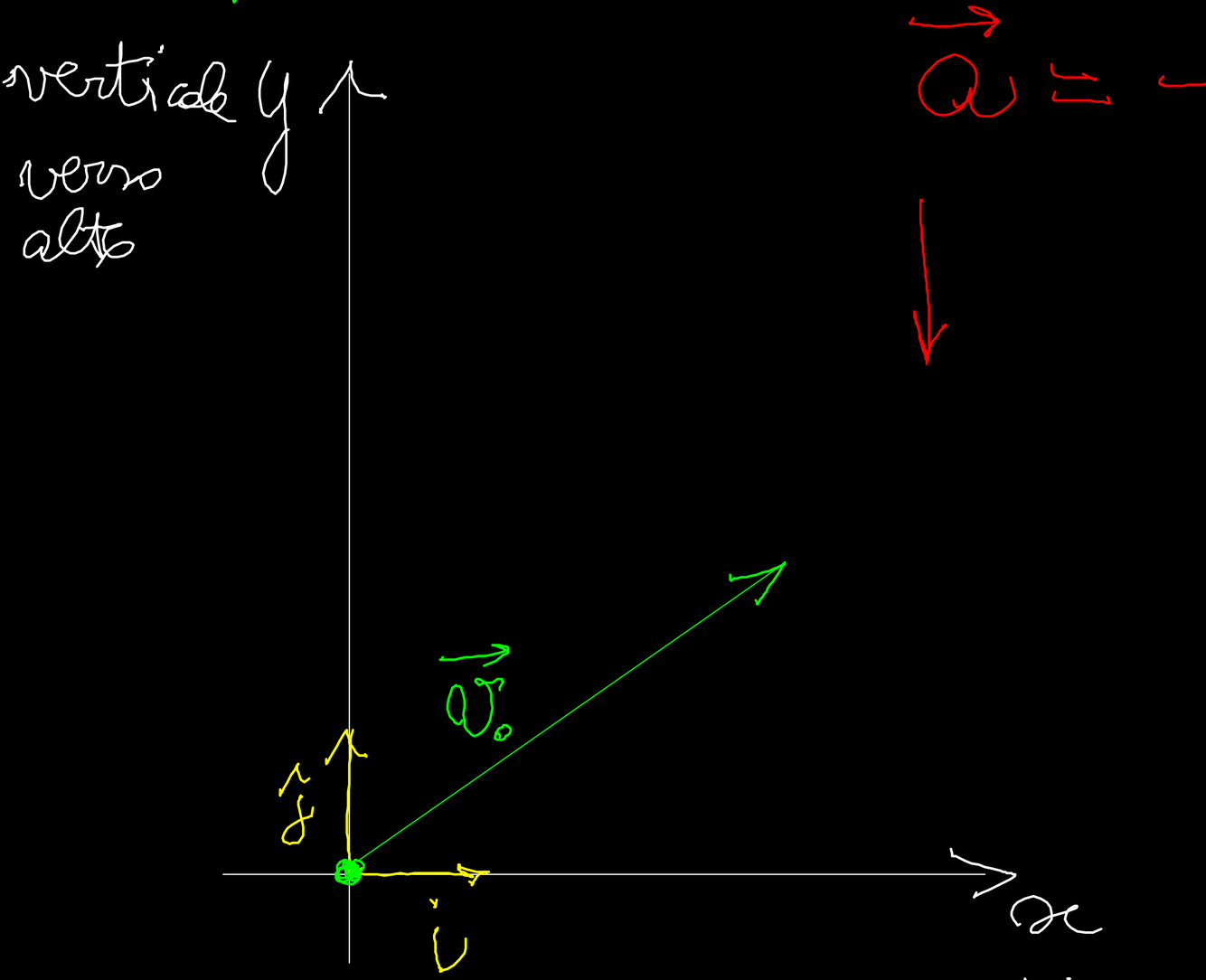
$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r}(t=0) = \vec{r}_0$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

Moto dei proiettili

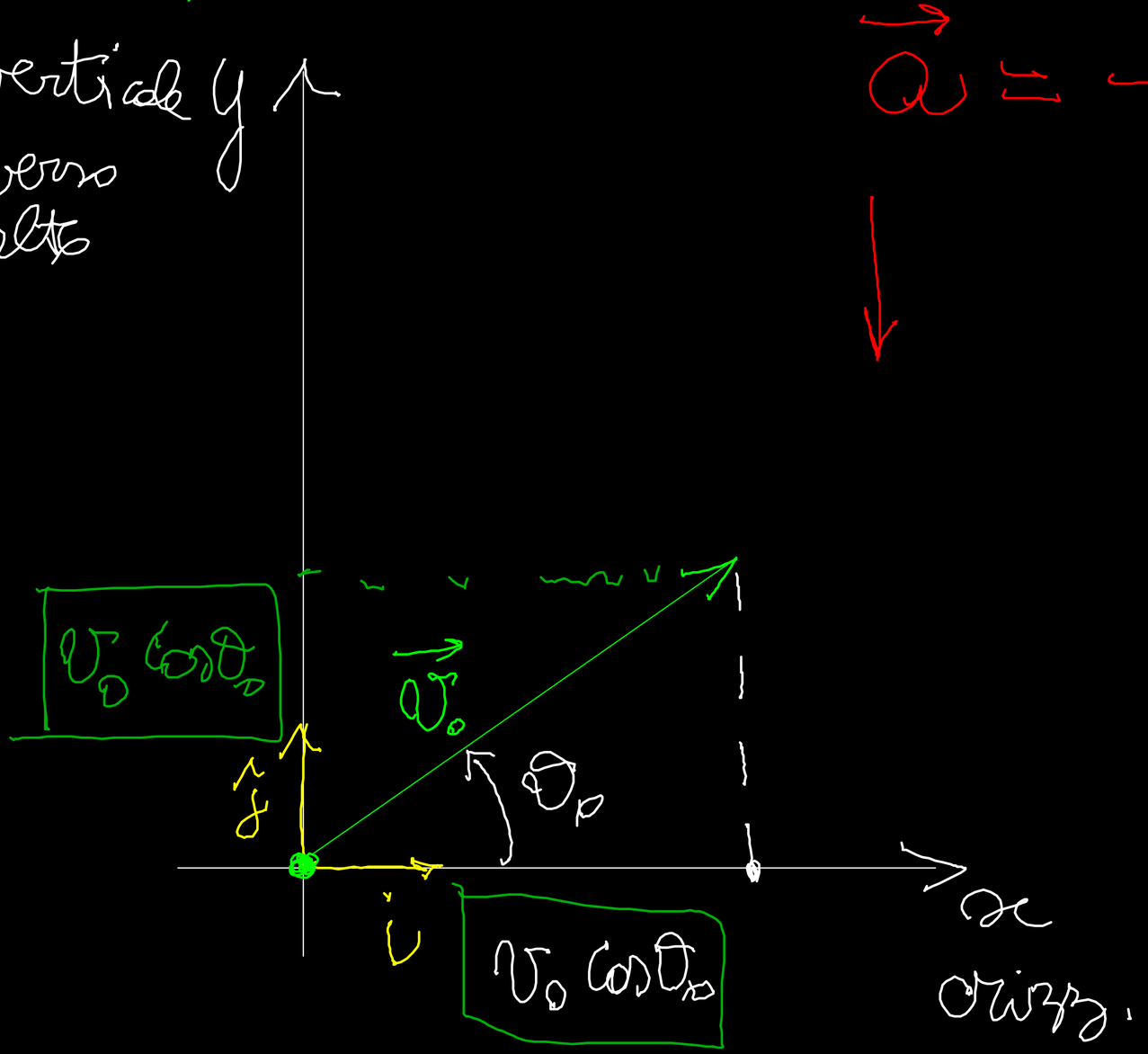


$$\vec{a} = -g\,\hat{j} \qquad g = 9.8\ \frac{m}{s^2}$$

$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} v_x = v_{x0} = \\ v_y = v_{y0} - gt \end{cases}$$

Moto dei proiettili



$$\vec{a} = -g\,\hat{j} \qquad g = 9.8\ \frac{m}{s^2}$$

$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} v_x = v_{x0} = \\ v_y = v_{y0} - gt \end{cases}$$

$$\begin{cases} v_x = v_0 \cos\theta_0 \\ v_y = v_0 \operatorname{sen}\theta_0 - gt \end{cases}$$

$$\begin{cases} x = (v_0 \cos\theta_0)\, t \\ y = (v_0 \operatorname{sen}\theta_0)\, t - \frac{1}{2} g t^2 \end{cases}$$